Karlsruhe Institute of Technology

(N)2HDM - Renormalization, Enhanced Higgs Decays and All That

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ITP Research Seminar

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Outline of the Talk

- Motivation
- Principle of Gauge Invariance
- Introduction to the (N)2HDM
  - Decoupling and Strong Coupling Regimes
- Automated One-Loop Calculations with (ewN)2HDECAY
- Renormalization of the (N)2HDM
  - Standard and Alternative Tadpole Scheme
  - Mixing Angle Renormalization
- Numerical Results

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- the Standard Model (SM) has theoretical shortcomings and does not provide explanations for all phenomena observed in nature
- consider the Two-Higgs-Doublet Model ("**2HDM**") and its **singlet extension**, *i.e.* the Next-to-Minimal-Two-Higgs-Doublet Model ("**N2HDM**")
- **motivations** for studying the 2HDM and N2HDM:
  - simple extensions of the SM
  - no constraints due to SUSY relations (cf. MSSM and NMSSM)
  - can provide a dark matter candidate (*"dark sectors" of the (N)2HDM*)
  - additional sources of CP violation (*complex (N)2HDM*)
  - **extended scalar sector**
    - interesting phenomenology
    - Higgs-to-Higgs (cascade) decays as interesting signatures

Motivation: Electroweak One-Loop Corrections (I)

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- **state-of-the-art** code for BRs of Higgs decays in the (N)2HDM: **HDECAY** and **N2HDECAY** (based on **HDECAY)**, containing:
  - off-shell decay modes for final-state massive vector bosons / heavy quarks
  - loop-induced decays into final-state gluon and photon pairs and $Z\gamma$
  - state-of-the-art QCD corrections, where applicable

[HDECAY: A. Djouadi, J. Kalinowski, and M. Spira, *Comput. Phys. Commun.* **108** (1998) 56-74;
A. Djouadi, J. Kalinowski, M. Mühlleitner, and M. Spira, *Comput. Phys. Commun.* **238** (2019) 214-231]

[N2HDECAY: M. Mühlleitner, M. O. P. Sampaio, R. Santos, J. Wittbrodt, *JHEP* **03** (2017) 094;
I. Engeln, M. Mühlleitner, and J. Wittbrodt, *Comput. Phys. Commun.* **234** (2019) 256-262]

Motivation: Electroweak One-Loop Corrections (II)

- **electroweak** corrections at one-loop **still missing**
- previous analysis has shown: they can be of **relevant size**

[MK, *Master's thesis* (2016), KIT; MK, R. Lorenz, M. M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
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- renormalization of **scalar mixing angles** is non-trivial: existing schemes are numerically unstable, process-dependent or **gauge-dependent**

  ➡ search for a suitable renormalization scheme of the mixing angles

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- **investigation** of the electroweak one-loop corrections:
  - size and relevance of the electroweak corrections
  - renormalization scheme dependence of the electroweak corrections

    ➡ **estimate of theoretical uncertainty** due to missing higher orders
  - size of the electroweak corrections relative to the decay width at tree level

    ➡ "**numerical stability**" of renormalization schemes

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  ➡ equations of motions, observables, ... **must not depend** on ξ

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- higher-order calculations: cancelation of gauge dependences becomes very **intricate**

Cancelation of Gauge Dependences (I)

- ξ encodes **redundant** (unphysical) degrees of freedom
  - ➡ observables, decay amplitudes, etc. **must not depend** on ξ
  - ➡ cancelation is ensured by BRST symmetry

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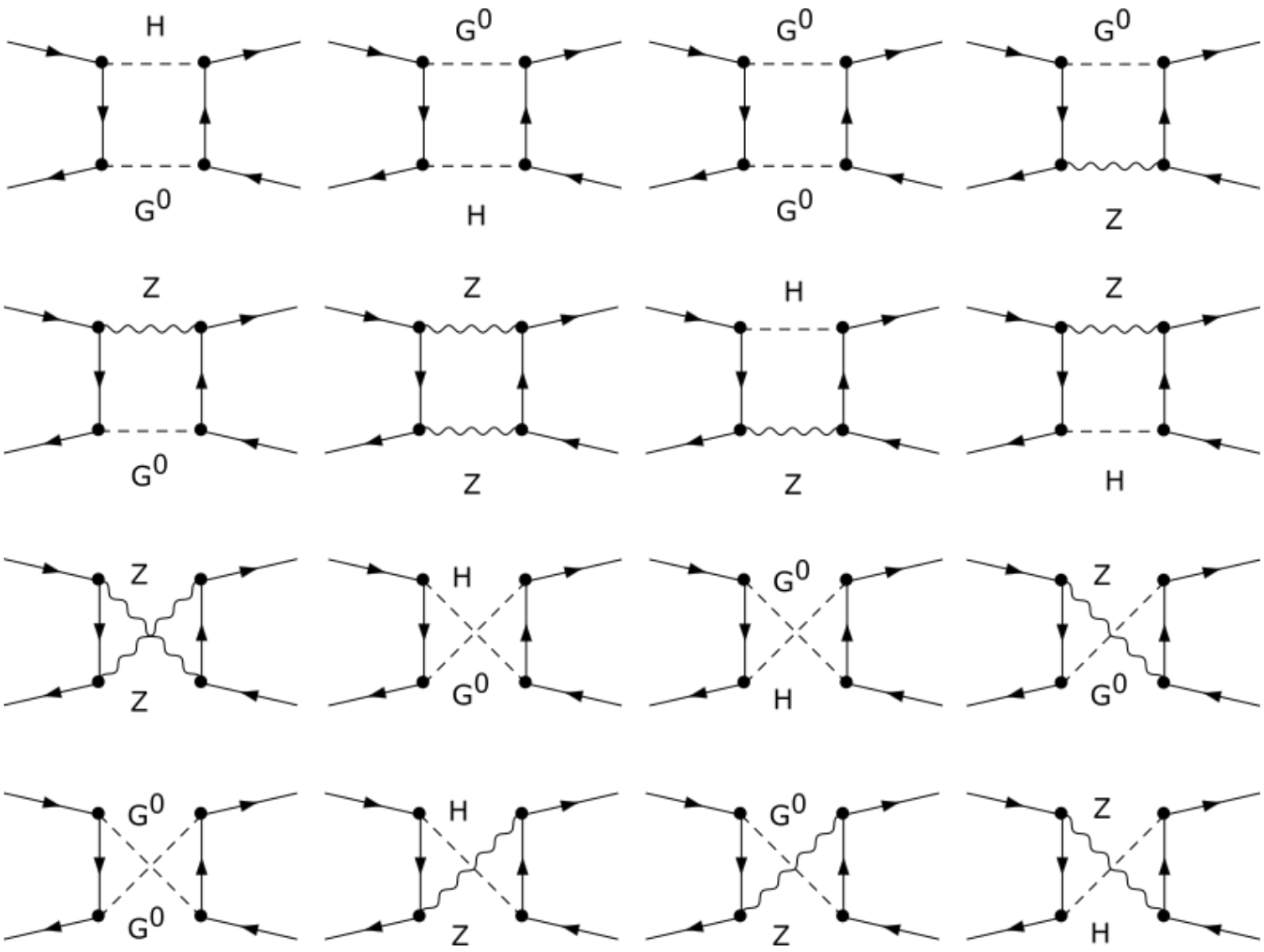
- for LO OS processes, cancelation of ξ dependences is straightforward
- at higher orders, the cancelation becomes **very intricate**

Cancelation of Gauge Dependences (I)



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Cancelation of Gauge Dependences (II)

- **sources** of **gauge dependences** at higher orders:
  - genuine **loop corrections**
  - external leg corrections (**wave-function renormalization constants**)
  - explicit **tadpole** contributions (**proper** treatment of the vacuum state)
  - **counterterms** of all independent parameters

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- **with** a **proper** vacuum treatment: all gauge dependences stemming from genuine loop corrections **fully cancel** against external leg corrections

[P. A. Grassi, P. Gambino, *Phys.Rev.* **D62** (2000) 076002]

➡ counterterms are necessarily **gauge-independent**

➡ **simplification** of the book-keeping of gauge dependences in a higher-order calculation

Cancelation of Gauge Dependences (III)

- **without** a proper vacuum treatment: gauge dependences need to be consistently **included** in counterterms to ensure an overall cancelation

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- possible **violation** of the cancelation of gauge dependences: renormalization conditions for **mixing matrices** and **mixing angles**
  - SM: CKM matrix ➡ solved

    [P. Gambino, P.A. Grassi, F. Madricardo, *Phys.Lett.* B**454** (1999) 98-104;
    A. Barroso, L. Brucher, R. Santos, *Phys.Rev.* D**62** (2000) 096003;
    B.A. Kniehl, F. Madricardo, M. Steinhauser, *Phys.Rev.* D**62** (2000) 073010;
    Y. Yamada, *Phys.Rev.* D**64** (2001) 036008;
    A. Denner, E. Kraus, M. Roth, *Phys.Rev.* D**70** (2004) 033002]

  - (N)2HDM: **scalar mixing angles** ➡ **?**

Introduction to the N2HDM: Potential

- **two** complex $SU(2)_L$ Higgs **doublets** and one real gauge **singlet**:

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix} , \qquad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix} , \qquad \Phi_s = v_s + \rho_s$$

- non-vanishing **vacuum expectation values** (VEVs) v_1, v_2, v_s with

$$v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$

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- scalar Lagrangian with **CP- and** $\mathbb{Z}_2$**-conserving** N2HDM potential:

$$\begin{aligned} V_{\mathrm{N2HDM}} = {} & m_{11}^2 \left(\Phi_1^\dagger \Phi_1\right) + m_{22}^2 \left(\Phi_2^\dagger \Phi_2\right) - m_{12}^2 \left[\left(\Phi_1^\dagger \Phi_2\right) + \left(\Phi_2^\dagger \Phi_1\right)\right] \\ & + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) \\ & + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right] \\ & + \frac{1}{2} m_s^2 \Phi_s^2 + \frac{1}{8}\lambda_6 \Phi_s^4 + \frac{1}{2}\lambda_7 \left(\Phi_1^\dagger \Phi_1\right)\Phi_s^2 + \frac{1}{2}\lambda_8 \left(\Phi_2^\dagger \Phi_2\right)\Phi_s^2 \end{aligned}$$

Introduction to the N2HDM: Parameters

- **twelve** real-valued potential parameters:
  - dimensionless $\lambda_i \ (i = 1, ..., 8)$
  - squared mass parameters $m_{11}^2,\ m_{22}^2, m_s^2$ and m_{12}^2

- difference w.r.t. NMSSM: constants **not fixed through SUSY relations**

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  - $\alpha_1, \alpha_2, \alpha_3$ for the CP-even sector
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  ➡ physical Higgs bosons and Goldstones: $(\underbrace{H_1, H_2, H_3}_{\text{CP-even}}, \underbrace{G^0, A}_{\text{CP-odd}}, \underbrace{G^\pm, H^\pm}_{\text{charged}})$

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$$m^2_{\phi_{\text{heavy}}} \approx c^2_{\phi_{\text{heavy}}} \frac{m^2_{12}}{\sin\beta\cos\beta} + f(\lambda_i)v^2$$

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where $f(\lambda_i)$ is a linear combination of the λ_i and

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- two interesting limits in case that $m^2_{\phi_{\text{heavy}}}$ becomes **large**:
  - **decoupling**: $\frac{m^2_{12}}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2$ **for all** heavier Higgs bosons
    - $m^2_{\phi_{\text{heavy}}}$ dominated by large $m^2_{12}/\sin\beta\cos\beta$, independent of the λ_i
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  - **strong coupling**: $\frac{m^2_{12}}{\sin\beta\cos\beta} \lesssim f(\lambda_i)v^2$ for **at least one** heavier Higgs boson
    - large λ_i required for large $m^2_{\phi_{\text{heavy}}}$

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- **trilinear** and **quartic** Higgs couplings can **become small**

- **decoupling theorem**: loop effects due to heavy Higgs bosons vanish in the limit $m_{\phi_{\text{heavy}}} \to \infty$ [T. Appelquist, J. Carazzone, *Phys. Rev. D* **11** (1975) 2856]

- reflects the **decoupling limit in the MSSM** where the Higgs couplings are given in terms of gauge couplings g and g' due to SUSY relations

Wrong-Sign Limit of the 2HDM

- even with large $\frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2$, decoupling is **not always guaranteed**

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- in the wrong-sign regime of the 2HDM, decoupling is strongly **disfavored** and **strong coupling easily arises**

[P. M. Ferreira, J. F. Gunion, H. E. Haber, R. Santos, *Phys. Rev. D* **89** (2014) 115003]

➡ for the analyses, **distinguish wrong-sign** and **correct-sign** regimes **within** the "decoupling limit"

Decoupling and Correct-/Wrong-Sign Limit

- consider *e.g.* the ratio λ_{HHh}/m_H^2 appearing in the **NLO corrections**
- apply the **SM limit** $\sin(\beta-\alpha)\to 1$ and the **decoupling limit**

$$m_H^2 \approx \frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2 \quad \text{and} \quad m_H^2 \gg m_h^2$$

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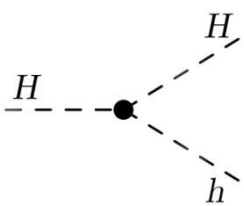
- in these limits, we find:



$$\frac{\lambda_{HHh}}{m_H^2} = -\frac{1}{m_H^2 v}\frac{\sin(\beta-\alpha)}{\sin(2\beta)}\left[\sin(2\alpha)\left(2m_H^2+m_h^2\right) - \frac{m_{12}^2}{\sin\beta\cos\beta}\left(3\sin(2\alpha)+\sin(2\beta)\right)\right]$$

$$\begin{cases} \approx 0 & (\text{correct-sign limit},\ \sin(\beta-\alpha)\to 1) \\ \approx \frac{2}{v} & (\text{wrong-sign limit},\ \sin(\beta-\alpha)\to 1,\ \sin(\alpha+\beta)\to 1,\ \tan\beta \gg 1) \end{cases}$$

[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

➡ **decoupling** in the **correct-sign** regime (decoupling theorem)

➡ **no decoupling** in the **wrong-sign** regime (**non-decoupling** effects)

Strong Coupling Limit of the 2HDM

- **strong coupling**: $\frac{m_{12}^2}{\sin\beta\cos\beta} \lesssim f(\lambda_i)v^2$ **for at least one** $\phi_{\text{heavy}} \in \{H, A, H^\pm\}$
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  - large λ_i required for large $m^2_{\phi_{\text{heavy}}}$
- **trilinear** and **quartic** Higgs couplings **become large**
- decoupling theorem **does not apply**: loop effects due to heavy Higgs bosons do not vanish in the limit $m_{\phi_{\text{heavy}}} \to \infty$
- **reason**: radiative corrections due to heavy Higgs bosons develop a **power-law**-like behavior in $m_{\phi_{\text{heavy}}}$

➡ **large NLO corrections** due to **non-decoupling effects**

➡ for $H \to h\,h$: corrections grow with $m^4_{\phi_{\text{heavy}}}$

[S. Kanemura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Rev. D* **70** (2004) 115002;
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- **aim:** calculate all (N)2HDM Higgs decays **at one-loop (electroweak)**

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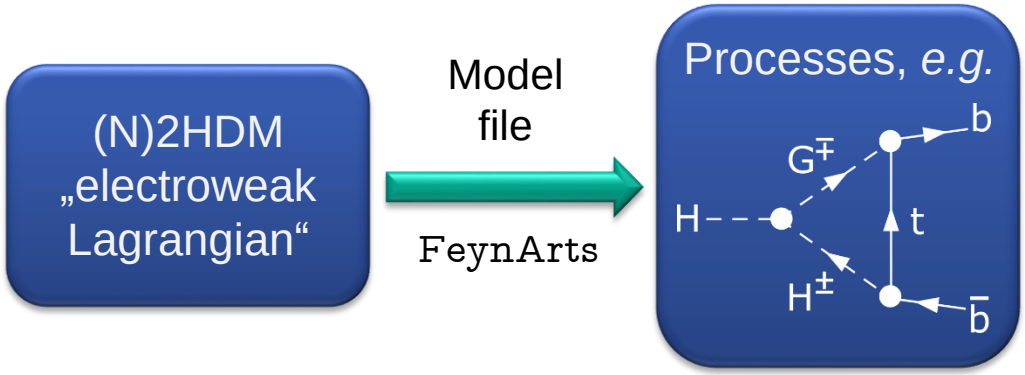

[FeynArts: T. Hahn, *Comput. Phys. Commun.* **140** (2001) 418;
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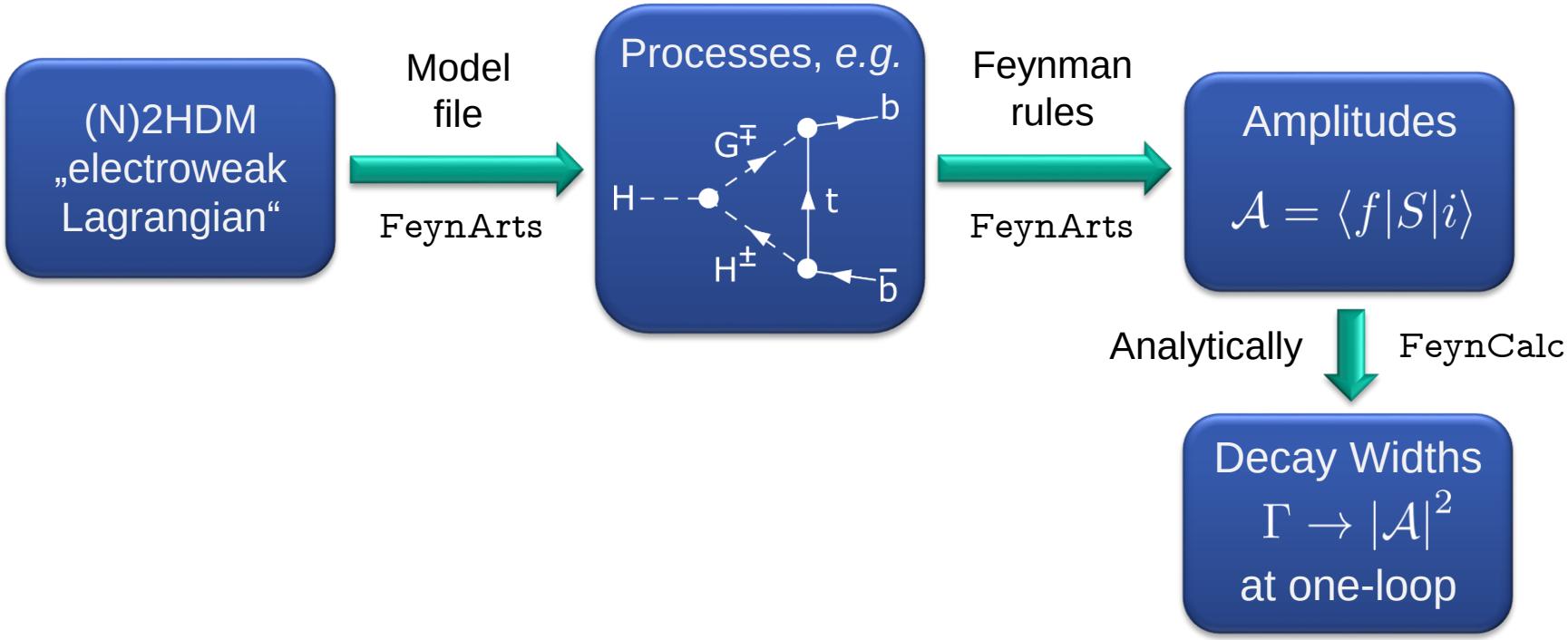

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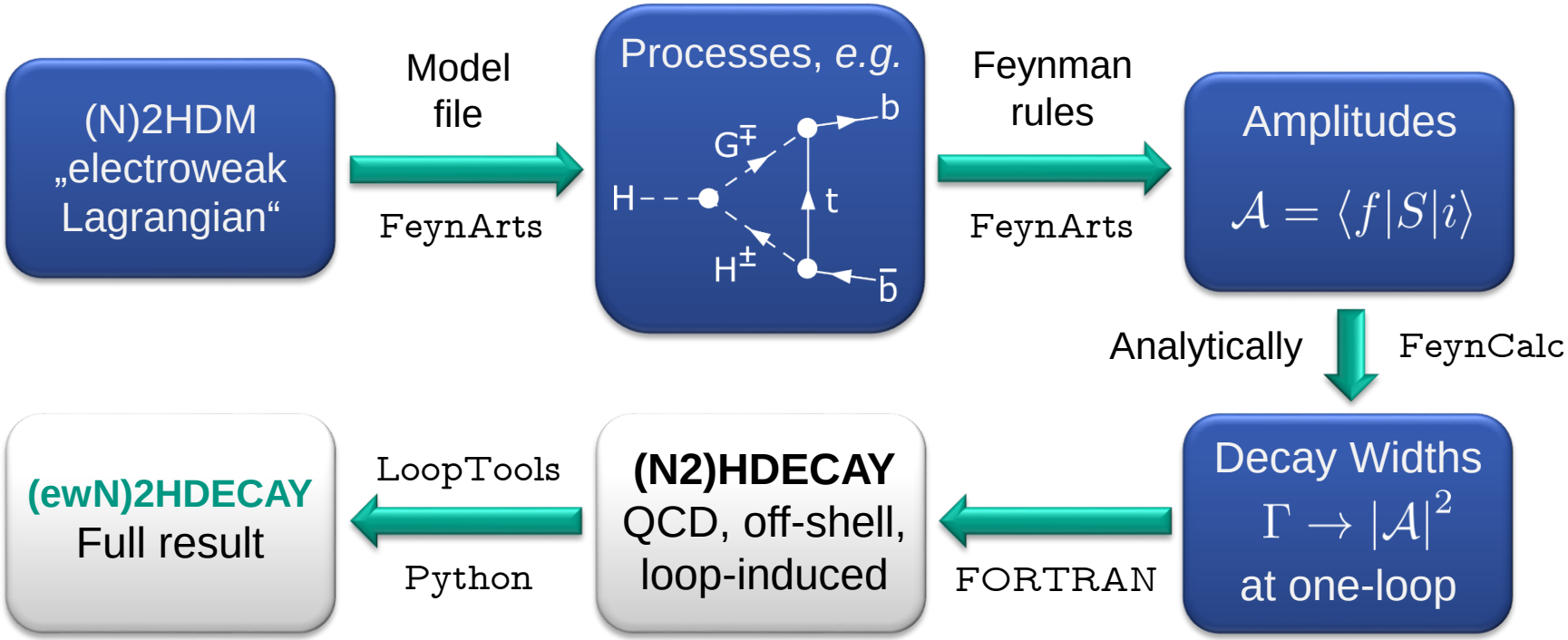

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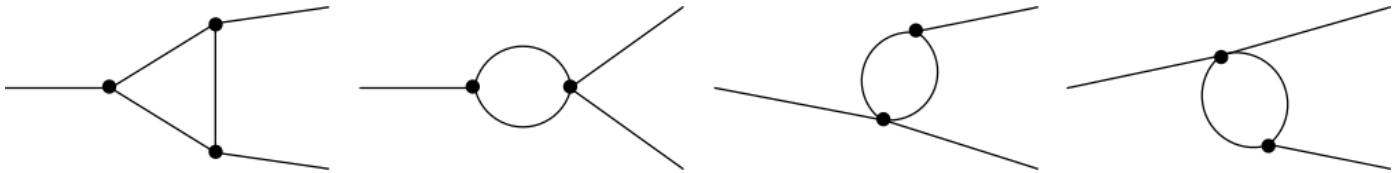
Electroweak Corrections at One-Loop Level (II)

- contributing topologies at one-loop level:



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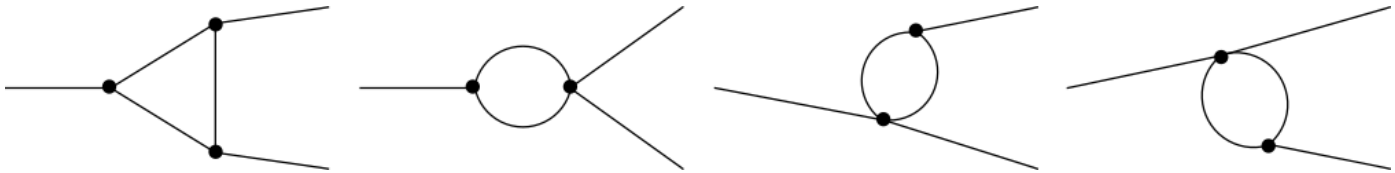

- 2HDM decay channels that are considered (OS, non-loop-induced):

- $h/H/A \to f\bar{f}$ $(f = c, s, t, b, \mu, \tau)$
- $h/H \to VV$ $(V = W^{\pm}, Z)$
- $h/H \to VS$ $(V = Z, W^{\pm},\ S = A, H^{\pm})$
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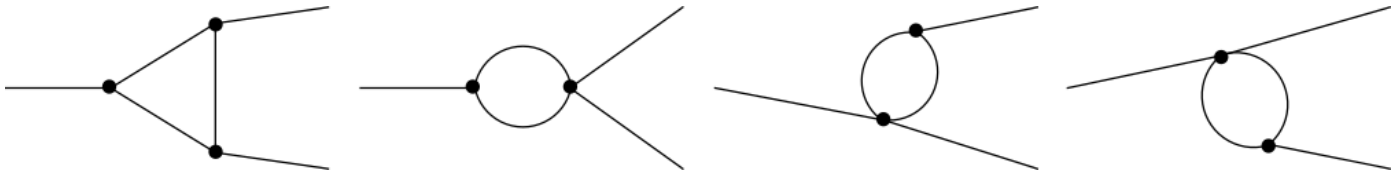

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➡ **semi-automated** calculation of the decays with **2HDMCalc**

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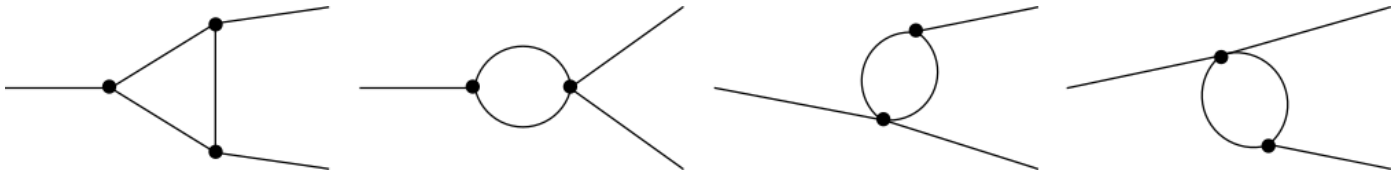

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  - $A \to V\,S \quad (V\,S = Z\,H_i, W^\pm H^\mp)$
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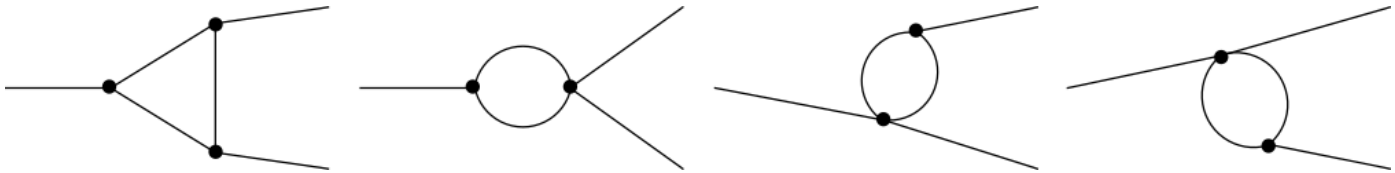

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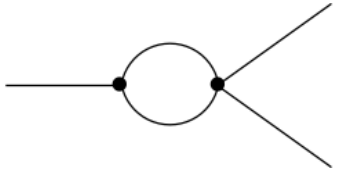

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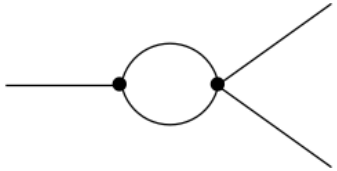

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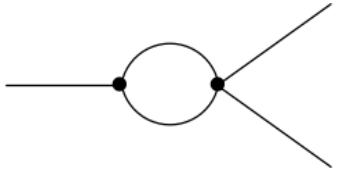

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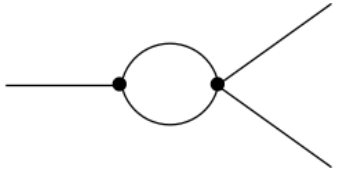

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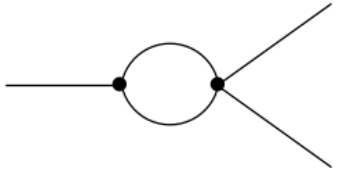

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- consistently remove the divergences via **renormalization**
- idea: split ‘bare’ parameters into **renormalized** values and **counterterms**

$$m_i^2 \to m_i^2 + \delta m_i^2$$

- counterterms need to be fixed via **renormalization conditions**

Renormalization of the (N)2HDM

- set of free parameters of the 2HDM (excluding CKM elements, …)

$$\left\{T_h,\ T_H,\ \alpha_{\rm em},\ m_W,\ m_Z,\ m_f,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},\ \alpha,\ \beta,\ m_{12}^2\ ,\cdots\ \right\}$$

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- renormalization program for the (N)2HDM:
  - tadpole terms ➡ standard / **alternative** tadpole scheme
  - mass counterterms ➡ on-shell
  - fine-structure constant ➡ at Z mass
  - soft-$\mathbb{Z}_2$-breaking scale m_{12}^2 ➡ $\overline{\rm MS}$
  - singlet VEV v_s ➡ $\overline{\rm MS}$
  - **scalar mixing angles** ➡ **?**

[MK, *Master's thesis* (2016), KIT;
MK, R. Lorenz, M. M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
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Renormalization: General Tadpole Conditions

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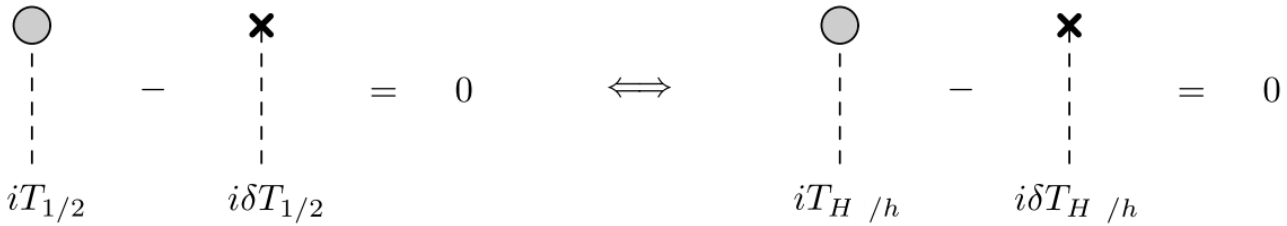

- conversion from gauge to **mass basis**:

$$\begin{pmatrix}\delta T_1\\ \delta T_2\end{pmatrix} = \begin{pmatrix}\mathrm{c}_\alpha & -\mathrm{s}_\alpha\\ \mathrm{s}_\alpha & \mathrm{c}_\alpha\end{pmatrix}\begin{pmatrix}\delta T_H\\ \delta T_h\end{pmatrix} = \begin{pmatrix}\mathrm{c}_\alpha\delta T_H - \mathrm{s}_\alpha\delta T_h\\ \mathrm{s}_\alpha\delta T_H + \mathrm{c}_\alpha\delta T_h\end{pmatrix}$$

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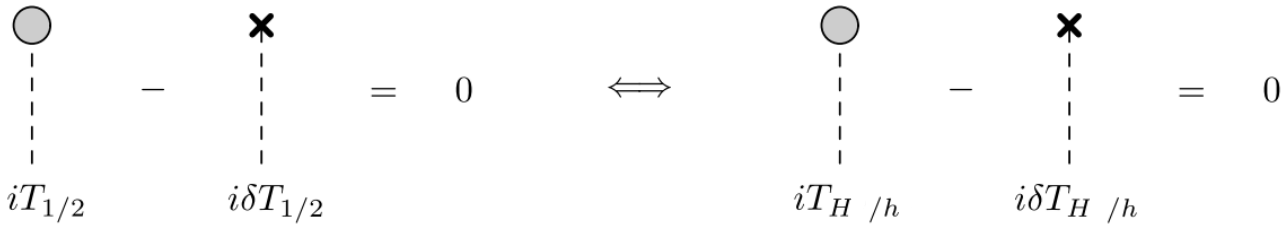

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- **purpose**: restoring the minimum conditions of the potential at NLO
- **textbook explanation**: **no tadpole diagrams** in NLO calculations

➡ **really?**

Renormalization: Standard Tadpole Scheme

- **standard scheme**: VEVs are derived from the **loop-corrected potential**
- VEVs in the mass relations produce correct one-loop OS masses, *e.g.*

$$m_W^2 = g^2 \frac{v^2}{4} \quad , \qquad m_A^2 = \frac{m_{12}^2}{\sin\beta\cos\beta} - \lambda_5 v^2$$

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- tadpole terms **appear explicitly** in the bare mass matrices

  ➡ mass matrix counterterms **contain** the **tadpole counterterms**:

$$\delta D_\phi^2 \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \boxed{\begin{pmatrix} \delta T_{\phi_1\phi_1} & \delta T_{\phi_1\phi_2} \\ \delta T_{\phi_1\phi_2} & \delta T_{\phi_2\phi_2} \end{pmatrix}}$$

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- **standard scheme**: VEVs are derived from the **loop-corrected potential**

- VEVs in the mass relations produce correct one-loop OS masses, *e.g.*

$$m_W^2 = g^2 \frac{v^2}{4} \quad , \qquad m_A^2 = \frac{m_{12}^2}{\sin\beta\cos\beta} - \lambda_5 v^2$$

- tadpole terms **appear explicitly** in the bare mass matrices

  ➡ mass matrix counterterms **contain** the **tadpole counterterms**:

$$\delta D_\phi^2 \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \boxed{\begin{pmatrix} \delta T_{\phi_1\phi_1} & \delta T_{\phi_1\phi_2} \\ \delta T_{\phi_1\phi_2} & \delta T_{\phi_2\phi_2} \end{pmatrix}}$$

- one-loop corrected potential is **gauge-dependent**

  ➡ **VEVs** are gauge-dependent

  ➡ **mass counterterms** become **gauge-dependent**

Renormalization: Alternative Tadpole Scheme (I)

- **alternative scheme**: VEVs represent the same minimum as at **tree level**

  [based on: J. Fleischer, F. Jegerlehner, *Phys. Rev.* D **23** (1981) 2001-2026]

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- correct minimum conditions at NLO require a **shift in the VEVs**

$$v_1 \;\rightarrow\; v_1 + \delta v_1 \;, \qquad v_2 \;\rightarrow\; v_2 + \delta v_2$$

- fixation of the shifts by **applying the minimum conditions**:

$$\begin{pmatrix} \delta v_1 \\ \delta v_2 \end{pmatrix} = \begin{pmatrix} \frac{T_H}{m_H^2} \mathrm{c}_\alpha - \frac{T_h}{m_h^2} \mathrm{s}_\alpha \\ \frac{T_H}{m_H^2} \mathrm{s}_\alpha + \frac{T_h}{m_h^2} \mathrm{c}_\alpha \end{pmatrix}$$

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- the shifts translate into **every CT**, **wave function renormalization constants** and **Feynman rules**

- alternative tadpole scheme **worked out for the (N)2HDM at one-loop**

Renormalization: Alternative Tadpole Scheme (II)

- example: **W boson mass**

$$m_W^2 = g^2 \frac{v^2}{4}$$

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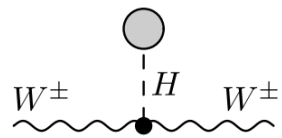

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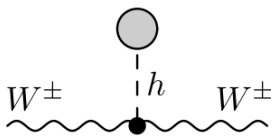

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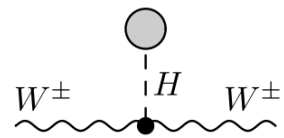

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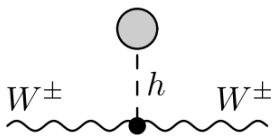

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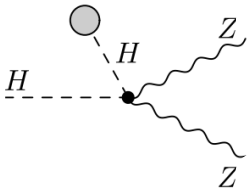
- example: **coupling between Higgs and Z bosons**

$$i g_{HZZ} = \frac{i g^2}{2 \mathrm{c}_W^2} \left(\mathrm{c}_\alpha v_1 + \mathrm{s}_\alpha v_2 \right) \quad , \qquad i g_{HHZZ} = \frac{i g^2}{2 \mathrm{c}_W^2}$$

$$i g_{HZZ} \;\rightarrow\; i g_{HZZ} + \frac{i g^2}{2 c_W^2} \left(c_\alpha \delta v_1 + s_\alpha \delta v_2 \right) = i g_{HZZ} + \Bigg($$



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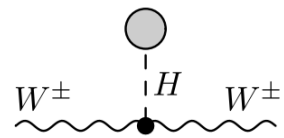

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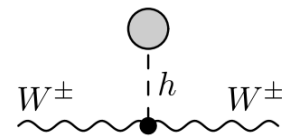

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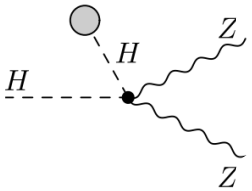

$$\Bigg)_{\text{trunc}}$$

- **effects** of the alternative tadpole scheme:
  - **tadpole diagrams are added everywhere** where they exist in the 2HDM
  - mass counterterms become **manifestly gauge-independent**
  - tadpole counterterms in the scalar sector are **removed**

Renormalization: Scalar Mixing Angles (I)

- renormalization of mixing angles α and β is **non-trivial** in the 2HDM
- simplest approach: $\overline{\mathrm{MS}}$ conditions for α and β (alternatively: λ_3)
  - can be **numerically unstable** in one-to-two-body decays
  - **divergences for degenerate masses / "dead corners"** of parameter space

[MK, R. Lorenz, M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
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- analyze renormalization schemes **for the 2HDM** w.r.t. **"three desirable criteria":** [A. Freitas, D. Stöckinger, *Phys. Rev.* D**66** (2002) 095014]
  - gauge independence
  - process independence
  - **numerical stability** (*i.e.* leads to **moderate NLO corrections**)

Intermezzo: Types of Numerical Instabilities

- measure for the **relative size** of the **NLO corrections**:

$$\Delta\Gamma \equiv \frac{\Gamma^{\mathrm{NLO}} - \Gamma^{\mathrm{LO}}}{\Gamma^{\mathrm{LO}}}$$

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  - due to **unsuitable renormalization schemes** for some parameters
    - *e.g.* $\overline{\mathrm{MS}}$: finite parts of $\delta\alpha, \delta\beta$ missing for cancellation of large contributions
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  ➡ in this talk: **"numerical instability"** of the renormalization scheme

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  ➡ in this talk: **"numerical instability"** of the renormalization scheme

- in Higgs-to-Higgs decays in the (N)2HDM, $\Delta\Gamma$ may **additionally** become large due to certain **limits in the parameter space**

  ➡ **wrong-sign** limit, **strong coupling** limit

Renormalization: Scalar Mixing Angles (II)

- approach by S. Kanemura *et al.*: connect the definition of α and β with the **inverse propagator matrix** ("KOSY scheme")

[S. Kanemura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Rev. D* **70** (2004) 115002]

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = R^T_{\theta,0}\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix}_0 \approx R^T_{\delta\theta}R^T_{\theta}\sqrt{Z_{\widetilde{\phi}}}R_{\theta}R^T_{\theta}\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix} \approx \begin{pmatrix}1+\frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2}+\delta\theta\\ \delta C_{\phi_2}-\delta\theta & 1+\frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

- mixing angle counterterms **within the standard tadpole scheme**:

$$\delta\alpha = \frac{1}{2\left(m_H^2-m_h^2\right)}\mathrm{Re}\Big[\Sigma_{Hh}(m_H^2)+\Sigma_{Hh}(m_h^2)-2\delta T_{Hh}\Big]$$

$$\delta\beta = -\frac{1}{2m_{H^\pm}^2}\mathrm{Re}\Big[\Sigma_{G^\pm H^\pm}(m_{H^\pm}^2)+\Sigma_{G^\pm H^\pm}(0)-2\delta T_{G^\pm H^\pm}\Big]$$

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- the KOSY scheme as described above leads to the inclusion of gauge-dependent contributions in the partial decay widths [MK, *Master's thesis* (2016), KIT]

➡ gauge dependences **need to be removed**

[cf. S. Kanemura, M. Kikuchi, K. Sakurai, K. Yagyu, *Phys. Rev.* D **96** (2017) 035014]

Renormalization: Scalar Mixing Angles (III)

- gauge-independent **“OS approach”**: use the **pinch technique** (PT)
- **PT-based definition** of the scalar mixing angle counterterms:
  use the pinched scalar self-energies instead of the usual ones
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- **PT-based definition** of the scalar mixing angle counterterms:
  use the pinched scalar self-energies instead of the usual ones
- necessary requirement: use the **alternative tadpole scheme**
- properties of the pinched scheme:
  - **process-independent**, symmetric in the fields
  - manifestly **gauge-independent** per construction
    ➡ gauge-independent NLO **amplitudes**
  - **numerically stable** (depending on the point in parameter space)
    ➡ proposed solution for renormalizing $\delta\alpha$ and $\delta\beta$ in the 2HDM

[MK, R. Lorenz, M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143]

Renormalization: Scalar Mixing Angles (IV)

- gauge-independent approach: **process-dependent schemes**

  [A. Freitas, D. Stöckinger, *Phys. Rev.* D**66** (2002) 095014;
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- idea: impose the gauge-invariant condition

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- properties of process-dependent schemes:
  - **process-dependent** per construction
  - **gauge-independent**
  - **potentially numerically unstable** over large parameter ranges

Renormalization: Scalar Mixing Angles (V)

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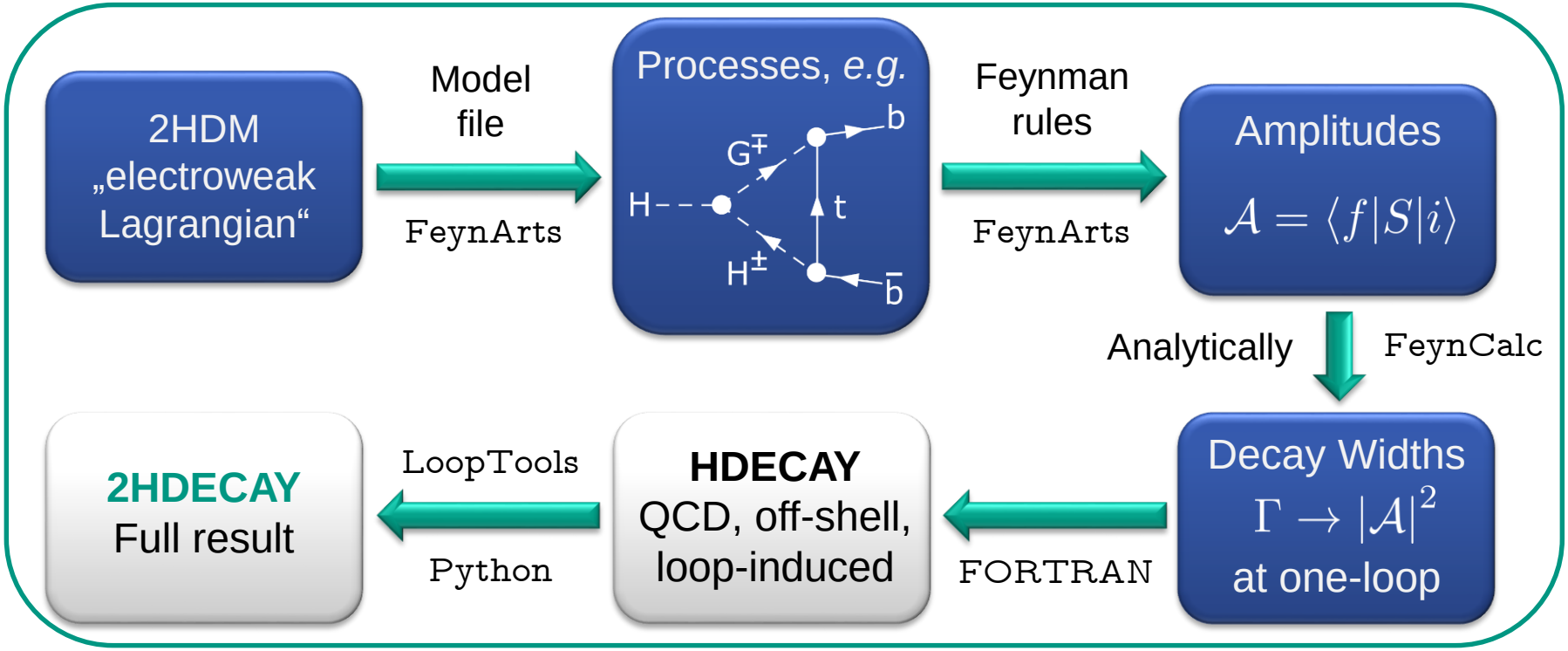
- generalization to the **N2HDM** is straightforward
- the CP-odd and charged mixing angle β is **analogously** renormalized
- for the three CP-even mixing angles $\alpha_1, \alpha_2, \alpha_3$, we consider several different schemes:
  - $\overline{\mathrm{MS}}$ scheme
  - adapted KOSY schemes
  - **PT-based schemes**

Implementation: 2HDECAY (I)



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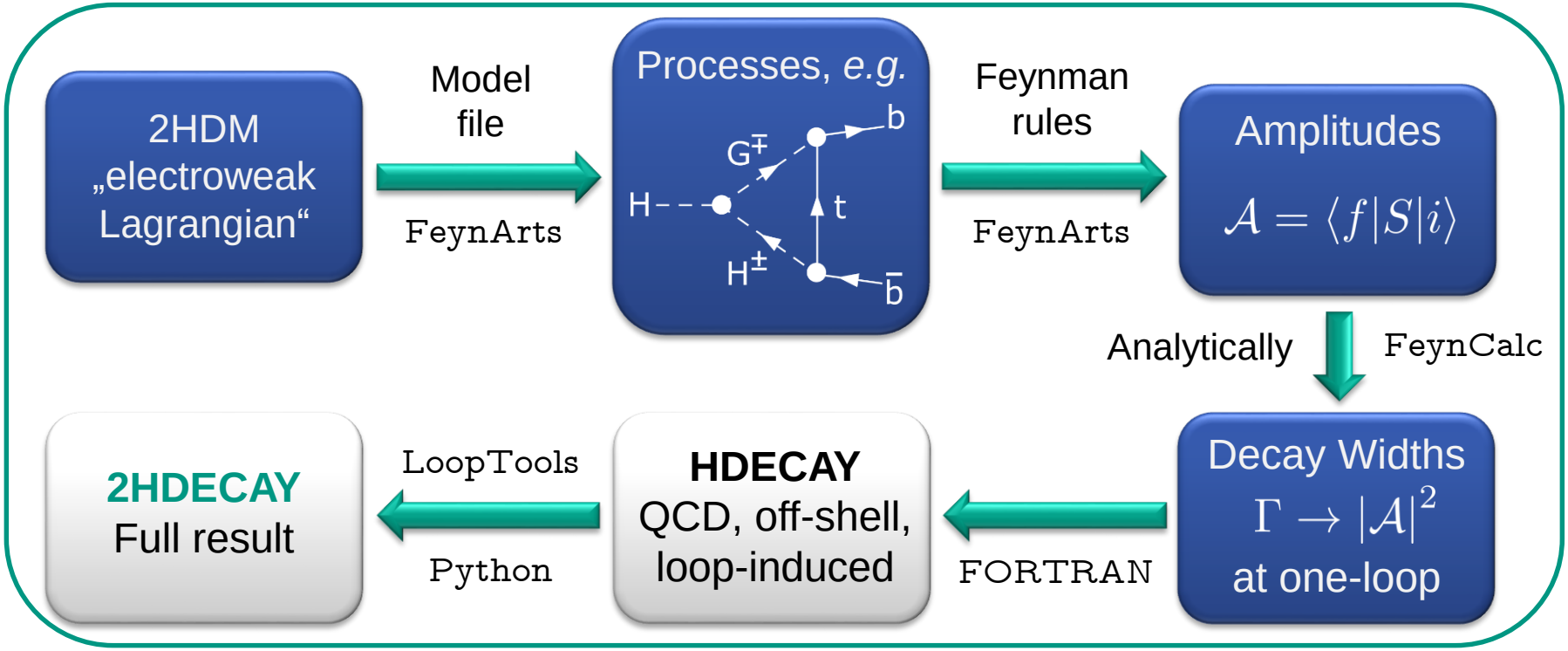

2HDECAY: “**2**HDM **HDECAY**“

A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

[MK, M. Mühlleitner, M. Spira, *Computer Physics Communications* **246** (2020) 106852]

⇩ **https://github.com/marcel-krause/2HDECAY**

Implementation: 2HDECAY (II)

- 17 renormalization schemes are implemented in **2HDECAY**:

Input ID	Tadpole scheme	$\delta\alpha$	$\delta\beta$	Label
1	standard	KOSY	KOSY (odd)	KOSY^o(std)
2	standard	KOSY	KOSY (charged)	KOSY^c(std)
3	alternative (FJ)	KOSY	KOSY (odd)	KOSY^o
4	alternative (FJ)	KOSY	KOSY (charged)	KOSY^c
5	alternative (FJ)	p_*-pinched	p_*-pinched (odd)	p_*^o
6	alternative (FJ)	p_*-pinched	p_*-pinched (charged)	p_*^c
7	alternative (FJ)	OS-pinched	OS-pinched (odd)	OS^o
8	alternative (FJ)	OS-pinched	OS-pinched (charged)	OS^c
9	alternative (FJ)	proc.-dep. 1	proc.-dep. 1	proc1
10	alternative (FJ)	proc.-dep. 2	proc.-dep. 2	proc2
11	alternative (FJ)	proc.-dep. 3	proc.-dep. 3	proc3
12	alternative (FJ)	physical OS1	physical OS1	OS1
13	alternative (FJ)	physical OS2	physical OS2	OS2
14	alternative (FJ)	physical OS12	physical OS12	OS12
15	alternative (FJ)	rigid symmetry (BFM)	BFMS	BFMS
16	standard	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$(std)
17	alternative (FJ)	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$

[MK, M. Mühlleitner, M. Spira, *Computer Physics Communications* **246** (2020) 106852]

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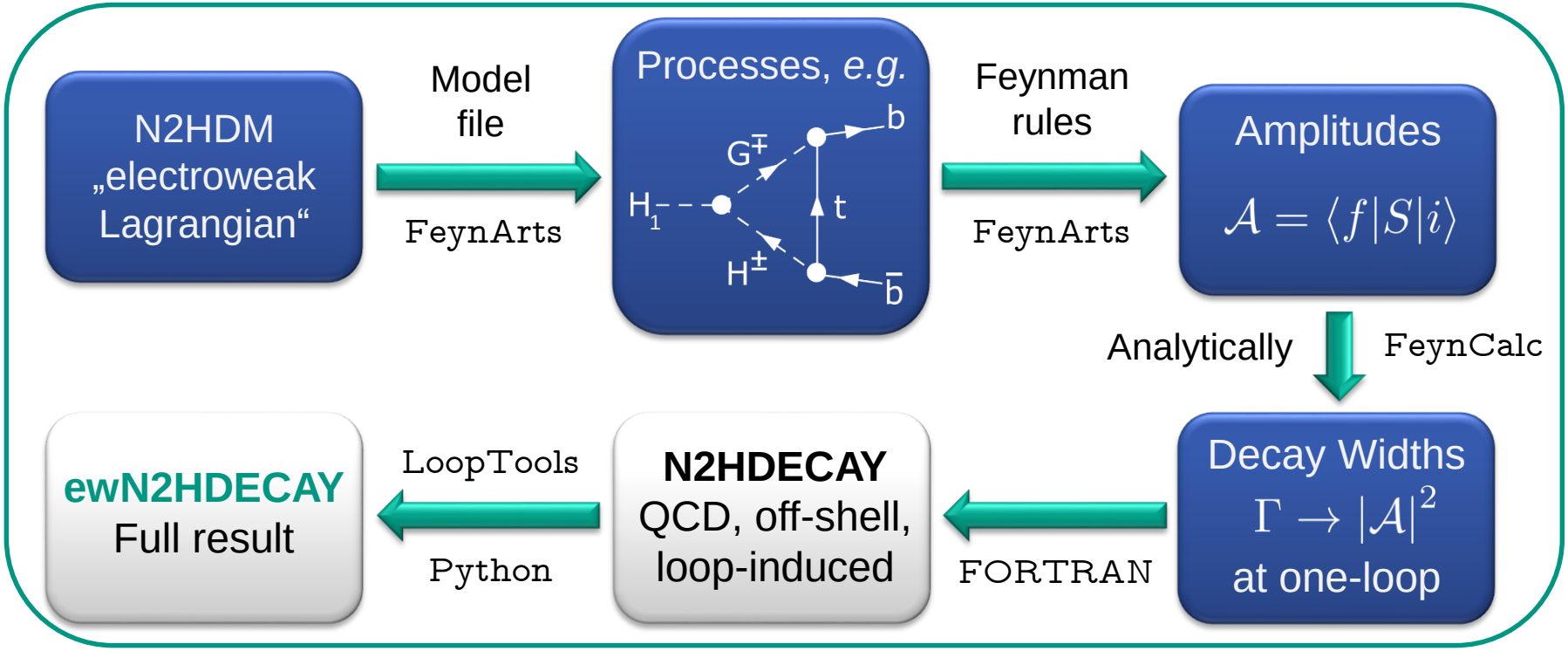

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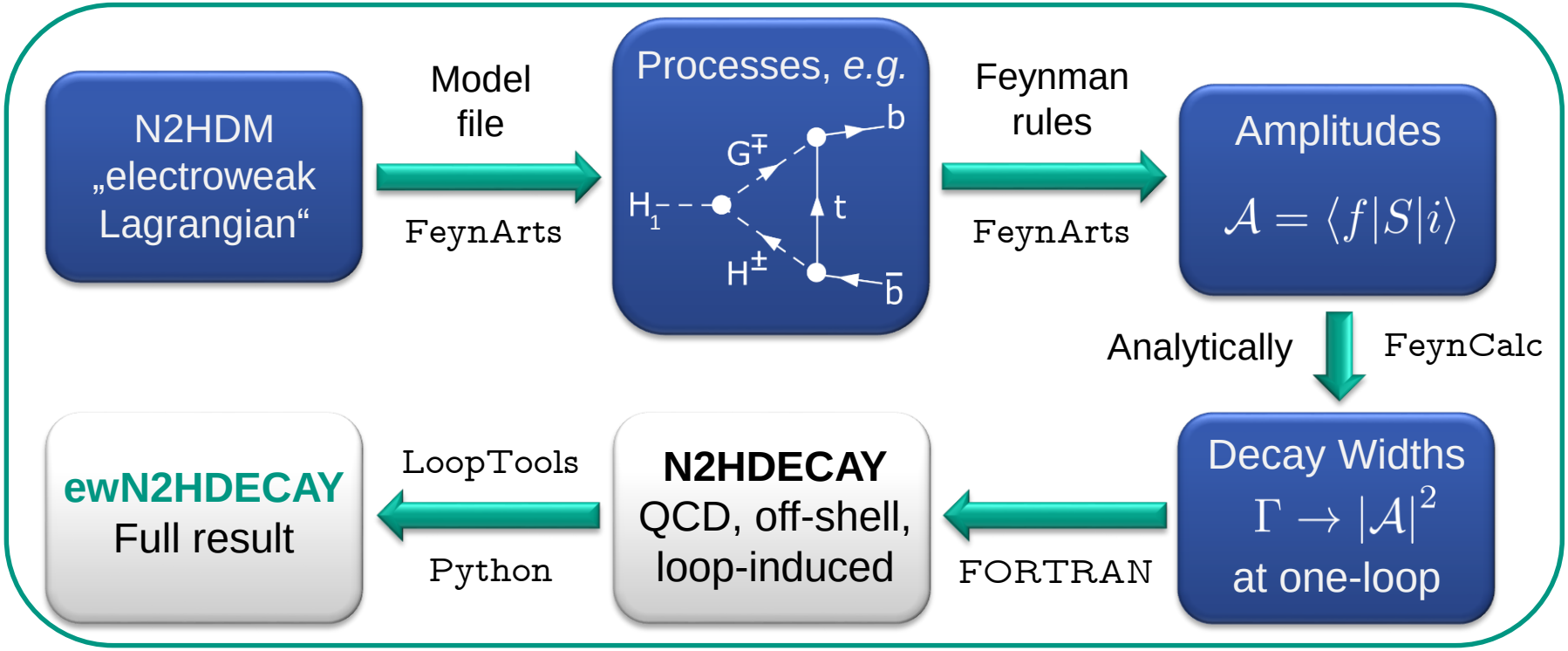

ewN2HDECAY: “**e**lectro**w**eak **N2HDECAY**“

A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Next-to-Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

[MK, M. Mühlleitner, *Computer Physics Communications* **2019**, *arXiv:1904.02103*]

⇩ **https://github.com/marcel-krause/ewN2HDECAY**

Implementation: ewN2HDECAY (II)

- 10 renormalization schemes are implemented in **ewN2HDECAY**:

Input ID	Tadpole scheme	$\delta\alpha$	$\delta\beta$	Label
1	standard	Ad. KOSY	Ad. KOSY (odd)	KOSY^o(std)
2	standard	Ad. KOSY	Ad. KOSY (charged)	KOSY^c(std)
3	alternative (FJ)	Ad. KOSY	Ad. KOSY (odd)	KOSY^o
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5	alternative (FJ)	p_*-pinched	p_*-pinched (odd)	p_*^o
6	alternative (FJ)	p_*-pinched	p_*-pinched (charged)	p_*^c
7	alternative (FJ)	OS-pinched	OS-pinched (odd)	OS^o
8	alternative (FJ)	OS-pinched	OS-pinched (charged)	OS^c
9	standard	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$(std)
10	alternative (FJ)	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$

[MK, M. Mühlleitner, *Computer Physics Communications* **2019**, *arXiv:1904.02103*]

⇩ **https://github.com/marcel-krause/ewN2HDECAY**

Numerical Analysis: Input Parameters (I)

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- exemplarily, we consider a **type II 2HDM** in the following
- keep in mind: the 2HDM contains **a lot of free parameters:**

$$\{m_H, m_A, m_{H^\pm}, m_{12}^2, \tan\beta, \alpha\}$$

➡ **scanning** through the parameter space is possible

- chosen parameter points respect **several experimental and theoretical constraints**

Numerical Analysis: Input Parameters (II)

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$$m_H \geq 2m_h \quad \text{and large} \quad m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin\beta\cos\beta}} \pm 5\,\%$$

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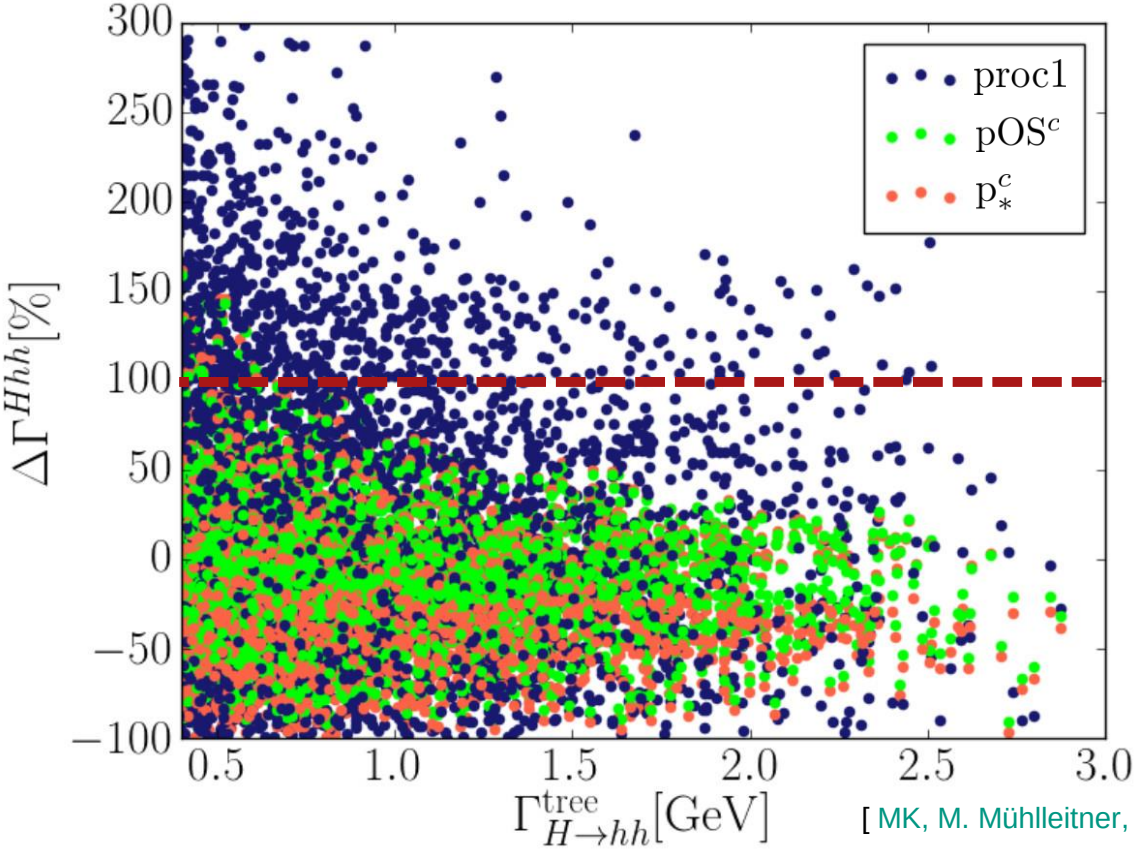
- **aim**: distinguish large NLO corrections due to the strong coupling regime from **numerical instability** due to the chosen renormalization scheme

Numerical Analysis: Condition I



Condition I: $m_H \geq 2m_h$

- $\mathrm{proc1}$: process-dependent 1
- pOS^c : “on-shell pinched”
- p^c_* : “p*-pinched”

relative size of NLO corrections:

$$\Delta\Gamma \equiv \frac{\Gamma_{\rm NLO}-\Gamma_{\rm LO}}{\Gamma_{\rm LO}}$$

[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

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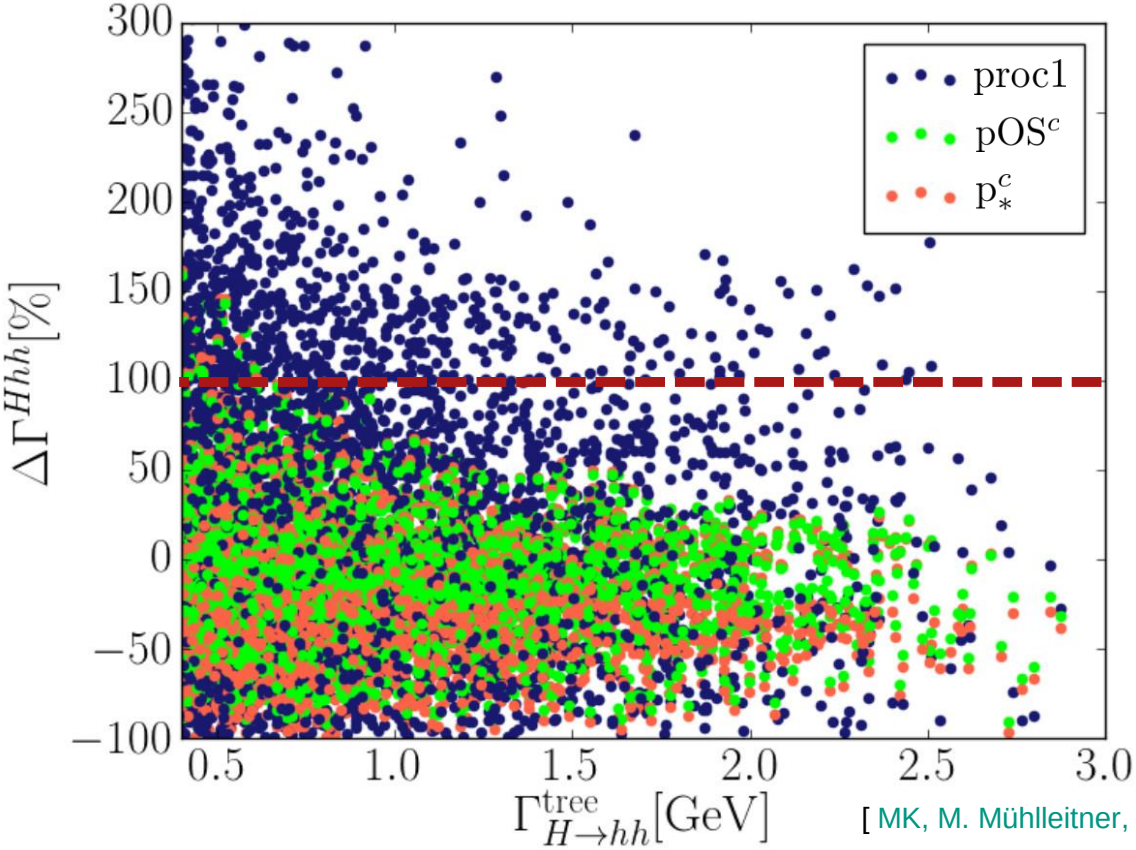

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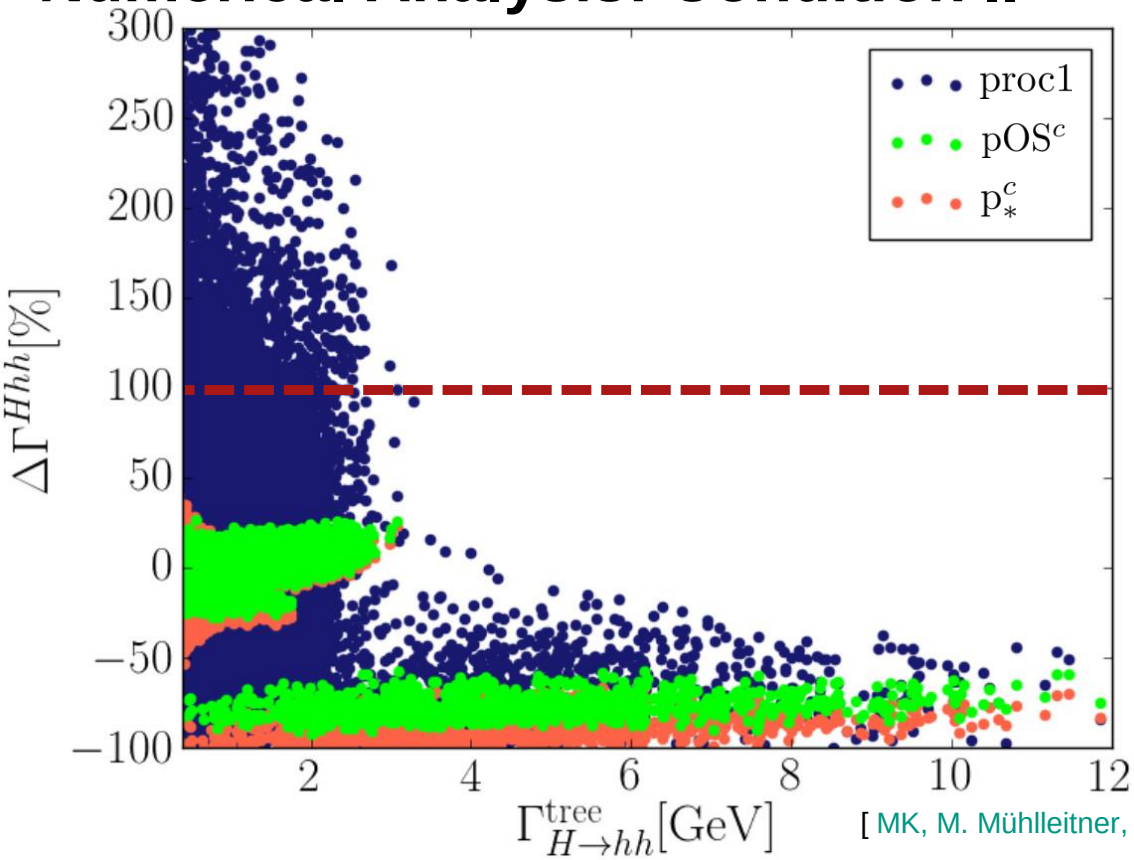
- process-dependent scheme: typically **huge** NLO corrections
- pinched schemes: well-behaving for **large parameter ranges**, but also **large NLO corrections possible** ➡ **numerical instability?**

Numerical Analysis: Condition II



Condition II: $m_H \geq 2m_h$ **and**

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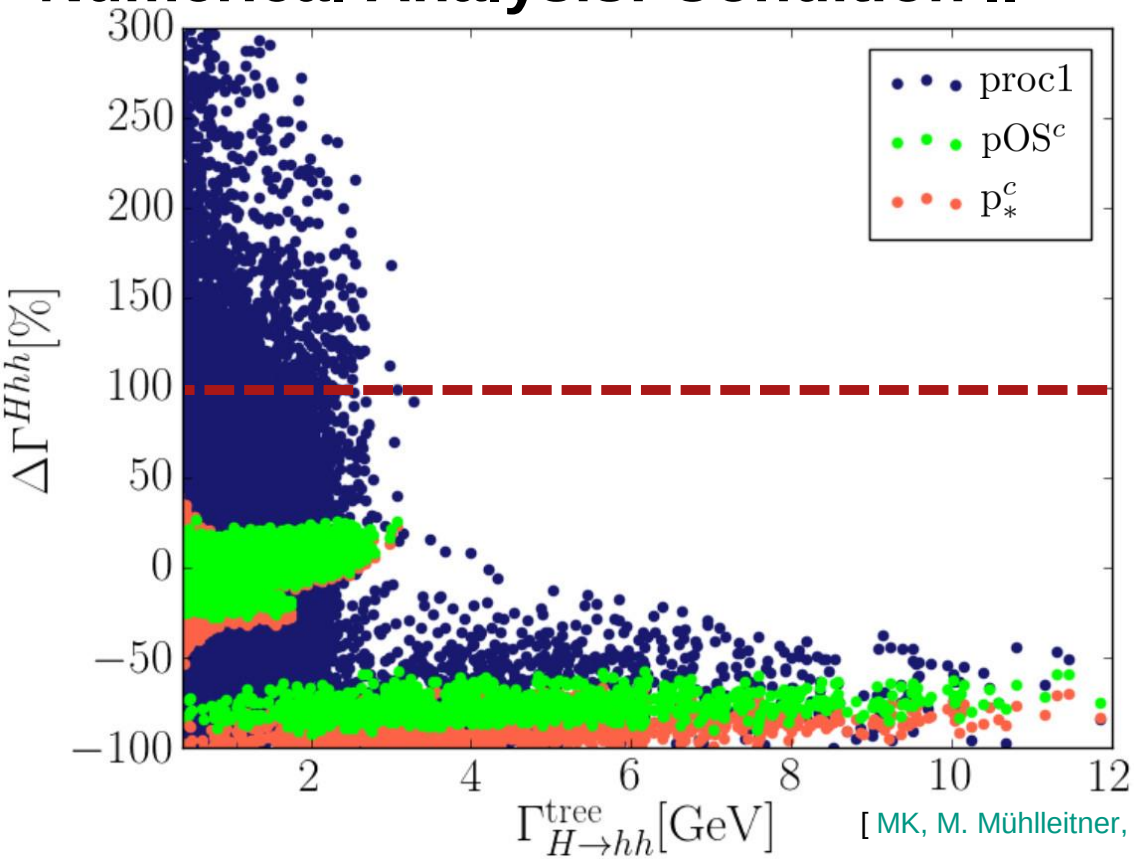

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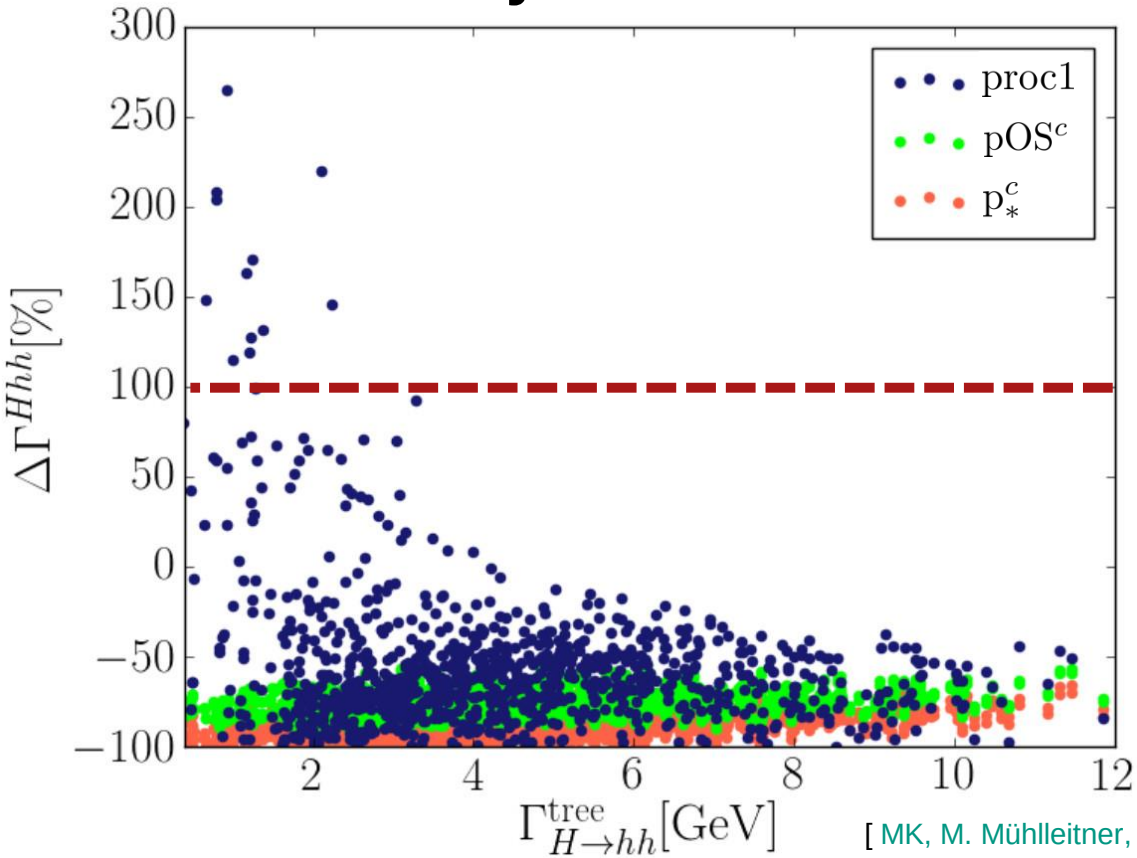
- process-dependent scheme: **still** typically **huge** NLO corrections
- pinched schemes: one **well-behaving regime** and one regime with **large NLO corrections**

➡ **numerical instability** or still **strong coupling**?

Numerical Analysis: Condition II



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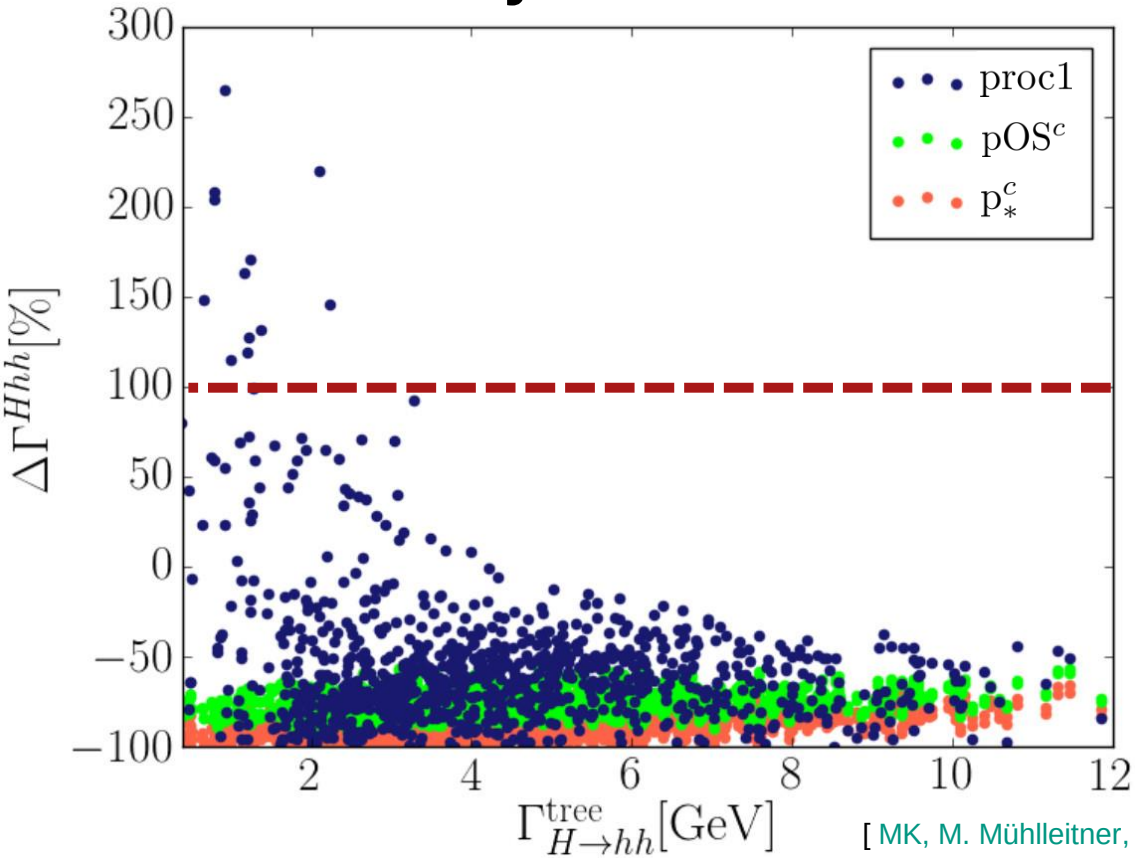

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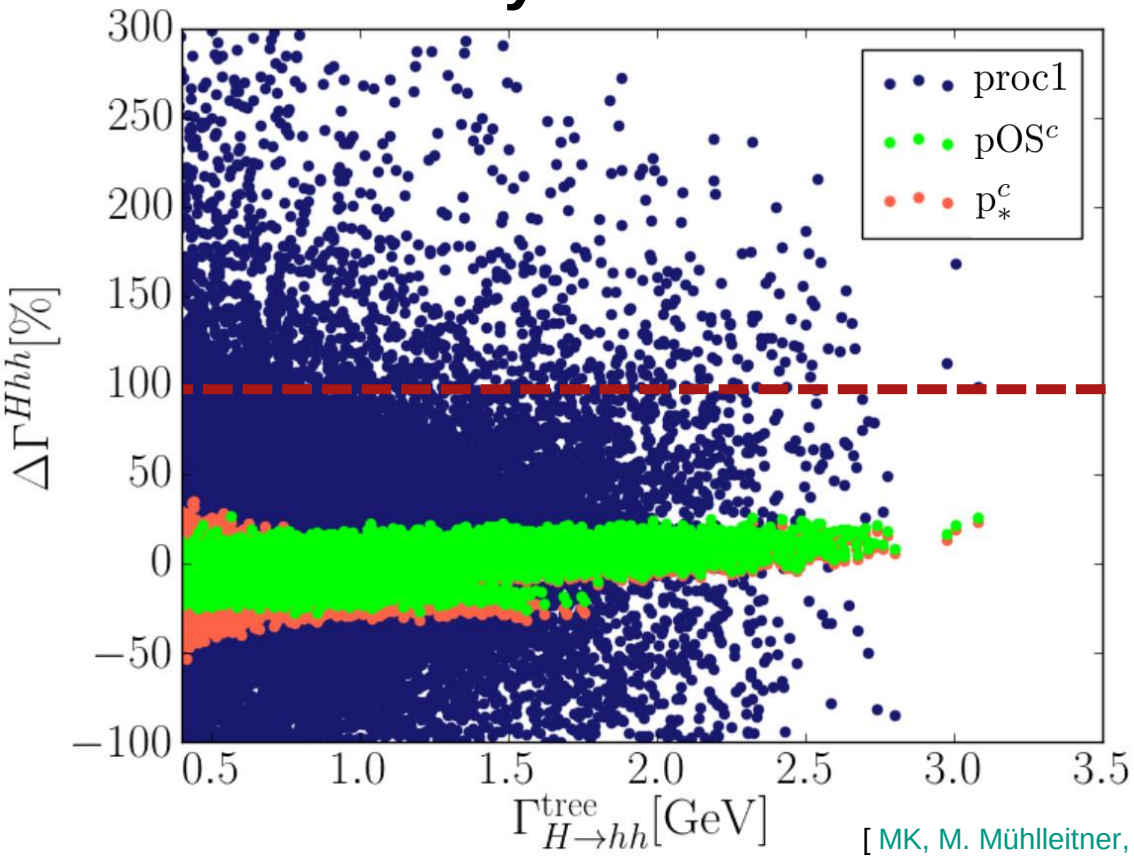
[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

- all schemes: mostly **large NLO corrections**
  - ➡ decoupling is not possible in wrong-sign type II 2HDM
  - ➡ **non-decoupling effects** increase NLO corrections

Numerical Analysis: Condition II



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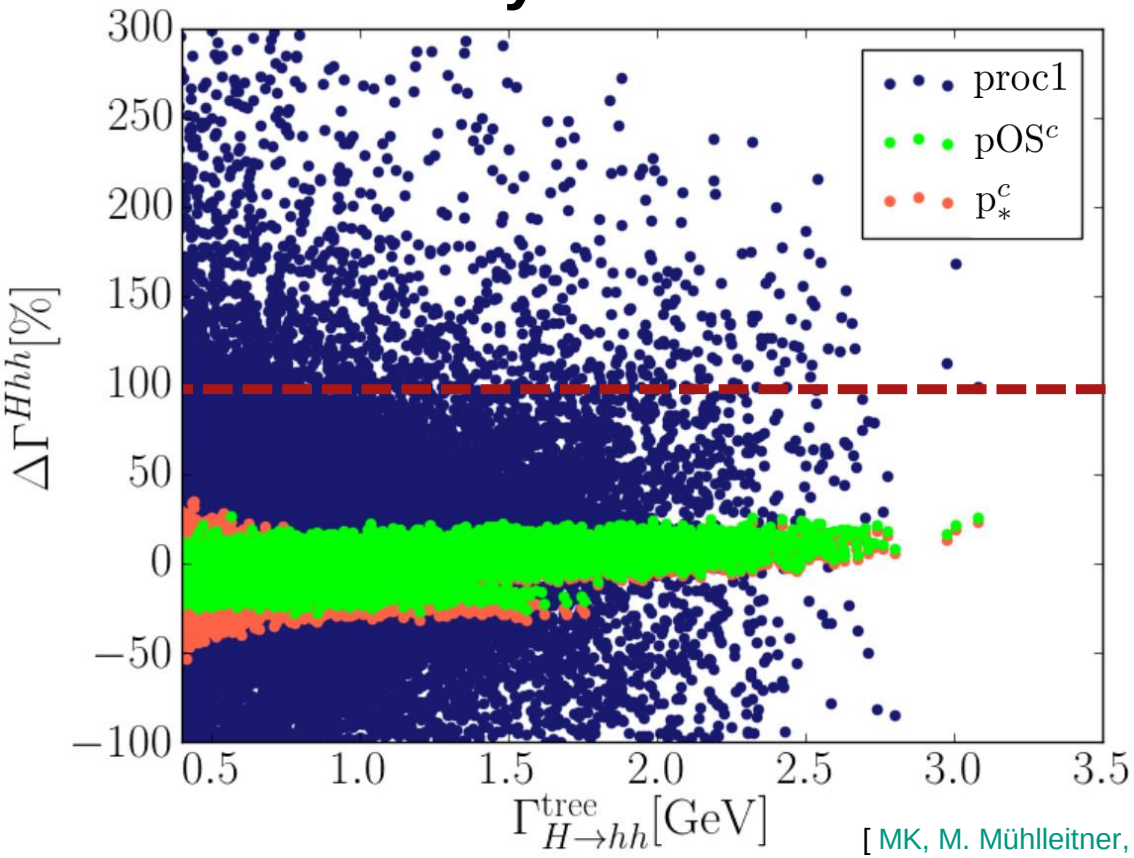

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[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

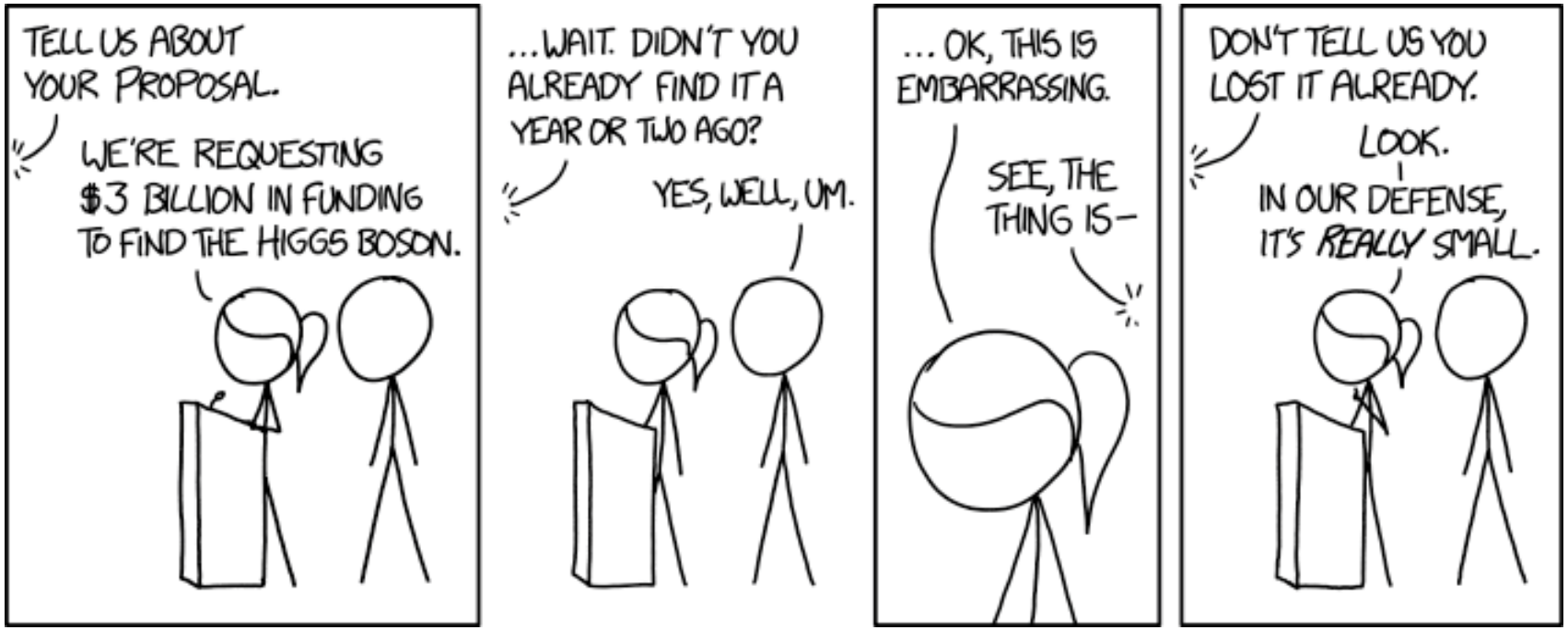
- process-dependent scheme: still **huge NLO corrections**
  - ➡ **numerical instability** of the scheme
- PT-based schemes: mostly **moderate NLO corrections**
  - ➡ **numerically stable scheme**

Conclusions and Outlook

- renormalization can **spoil gauge independence** in the (N)2HDM
- a gauge-independent, process-independent and numerically stable scheme for $\delta\alpha$ and $\delta\beta$ worked out **for the first time for the (N)2HDM**
- NLO corrections to Higgs-to-Higgs decays can become **large**
  - due to chosen renormalization schemes ("**numerical instability**")
  - if the LO width becomes very small
  - due to parametrically enhanced contributions from VCs, CTs and WFRCs
  - in certain limits of the (N)2HDM due to **non-decoupling effects**
- analyses of the NLO corrections performed with **(ewN)2HDECAY**: several different renormalization schemes included
- for correct-sign decoupling: **moderate corrections** for certain schemes

  ➡ **numerically stable** schemes
- dedicated phenomenological studies in the very near future: **stay tuned!**

So long, and thanks for all the fish!



[source: https://xkcd.com/1437]

Backup slides

Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet** $(\phi_1,\ \phi_2)$
- wave-function renormalization constants (WFRCs):

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = \sqrt{Z_\phi}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \approx \left(1_{2\times 2} + \frac{\delta Z_\phi}{2}\right)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \quad , \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix}\frac{\delta Z_{\phi_1\phi_1}}{2} & \frac{\delta Z_{\phi_1\phi_2}}{2}\\ \frac{\delta Z_{\phi_2\phi_1}}{2} & \frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}$$

- two-point correlation function for the doublet with momentum p^2:

$$\widehat{\Gamma}_\phi(p^2) := \begin{pmatrix}\widehat{\Gamma}_{\phi_1\phi_1}(p^2) & \widehat{\Gamma}_{\phi_1\phi_2}(p^2)\\ \widehat{\Gamma}_{\phi_1\phi_2}(p^2) & \widehat{\Gamma}_{\phi_2\phi_2}(p^2)\end{pmatrix}$$

$$= i\sqrt{Z_\phi}^\dagger\left[p^2 1_{2\times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2\right]\sqrt{Z_\phi} \approx i\left[p^2 1_{2\times 2} - D_\phi^2 + \widehat{\Sigma}_\phi(p^2)\right]$$

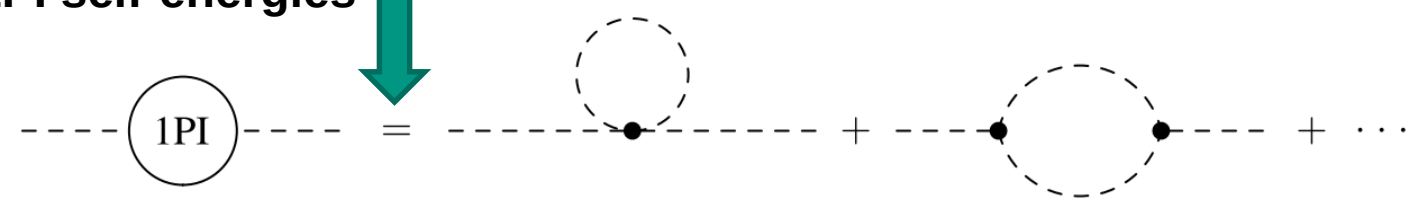
mass matrices

mass CTs

renormalized self-energies

1PI self-energies

$i\Sigma(p^2) :=$



Renormalization: On-Shell Conditions (II)

- **on-shell conditions**:
  - **mixing** of fields **vanishes** for $p^2 = m^2_{\phi_i}$
  - squared **masses** $m^2_{\phi_i}$ are the real parts of the **poles** of the propagator
  - **field normalization**: residue of the propagator at its pole equals i

- fixation of **diagonal** mass counterterms:

$$\mathrm{Re}\left[\delta D^2_{\phi_1\phi_1}\right] = \mathrm{Re}\left[\Sigma_{\phi_1\phi_1}(m^2_{\phi_1})\right] \quad , \quad \mathrm{Re}\left[\delta D^2_{\phi_2\phi_2}\right] = \mathrm{Re}\left[\Sigma_{\phi_2\phi_2}(m^2_{\phi_2})\right]$$

- fixation of WFRCs:

$$\delta Z_{\phi_1\phi_1} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_1\phi_1}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_1}} \quad , \quad \delta Z_{\phi_2\phi_2} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_2\phi_2}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_2}}$$

$$\delta Z_{\phi_1\phi_2} = \frac{2}{m^2_{\phi_1}-m^2_{\phi_2}}\mathrm{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta D^2_{\phi_1\phi_2}\right] \quad , \quad \delta Z_{\phi_2\phi_1} = \frac{2}{m^2_{\phi_2}-m^2_{\phi_1}}\mathrm{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_1}) - \delta D^2_{\phi_1\phi_2}\right]$$

- the **specific form** of the $\delta D^2_{\phi_i\phi_j}$ **depends on the tadpole scheme**

Renormalization: Alternative Tadpole Scheme (III)

- technical note: distinguish between tadpole renormalization and renormalization of the **other physical parameters**
- at one-loop, the proper **renormalized VEV** is given by the tree-level VEV:

$$v^{\text{ren}}\big|_{\text{FJ}} = v^{\text{tree}} = \frac{2m_W}{g}\bigg|^{\text{tree}}$$

- the effect of the shifts δv_i **were already applied**
- at NLO, the other tree-level parameters m_W and g still **have to be renormalized**:

$$\frac{2m_W}{g}\bigg|^{\text{tree}} \to \frac{2m_W}{g}\bigg|^{\text{ren}}_{\text{FJ}} + \underbrace{\frac{2m_W}{g}\left(\frac{\delta m_W^2}{2m_W^2} - \frac{\delta g}{g}\right)\bigg|_{\text{FJ}}}_{\equiv \Delta v}$$

- the quantity Δv combines the effect of the CTs of m_W and g

Renormalization: Alternative Tadpole Scheme (IV)

- **generalization** to more complicated Higgs models, *e.g.* the singlet extensions of the SM ("HSM") or N2HDM is straightforward
- the shifts δv_i (including δv_S) are connected to the tadpole diagrams
- after performing the shifts, Δv_S still **has to be renormalized**
- in the standard tadpole scheme: Δv_S is **protected** from UV divergences

  [M. Sperling, D. Stöckinger, A. Voigt, *J. High Energ. Phys.* **1307** (2013) 132;
  F. Bojarski, G. Chalons, D. Lopez-Val, T. Robens, *J. High Energ. Phys.* **2016** (2016) 147]

  ➡ freedom of choice: set $\Delta v_S = 0$

- in the alternative tadpole scheme: Δv_S **becomes UV-divergent**

  [MK, D. Lopez-Val, M. M. Mühlleitner, R. Santos, *J. High Energ. Phys.* **2017** (2017) 77]

  ➡ renormalization through $\overline{\mathrm{MS}}$, process-dependent scheme, …

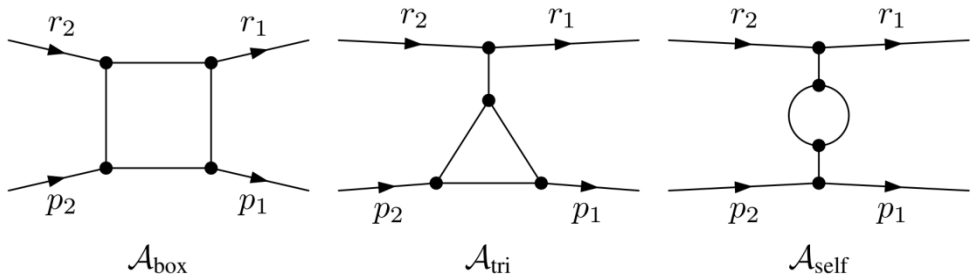
Renormalization: Scalar Mixing Angles (V)

- gauge-independent **"physical OS approach"**: use S matrix elements through a process [A. Denner, S. Dittmaier, J.-N. Lang, *JHEP* **2018** (2018) 104]
- idea: introduce **two right-handed fermion singlets** ν_{iR} with additional $\mathbb{Z}_2$ symmetries to prevent generation mixing
  ➡ massive neutrinos with Yukawa couplings y_{ν_i}
- renormalization of $\delta\alpha$ and $\delta\beta$ through requirement that ratios of tree-level and one-loop S matrix elements are equivalent, *e.g.*:

$$\frac{\mathcal{A}_1^{H\nu_i\nu_i}}{\mathcal{A}_1^{h\nu_i\nu_i}} \equiv \frac{\mathcal{A}_0^{H\nu_i\nu_i}}{\mathcal{A}_0^{h\nu_i\nu_i}} \quad (i = 1, 2)$$

- after renormalization: **recover the 2HDM** by decoupling the singlets
- properties of the "physical OS approach":
  - CTs are defined purely through gauge-independent S matrix elements
    ➡ manifestly **gauge-independent** per construction
  - **numerically stable** (depending on the point in parameter space)

Pinch Technique: Introduction (I)



$$s = (r_1 + p_1)^2 = (r_2 + p_2)^2$$
$$t = (r_1 - r_2)^2 = (p_1 - p_2)^2$$

- we consider a **fermion scattering process** at one-loop QCD:

$$\mathcal{A}_{\text{full}}(s,t,m_1,m_2) = \mathcal{A}_{\text{box}}(s,t,m_1,m_2;\xi) + \mathcal{A}_{\text{tri}}(t,m_1,m_2;\xi) + \boxed{\mathcal{A}_{\text{self}}(t;\xi)}$$

- the gauge dependences **have to cancel** within the individual topologies
  - ➔ rearrangement of the contributions is **always possible**
  - ➔ rearrangement shows that **all** gauge dependences have **self-energy-like** or triangle-like form

$$\mathcal{A}_{\text{full}}(s,t,m_1,m_2) = \tilde{\mathcal{A}}_{\text{box}}(s,t,m_1,m_2) + \tilde{\mathcal{A}}_{\text{tri}}(t,m_1,m_2) + \tilde{\mathcal{A}}_{\text{self}}(t) \quad ,$$

$$\mathcal{A}_{\text{tri}}(t,m_1,m_2;\xi) \;\rightarrow\; \tilde{\mathcal{A}}_{\text{tri}}(t,m_1,m_2) + f_{\text{self}}(t;\xi) \quad , \quad \text{etc.}$$

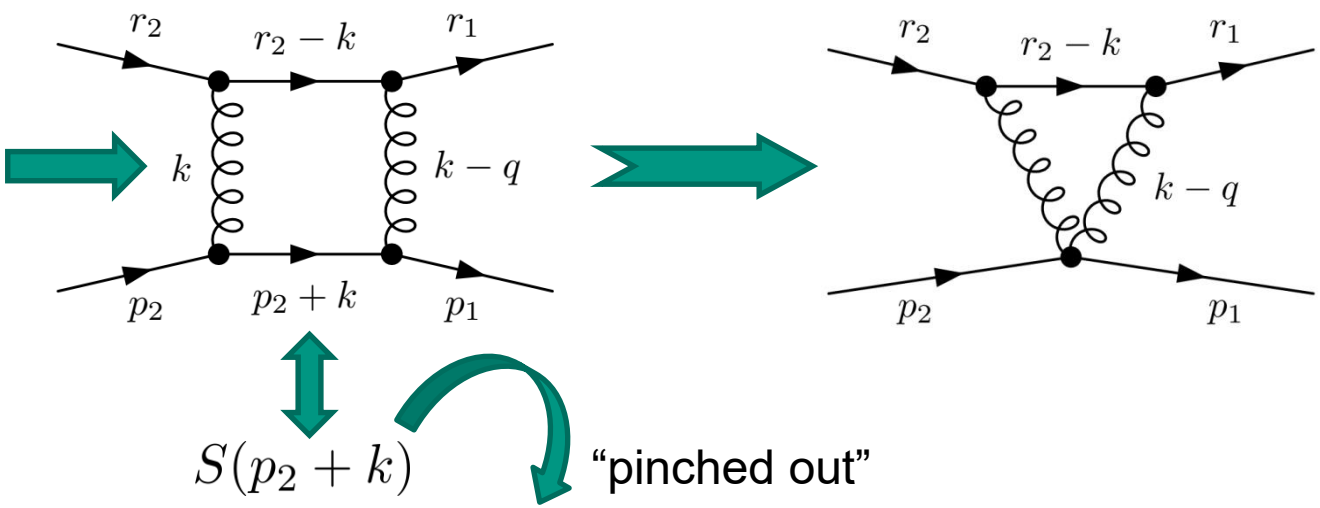
Pinch Technique: Introduction (II)

- determination of the gauge-dependent contributions: "**pinching**"
- main idea: trigger the **elementary Ward identity** for the loop momentum

$$\not{k} = (\not{k} + \not{p} - m) - (\not{p} - m) = \underbrace{S^{-1}(k+p) - S^{-1}(p)}_{\text{inverse fermion propagators}}$$

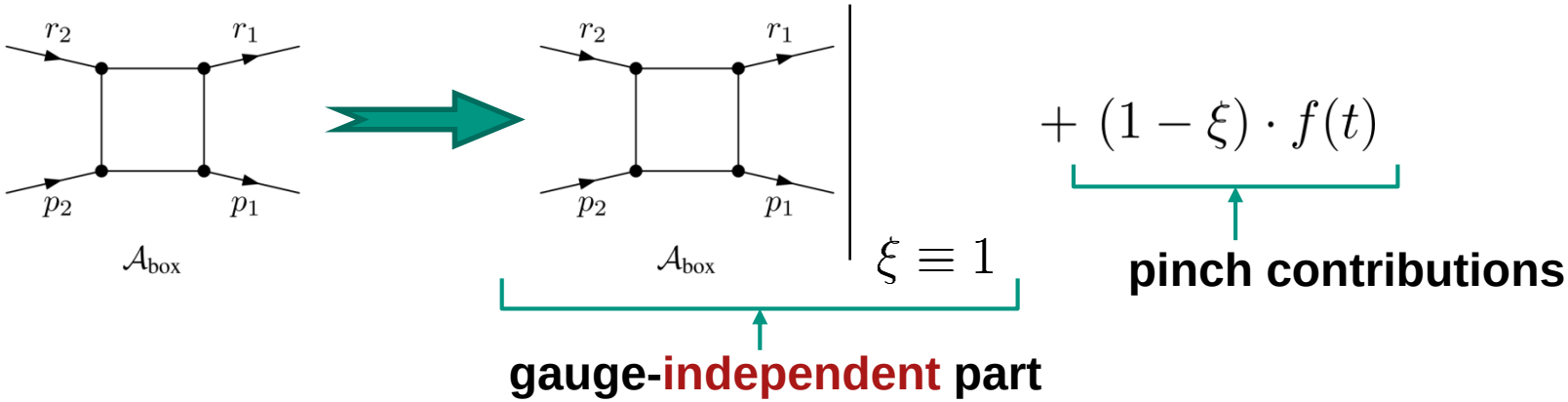
- right expression: vanishes OS between spinors
- left expression: **cancels** ("pinches out") an **internal fermion** propagator

$$\not{k} \equiv S^{-1}(p_2 + k) - S^{-1}(p_2)$$



Pinch Technique: Results (I)

- (almost) all pinch contributions are **proportional** to $(1-\xi)$
- the non-pinched contributions are **equivalent** to diagrams calculated in **Feynman-'t Hooft gauge**, *i.e.* for $\xi \equiv 1$



- the **pinch contributions** are **self-energy like**, *i.e.* functions of only t
  ➔ **reallocation** of pinch contributions to the **gluon self-energy** possible

Pinch Technique: Results (II)

- sum of all pinch contributions ➔ **cancelation of gauge dependences**

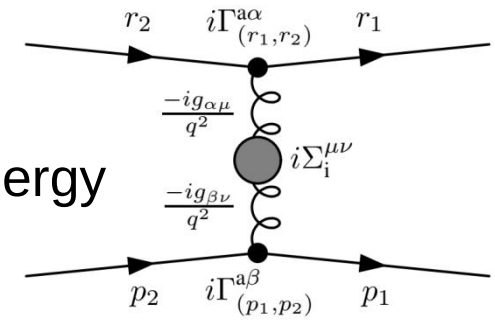
	$g_s^2 t(1-\xi)^2 \int_k \frac{k^\mu k^\nu}{k^4(k+q)^4}$	$g_s^2 t(1-\xi) \int_k \frac{k^\mu k^\nu}{k^4(k+q)^2}$	$g_s^2 t(1-\xi) \int_k \frac{g^{\mu\nu}}{k^2(k+q)^4}$	$g_s^2 t(1-\xi) \int_k \frac{g^{\mu\nu}}{k^4}$
$i\Sigma^{\mu\nu}_{\text{box}}$	$t\frac{C_A}{2}$	0	$-tC_A$	0
$i\Sigma^{\mu\nu}_{\text{tri1}}$	0	0	0	$C_A - 2C_f$
$i\Sigma^{\mu\nu}_{\text{tri2}}$	$-tC_A$	$2C_A$	$2tC_A$	$-2C_A$
$i\Sigma^{\mu\nu}_{\text{self,q}}$	0	0	0	$2C_f$
$i\Sigma^{\mu\nu}_{\text{self,g}}$	$t\frac{C_A}{2}$	$-2C_A$	$-tC_A$	C_A
Sum	0	0	0	0

$(q^2 \equiv t)$

C_A, C_f : Casimir operators

- **main results** from the application of the pinch technique:
  - demonstration of **intricate cancelation** of gauge dependences
  - cancelation is **not accidental**, but follows from **Ward identities**

Gauge-Independent Self-Energies via PT



- all pinch contributions are self-energy-like
  ➔ **reallocate** pinch contributions to the gluon self-energy

- the pinched self-energy is equivalent to the one evaluated for $\xi \equiv 1$ after the cancelation of all gauge dependences
  ➔ Feynman-‘t Hooft-gauge is a **special gauge choice**

- **interesting properties** of the pinched gluon self-energy:
  - analogy to the gluon self-energy given by the **Background Field Method**
  - **uniquely defined** by the pinch technique framework
  - manifestly **gauge-independent** ➔ allows for gauge-independent **counterterms**
  - obeys **QED-like Ward identities** instead of complicated Slavnov-Taylor identities

[for more details cf. *e.g.* D. Binosi, J. Papavassiliou, Phys. Rep. **479** (2009) 1]

Applications of the Pinch Technique

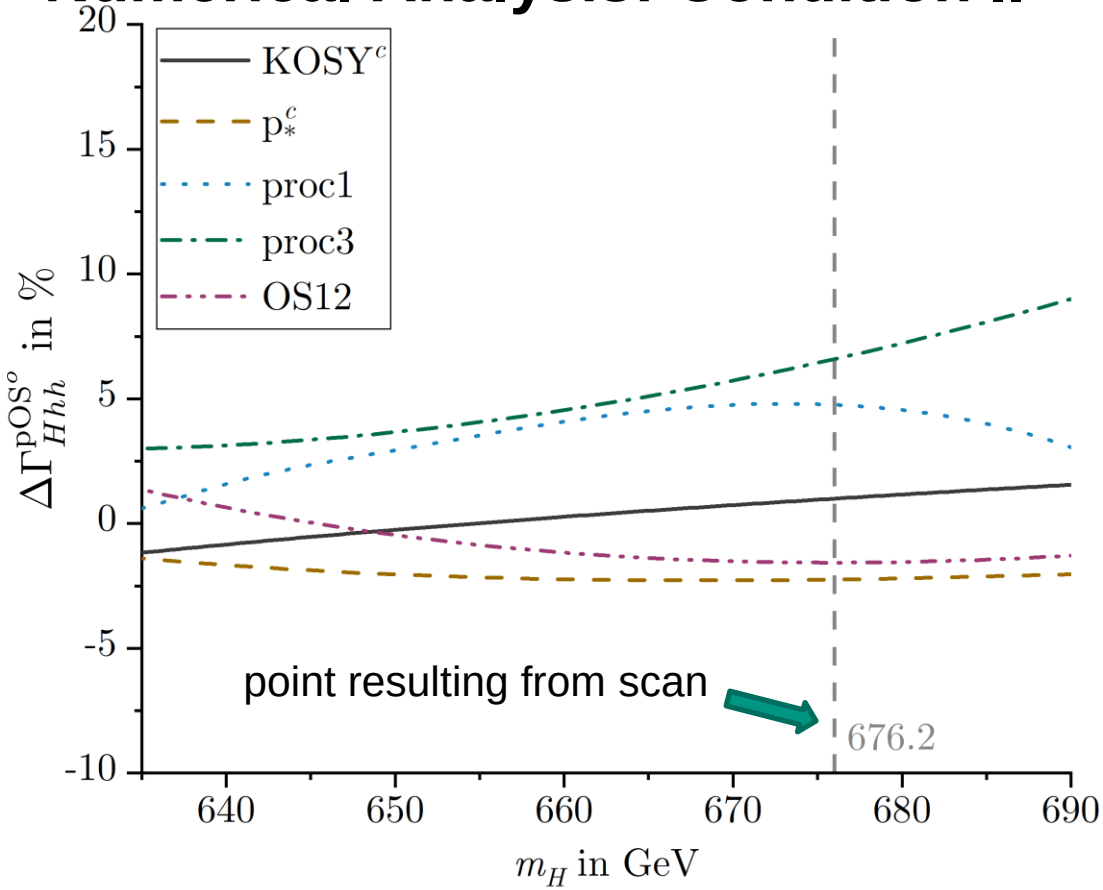
- the pinch technique can be applied to *e.g.* the SM, MSSM, **(N)2HDM**, …

- for consistency: **tadpole diagrams** have to be taken into account
  ➔ "**alternative tadpole scheme**" is **needed**

- applications of the pinched self-energies:
  - definition of **gauge-independent counterterms**
  - general analysis of gauge dependence cancelations [D. Binosi, J. Papavassiliou, Phys. Rev. **D65** (2002) 085003]
  - generalization to **all orders** [D. Binosi, J. Phys. **G30** (2004) 1021]
  - construction of **QED-like Ward identities** for *e.g.* QCD
  - gauge-independent definition of **electroweak parameters**
  - consistent resummation for resonant transition amplitudes
  - extraction of gauge-independent part of **BFM** self-energies

  [D. Binosi, J. Papavassiliou, Phys. Rep. **479** (2009) 1; J. Papavassiliou, Phys. Rev. **D50** (1994) 5958]

Numerical Analysis: Condition II



Condition II: $m_H \geq 2m_h$ **and**

$$m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin\beta\cos\beta}} \pm 5\,\%$$

and correct-sign only; vary m_H

- KOSY^c: Kanemura *et al.*
- p^c_* : "p*-pinched"
- $\mathrm{proc1/3}$: proc.-dep.1/3
- $\mathrm{OS12}$: physical OS scheme O12

relative difference between pOS^o **and the other schemes:**

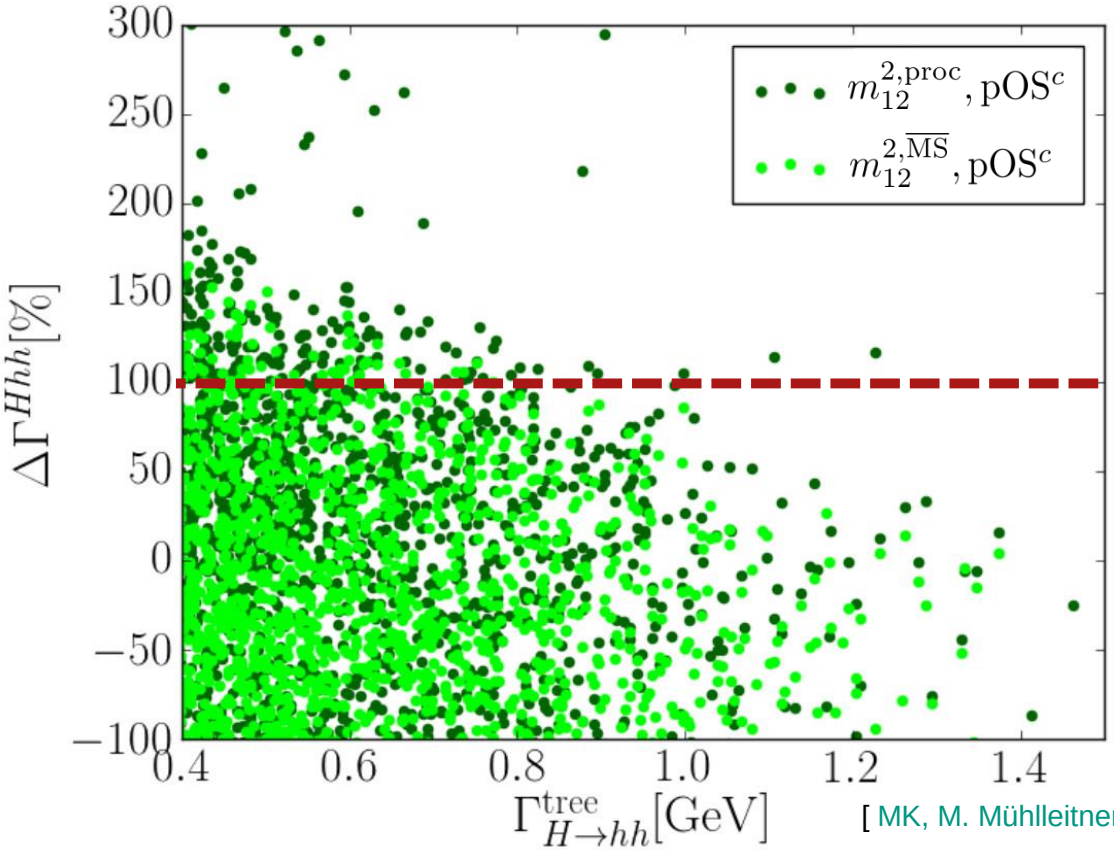
$$\Delta\Gamma^{\mathrm{pOS}^o} \equiv \frac{\Gamma^x_{\mathrm{NLO}} - \Gamma^{\mathrm{pOS}^o}_{\mathrm{NLO}}}{\Gamma^{\mathrm{pOS}^o}_{\mathrm{NLO}}}$$

⇩ **https://github.com/marcel-krause/2HDECAY**

- parameters are **converted** from **reference scheme** pOS^o **to all others**
- relative difference over large range of m_H between **-2% and 6%**

➡ **moderate uncertainty** for considered parameter point and decay

Numerical Analysis: Condition III



Condition III: $m_H \geq 2m_h$ **and** $m_H \geq 2m_A$

- pOS^c : "on-shell pinched"
- $\overline{\mathrm{MS}}$: m_{12}^2 ren. via $\overline{\mathrm{MS}}$
- proc: m_{12}^2 ren. via $H \to A\,A$

relative size of NLO corrections:

$$\Delta\Gamma \equiv \frac{\Gamma_{\mathrm{NLO}} - \Gamma_{\mathrm{LO}}}{\Gamma_{\mathrm{LO}}}$$

[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

- **no decoupling regime** due to additional OS condition $m_H \geq 2m_A$
- large NLO corrections for both the $\overline{\mathrm{MS}}$ and proc.-dep. scheme for m_{12}^2

➡ both **numerical instability** **and** **strong coupling** at work