NLO Yukawa and self-coupling corrections to $gg \to HH$

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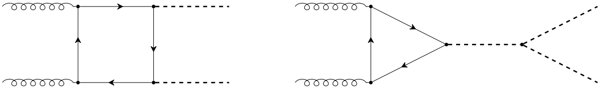
G. Heinrich, S. Jones, M. Kerner, T. Stone, A. Vestner | March 7th, 2024

ITP - KIT, IPPP

Why calculate higher orders to $gg \to HH$

- Sensitivity to Higgs selfcoupling λ

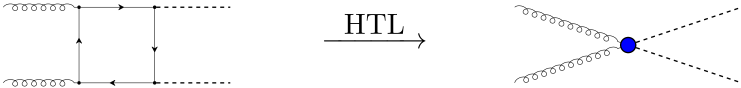


- Match expected experimental uncertainty at HL-LHC, corrections impact the extracted constraints
- Sizeable effects on differential cross sections expected
- Les Houches Wishlist $\geq$ 2015

Wishlist	known dσ	desired dσ
2015	N^2LO_{HTL}, NLO_{QCD}	N^2LO_{HTL}+NLO_{QCD}+NLO_{EW}
2021	$N^3LO_{\mathrm{HTL}} \otimes NLO_{\mathrm{QCD}}$	NLO_{EW}

A rudimentary history

- LO is already at loop level ⇒ Challenging calculation for NLO
- LO was already calculated 1988 (Glover and van der Bij 1988)
- First full m_t dependent NLO QCD result from 2016 (Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, et al. 2016)
- First full NLO EW result from 2023 (Bi et al. 2023)
- Simplification via expansions or heavy top limit is possible



- On the way to higher orders numerous combinations of these techniques are used, e.g. (Baglio et al. 2019), (Grazzini et al. 2018), (Mühlleitner et al. 2022), (Davies, Schönwald, et al. 2023), (Bagnaschi et al. 2023)

Our higher order calculation toolchain

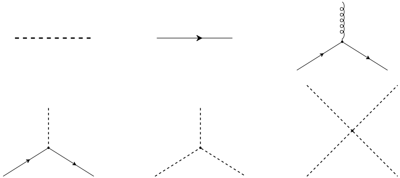
1. Produce contributing diagrams (QGRAF)
2. Project onto form factors (Mathematica)
3. Reduce the number of integrals (Reduze, Ratracer)
4. Integrate the remaining master integrals (pySecDec)
5. Perform the Renormalization (blood, sweat and tears)
6. Put everything back together

The Lagrangian

- Gaugeless limit ⇒ Weak bosons decouple
- Unitary gauge ⇒ Goldstone bosons decouple

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu H\right)^\dagger\left(\partial^\mu H\right) + \frac{\mu^2}{2}(v+H)^2 - \frac{\lambda}{16}(v+H)^4 + i\bar{t}\not{D}t - y_t\frac{v+H}{\sqrt{2}}\bar{t}t$$

Yields Feynman rules for:

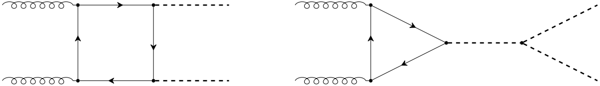


Reparametrize in terms of m_H, m_t and v.

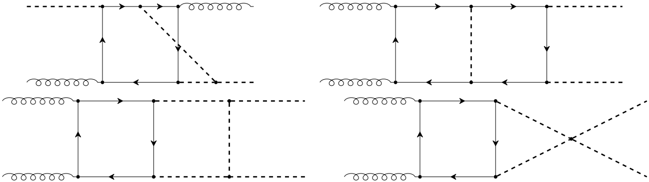
Contributing Diagrams

Karlsruhe Institute of Technology

LO



NLO (examples)



Automated by the tool `QGRAF`. (Nogueira 1993)

Formfactors

Separate the matrix element into tensor structures and Form Factors

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

Form factors can be obtained by using projectors

$$\mathcal{P}_i^{\mu\nu} T_{j,\mu\nu} = \delta_{ij}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_1^\nu p_2^\mu}{p_T^2 p_1 \cdot p_2} - \frac{2p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2p_2 \cdot p_3 p_1^\nu p_3^\mu}{p_T^2 p_1 \cdot p_2} + \frac{2p_3^\mu p_3^\nu}{p_T^2}$$

with

$$p_T = \sqrt{\frac{ut - m_H^4}{s}}$$

IBP Reduction

The expression for the matrix element contains unnecessary many integrals.

- Use integration by parts to relate different integrals to each other:

$$\int \prod_{\ell=1}^{L} \mathrm{d}^D k_\ell \frac{\mathrm{d}}{\mathrm{d}k_i^\mu} \left[\eta^\mu \mathcal{I}(\vec{\eta})\right] = 0$$

- Choose a suitable basis of master integrals M.I.
- Solve the resulting system of linear equations
- Have obtained a fully symbolic reduction to M.I.s retaining dependence on s, t, m_t and m_H using `ratracer` (Vitaly Magerya 2022)
- Faster reduction: fix as many open parameters as possible, e.g.

$$\frac{m_H^2}{m_t^2} = \frac{12}{23} \, .$$

This is calculated with `reduze`. (Manteuffel and Studerus 2012)

The Master Integrals

- The total number of remaining master integrals is 492
- d-factorizing integrals, i.e. parts depending on dimensionality d are separated from parts containing the kinematic dependence
- Still, too many mass scales to solve analytically
- Numerical evaluation using `pySecDec` is feasible (Heinrich, Jones, Kerner, V. Magerya, et al. 2024)

Tadpole Renormalization

- At higher orders the vev gets shifted.
- Fleischer-Jegerlehner tadpole scheme: (Fleischer and Jegerlehner 1981)

$$H + v \to H + v + \Delta v$$

- Require the tadpole diagrams to vanish also at NLO through the tadpole counterterm

$$\delta T = -T_H$$

- Identify $\delta T = -\Delta v m_H^2$
- This corresponds to a redistribution of tadpole contributions.

Counterterms

The NLO matrix element contains UV poles, which need to be cancelled by counterterms added to the LO matrix element

$$\mathcal{M}_{\mathrm{NLO}} = \frac{\mathcal{M}_{\mathrm{NLO,div}}}{\varepsilon} + \mathcal{M}_{\mathrm{NLO,fin}}$$

$$\mathcal{M}_{\mathrm{CT}} = \frac{\mathcal{M}_{\mathrm{CT,div}}}{\varepsilon} + \mathcal{M}_{\mathrm{CT,fin}}$$

They lead to additional Feynman rules:

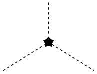
$$H_0 = \sqrt{Z_H}H = \sqrt{1+\delta_H}H$$

$$t_0 = \sqrt{Z_t}t = \sqrt{1+\delta_t}t$$

$$m_{H,0}^2 = m_H^2(1+\delta m_H^2)$$

$$m_{t,0} = m_t(1+\delta m_t)$$

$$v_0 + \Delta v = v(1+\delta_v) + \Delta v$$



$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$

etc.

Values are fixed through the on-shell renormalization scheme.

Conclusion

Where we are:

- The computationally challenging fully symbolic reduction is done
- Code for 2-loop evaluation is ready
- Renormalization is still under construction

Where to go:

- Include the full EW corrections and cross-check the result
- Investigate the effects of the bottom quark
- Implement an EFT framework to achieve variations on the couplings

Deriving the Tensor structure

General structure:

$$\mathcal{M}^{\mu\nu} = \quad a_{00} g^{\mu\nu} + a_{21} p_2^\mu p_1^\nu + a_{31} p_3^\mu p_1^\nu + a_{23} p_2^\mu p_3^\nu + a_{33} p_3^\mu p_3^\nu$$
$$+ a_{11} p_1^\mu p_1^\nu + a_{22} p_2^\mu p_2^\nu + a_{12} p_1^\mu p_2^\nu + a_{13} p_1^\mu p_3^\nu + a_{32} p_3^\mu p_2^\nu$$

Further constraints from Ward identities:

$$\epsilon_{1,\mu} p_1^\mu = 0 \qquad \epsilon_{2,\nu} p_2^\nu = 0$$

Basic example of Sector Decomposition

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1}$$

Diverging for $x \to 0$ and $y \to 0$

$$\mathfrak{I} = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y\right)^{-1} \left[\Theta(x-y) + \Theta(y-x)\right]$$

Variable transformation $y = xt$ and $x = yt$

$$\mathfrak{I} = \int_0^1 \frac{\mathrm{d}x}{x^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{b\epsilon}\left(1 + (1-x)t\right)} + \int_0^1 \frac{\mathrm{d}x}{y^{1+(a+b)\epsilon}} \int_0^1 \frac{\mathrm{d}t}{t^{1+a\epsilon}\left(1 + (1-y)t\right)}$$

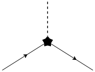
Both limits $x \to 0$ and $y \to 0$ are independent

On-Shell Renormalization

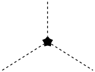
$$0 = \Big[\Sigma_i(\hat{p})\Big]_{\not{p}=m_i} \qquad\qquad 0 = \Big[\frac{\mathrm{d}}{\mathrm{d}\not{p}}\Sigma_i(\not{p})\Big]_{\not{p}=m_i}$$

$$= -i\left[(m_t - \not{p})\delta_t + m_t\delta m_t - \frac{m_t}{v m_H^2}\delta T\right]$$

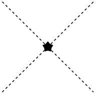
$$= -i\left[(m_H^2 - p^2)\delta_H + m_H^2\delta m_H^2 - 3\frac{\delta T}{v}\right]$$



$$= -i\frac{m_t}{v}\left(\delta m_t + \frac{\delta_H}{2} + \delta_t - \delta_v\right)$$



$$= -i3\frac{m_H^2}{v}\left(\delta m_H^2 + \frac{3}{2}\delta_H - \delta_v - \frac{\delta T}{v m_H^2}\right)$$



$$= -i3\frac{m_H^2}{v^2}(\delta m_H^2 + 2\delta_H - 2\delta_v)$$

Further references for previous work on $gg \to HH$

(Still not exhaustive)

- LO: (Glover and van der Bij 1988)
- QCD:
  - NLO: (Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, et al. 2016), (Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, and Zirke 2016), (Baglio et al. 2019), (Davies, Heinrich, et al. 2019), (Heinrich, Jones, Kerner, Luisoni, et al. 2019), (Bagnaschi et al. 2023)
  - NNLO: (Florian et al. 2016), (Grazzini et al. 2018)
  - N^3LO: (Chen et al. 2020), (Ajjath and Shao 2023)
- EW:
  - NLO: (Mühlleitner et al. 2022), (Davies, Schönwald, et al. 2023), (Bi et al. 2023)

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