Karlsruhe Institute of Technology

AVICENNA STUDIENWERK

RelExt: A New Tool for Searching Dark Matter Parameter Spaces

04.04.2025

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Outline

- Motivation
  - Dark Matter Evidences
  - Relic Density
- The Code
- Example: CP in the Dark
- Conclusions and Outlook

Motivation

Evidences for Dark Matter

- Observed discrepancies in stellar rotational curves
  [Rubin et al., 1980]
- Gravitational lensing effects in the Bullet Cluster
  [Clowe et al., 2006]
- Temperature fluctuations in the Cosmic Microwave Background (CMB) [PLANCK, 2018]

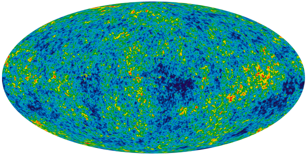

Figure: Taken from [WMAP, 2012].

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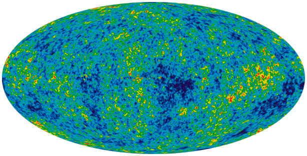

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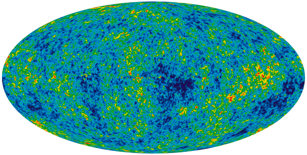

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Dark Matter Relic Density

Observed DM Relic Density

From the experimental data of the PLANCK experiment we obtain the DM relic density

$$\Omega_{\text{exp}} h^2 = 0.1200 \pm 0.0012$$

with $h^2 = \frac{H^2}{(100 \frac{km}{sMpc})^2}$.

- Several tools exist for relic density calculations, e.g.:
  - MicrOMEGAs [Belanger et al., 2002], [Belanger et al., 2002], [Belanger et al., 2002] , DarkSUSY [Bringmann et al., 2018] , MadDM [Backovic et al., 2014],[Backovic et al., 2015],[Ambrogi et al., 2019] , SuperIso Relic [Arby et al., 2019]

RelExt

RelExt

- **Efficiently scans** parameter spaces to identify compatible relic density values.
- Computes relic density for freeze-out (co-)annihilation processes.
- Automatically generates:
  - (Co-)annihilation amplitudes
  - Thermally averaged cross sections
  - Total decay widths for s-channel mediators
- Supports multiple pre-installed and user-defined models with a discrete **$\mathbf{Z}_2$ symmetry** (requires FeynRules model files).
- Compatible with other tools:
  - **ScannerS** [Azevedo et al., 2018] : checks all relevant theoretical and experimental constraints
  - **BSMPTv3** [Basler et al., 2019],[Basler et al., 2021],[Basler et al., 2024]: phase history and possible gravitational wave signals

Scan Methods

- Monte Carlo Grid Search (MCGS)
  - \- Majority of points of the initial run do not fulfill relic density constraint
  - \+ Fast generation of points
  - \+ Gets better for each run
- Single Parameter Search (SPS)
  - \- Adjustment of only one parameter of the model
  - \+ Fast generation of points which fulfill the relic density constraints
  - \+ Easy way to investigate the impact of parameters on the relic density
- Multi Parameter Search (MPS)
  - \- Slow computation
  - \+ Multiple parameters of the models are simultaneously adjusted

Example

CP in the Dark (CPVDM) [Azevedo et al., 2018]

- Scalar sector: two complex scalar doublets Φ_1, Φ_2 and one real scalar singlet Φ_S
- Imposing a discrete $\mathbb{Z}_2$ symmetry to the Lagrangian

$$\Phi_1 \to \Phi_1\,, \quad \Phi_2 \to -\Phi_2\,, \quad \Phi_s \to -\Phi_s$$

- The scalar potential is given by

$$\begin{aligned} V_{\text{Scalar}} = \; & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2\right)^2 \\ + \; & \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right] \\ + \; & \frac{1}{2} m_s^2 \Phi_s^2 + \frac{\lambda_6}{8} \Phi_s^4 + \frac{\lambda_7}{2} \Phi_1^\dagger \Phi_1 \Phi_s^2 + \frac{\lambda_8}{2} \Phi_2^\dagger \Phi_2 \Phi_s^2 \\ & + (A \Phi_1^\dagger \Phi_2 \Phi_s + \text{h.c.}), \quad \lambda_i \in \mathbb{R}\ (i \in [1,8]),\ A \in \mathbb{C} \end{aligned}$$

Scalar Fields of CPVDM

After electroweak symmetry breaking, the scalar fields are parametrized as

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\rho_1 + i\eta) \end{pmatrix}, \quad \Phi_s = \rho_s \ ,$$

in terms of the SM-like Higgs boson h, the CP-even and CP-odd fields, ρ_i ($i \in \{1, s\}$) and η, respectively, the charged and neutral Goldstone bosons G^+ and G^0, and the charged field H^+.

Scalar fields of CPVDM

Dark sector mass eigenstates $h_i (i = 1, 2, 3)$ are obtained by orthogonal rotation matrix R which is parametrized by the angles $\alpha_i (i = 1, 2, 3)$ such that

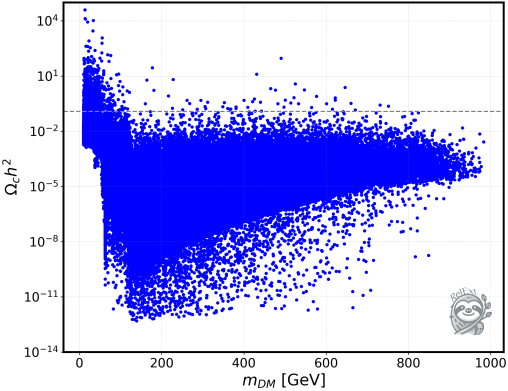
$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \rho_1 \\ \eta \\ \rho_s \end{pmatrix},$$

where $m_{h_1} \leq m_{h_2} \leq m_{h_3}$. The input parameter are then given by

$$m_{h_1}, \quad m_{h_2}, \quad m_{H^+}, \quad \alpha_1, \quad \alpha_2, \quad \alpha_3, \quad \lambda_2, \quad \lambda_6, \quad \lambda_8, \quad m_{22}, \quad m_s.$$

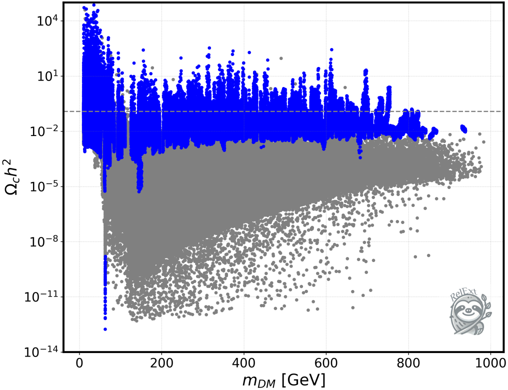
Monte Carlo Grid Search (MCGS) - Run 1

- Blue points: $2 \cdot 10^5$ Points obtained by MCGS
- Gray dashed line: observed relic density $\Omega_{\text{exp}} h^2 = 0.12$



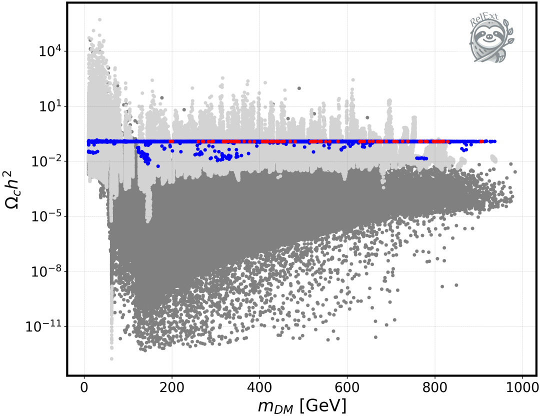
Monte Carlo Grid Search (MCGS) - Run 2

- Gray points: points from previous run
- Blue points: Second MCGS from the best cells of previous run



Multi Parameter Search (MPS)

- Blue points: MCGS and MPS
- Red Points: Points validated against all relevant constraints via **ScannerS**



Computation Time

- Fraction within 2σ relic density limit (middle), points within the limit per CPU time (right)
- Compared Methods: MCGS (random points), MCGS (best cells), MCGS(improved Grid) + MPS
- Computational setup: 4 cores on an AMD EPYC 7351 (1.2-2.4 GHz) and 1 GB RAM

Method	% within 2σ	[good points]/[CPU time]
MCGS (random points), $m_{\text{DM}} > m_h$	0.%	$\sim$0 s^{-1}
MCGS (random points), $m_{\text{DM}} < m_h$	0.027%	$\sim$0.0012 s^{-1}
MCGS (best cells), $m_{\text{DM}} > m_h$	0.29%	$\sim$0.0178 s^{-1}
MCGS (best cells), $m_{\text{DM}} < m_h$	3.66%	$\sim$0.135 s^{-1}
MCGS (best cells) + MPS, $m_{\text{DM}} > m_h$	97.5%	$\sim$0.065 s^{-1}
MCGS (best cells) + MPS, $m_{\text{DM}} < m_h$	97.3%	$\sim$0.085 s^{-1}

Download

Download and Installation

Download: `https://github.com/jplotnikov99/RelExt`

Requirements:

at least `CMake 3.22`

at least `GNU Make 4.3`

at least `C++ 11.4.0`

For own model implementation you need also:

at least `Mathematica 12.00`

`FeynRules` [Alloul et al. 2014]

`FeynCalc` [Mertig et al. 1991] / `FeynArts` [Hahn 2001]

Installation via the terminal in the `RelExt` directory:

1.: `mkdir build && cd build`

2.: `cmake .. && make`

Conclusions and Outlook

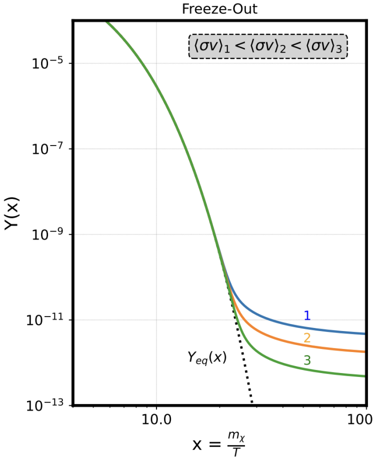
Conclusions and Outlook

- A new tool developed for efficient DM parameter searches
- Automated relic density computation for DM models featuring a discrete $\mathbb{Z}_2$ symmetry based on $2 \to 2$ (co-)annihilation processes
- Efficient scanning methods to search for parameter configurations that fulfill the DM relic density.
- Can be interfaced with tools like ScannerS and BSMPT
- Next steps: Further optimizations, inclusion of additional DM production mechanisms, next-to-leading order (NLO) calculations, computation of direct detection cross section

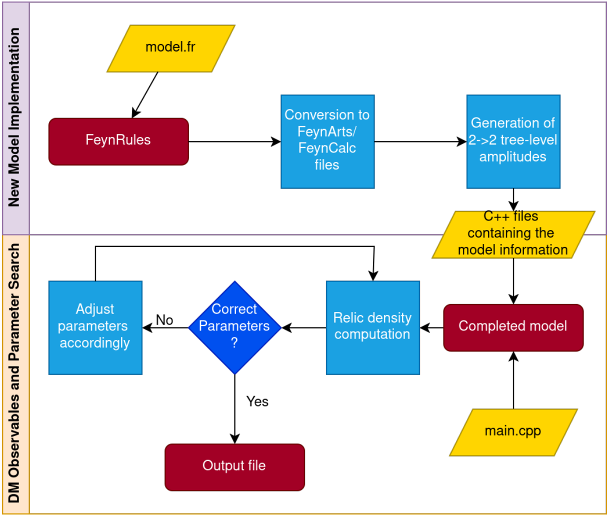
Thank you for listening!

Backup

Yield evolution in Freeze-Out

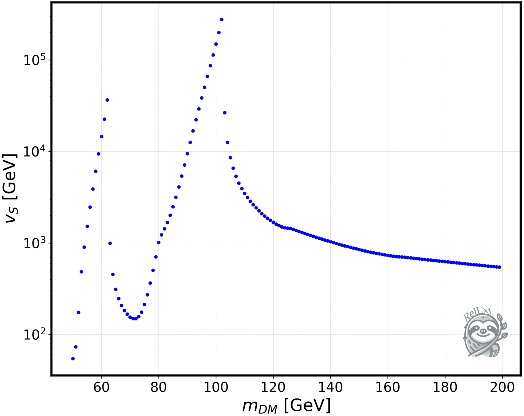


Flowchart of the Code

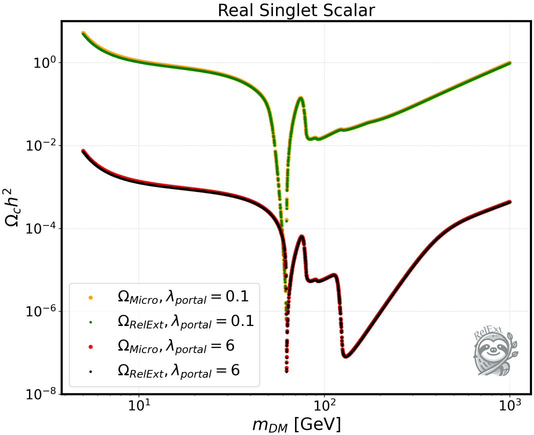


Single Parameter Search (SPS)

Singlet VEV v_S over the DM mass for the CxSM. All points generate the full measu relic density within the given uncertainty.



Comparison with MicrOMEGAs



Calculation of the Relic Density

Boltzmann-Equation

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -\sqrt{\frac{\pi}{45G}}\langle\sigma v\rangle\frac{g^{*}_{1/2}m_\chi}{x^2}(Y^2 - Y^2_{\text{eq}}), \quad x = \frac{m_\chi}{T}$$

Thermally averaged cross section

$$\langle\sigma v\rangle = \frac{2\pi^2 T\int_{2m_\chi^2}^{\infty}\mathrm{d}s\sqrt{s}(s-4m_\chi^2)K_1\left(\frac{\sqrt{s}}{T}\right)\sigma(s)}{\left(4\pi m_\chi^2 T K_2\left(\frac{m_\chi}{T}\right)\right)^2}$$

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Monte Carlo Grid Search

- A grid is placed over the parameter space with N_p parameters and N_b bins per parameter.
- **Weighting mechanism:**
  - Each computed relic density Ω_c is assigned a weight w based on its proximity to the target value Ω_d:

$$w = \begin{cases} \left(\frac{\Omega_d}{\Omega_c}\right)^2, & \text{if } \Omega_d < \Omega_c \\ \left(\frac{\Omega_c}{\Omega_d}\right)^2, & \text{if } \Omega_d > \Omega_c \end{cases}$$

- Keeps track of N_s best parameter points to improve search efficiency.
- Generates new points from top N_s cells with probability p_b, and random points with $p_r = 1 - p_b$.
- **Final output:** Stored best regions for future scans and the scanned parameters with the respective relic density.

Single Parameter Optimization

- Adjusts **one parameter** at a time to reach Ω_d.
- **Two possible scenarios:**
  - **Sign flip of $\Delta\Omega$** $\rightarrow$ uses **bisection method** to find the root.
  - **Local minimum detected** $\rightarrow$ switches to **gradient descent** to refine search.
- **If no solution within wanted uncertainty** ε, the method returns:
  - Last considered parameter and Ω_c, or
  - Minimum value if no solution found.

Multi Parameter Search

- Adjusts **multiple parameters simultaneously**.
- Each parameter x_i updated as:

$$x_i' = x_i(1 + \gamma_i)$$

  where γ_i is randomly chosen from $[-\gamma_{\text{max}}, \gamma_{\text{max}}]$
- If $\Delta\Omega$ increases, a new parameter set is tried.
- If $\Delta\Omega$ decreases, the update is applied iteratively until $\Delta\Omega$ increases again.
- **Stops when either:**
  - Ω_d is reached within uncertainty.
  - Maximum iteration limit is hit.