(N)2HDM - Renormalization, Enhanced Higgs Decays and All That

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Outline of the Talk

- Motivation
- Principle of Gauge Invariance
- Introduction to the (N)2HDM
  - Decoupling and Strong Coupling Regimes
- Automated One-Loop Calculations with (ewN)2HDECAY
- Renormalization of the (N)2HDM
  - Standard and Alternative Tadpole Scheme
  - Mixing Angle Renormalization
- Numerical Results
Motivation: 2HDM and Next-to-Minimal-2HDM

- the Standard Model (SM) has theoretical shortcomings and does not provide explanations for all phenomena observed in nature
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**motivations** for studying the 2HDM and N2HDM:

- simple extensions of the SM
- no constraints due to SUSY relations (cf. MSSM and NMSSM)
- can provide a dark matter candidate ("dark sectors" of the (N)2HDM)
- additional sources of CP violation (*complex (N)2HDM*)
- extended scalar sector
  - interesting phenomenology
  - Higgs-to-Higgs (cascade) decays as interesting signatures
Motivation: Electroweak One-Loop Corrections (I)

- predictions for branching ratios in the (N)2HDM to highest precision:
  - indirect search for new physics through the Higgs sector
  - distinguish models in case of discovery of new physics
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- **state-of-the-art** code for BRs of Higgs decays in the (N)2HDM: 
  **HDECAY** and **N2HDECAY** (based on **HDECAY**), containing:
  - off-shell decay modes for final-state massive vector bosons / heavy quarks
  - loop-induced decays into final-state gluon and photon pairs and $Z\gamma$
  - state-of-the-art QCD corrections, where applicable


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  search for a suitable renormalization scheme of the mixing angles
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  ➡️ Search for a suitable renormalization scheme of the mixing angles

- Investigation of the electroweak one-loop corrections:
  - Size and relevance of the electroweak corrections
  - Renormalization scheme dependence of the electroweak corrections
    ➡️ Estimate of theoretical uncertainty due to missing higher orders
  - Size of the electroweak corrections relative to the decay width at tree level
    ➡️ “Numerical stability” of renormalization schemes
Motivation: Gauge Parameter Independence

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- gauge theories imply the need for gauge-fixing *e.g.* via general $R_\xi$ gauges necessary for removal of redundant degrees of freedom

- the class of $R_\xi$ gauges form an **equivalence class** of the gauge theory equations of motions, observables, ... **must not depend** on $\xi$
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- the class of $R_\xi$ gauges form an equivalence class of the gauge theory equations of motions, observables, ... must not depend on $\xi$
- higher-order calculations: cancelation of gauge dependences becomes very intricate
Cancelation of Gauge Dependences (I)

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- at higher orders, the cancelation becomes **very intricate**
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- **sources** of **gauge dependences** at higher orders:
  - genuine **loop corrections**
  - external leg corrections (**wave-function renormalization constants**)
  - explicit **tadpole** contributions (**proper** treatment of the vacuum state)
  - **counterterms** of all independent parameters
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  - **counterterms** of all independent parameters

- **with a proper** vacuum treatment: all gauge dependences stemming from genuine loop corrections **fully cancel** against external leg corrections

  - counterterms are necessarily **gauge-independent**
  - **simplification** of the book-keeping of gauge dependences in a higher-order calculation

Cancelation of Gauge Dependences (III)

- **without** a proper vacuum treatment: gauge dependences need to be consistently **included** in counterterms to ensure an overall cancelation

  counterterms become **gauge-dependent**
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- **possible violation** of the cancelation of gauge dependences: renormalization conditions for **mixing matrices** and **mixing angles**

- SM: CKM matrix **solved**


- (N)2HDM: **scalar mixing angles** → ?
Introduction to the N2HDM: Potential

- **two** complex $SU(2)_L$ Higgs **doublets** and one real gauge **singlet**:

  \[
  \Phi_1 = \left( \frac{\omega_1^+}{\sqrt{2} (v_1 + \rho_1 + i\eta_1)} \right), \quad \Phi_2 = \left( \frac{\omega_2^+}{\sqrt{2} (v_2 + \rho_2 + i\eta_2)} \right), \quad \Phi_s = v_s + \rho_s
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- **non-vanishing vacuum expectation values (VEVs)** $v_1, v_2, v_s$ with

  \[
  v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2
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- **scalar Lagrangian with CP- and $\mathbb{Z}_2$-conserving** N2HDM potential:

\[
V_{\text{N2HDM}} = m_{11}^2 \left( \Phi_1^\dagger \Phi_1 \right) + m_{22}^2 \left( \Phi_2^\dagger \Phi_2 \right) - m_{12}^2 \left[ \left( \Phi_1^\dagger \Phi_2 \right) + \left( \Phi_2^\dagger \Phi_1 \right) \right] \\
+ \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right) \\
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- **twelve** real-valued potential parameters:
  - dimensionless $\lambda_i$ ($i = 1, \ldots, 8$)
  - squared mass parameters $m_{11}^2, m_{22}^2, m_s^2$ and $m_{12}^2$

- difference w.r.t. NMSSM: constants **not fixed through SUSY relations**
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- transformation to the Higgs mass basis via **scalar mixing angles**
  - $\alpha_1$, $\alpha_2$, $\alpha_3$ for the CP-even sector
  - $\beta$ for the CP-odd and charged sectors

  physical Higgs bosons and Goldstones: $(H_1, H_2, H_3, G^0, A, G^\pm, H^\pm)$
  - CP-even
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- **2HDM limit**: $\alpha_1 \to \alpha + \frac{\pi}{2}$, $\alpha_2 \to 0$, $\alpha_3 \to 0$, $v_s \to \infty$
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- **eight** real-valued potential parameters:
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- difference w.r.t. MSSM: constants **not fixed through SUSY relations**

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masses of **heavier** Higgs bosons $\phi_{\text{heavy}} \in \{H, A, H^\pm\}$ take the form

$$m_{\phi_{\text{heavy}}}^2 \approx c_{\phi_{\text{heavy}}}^2 \frac{m_{12}^2}{\sin \beta \cos \beta} + f(\lambda_i) v^2$$


where $f(\lambda_i)$ is a linear combination of the $\lambda_i$ and

$$c_{\phi_{\text{heavy}}} = \begin{cases} 
1 & \text{for } \phi_{\text{heavy}} \in \{A, H^\pm\} \\
\sin(\beta - \alpha) & \text{for } \phi_{\text{heavy}} = H 
\end{cases}$$
Introduction to the 2HDM: Parameters (II)

- The masses of heavier Higgs bosons $\phi_{\text{heavy}} \in \{H, A, H^\pm\}$ take the form

$$m^2_{\phi_{\text{heavy}}} \approx c^2_{\phi_{\text{heavy}}} \frac{m^2_{12}}{\sin \beta \cos \beta} + f(\lambda_i)v^2$$


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- Two interesting limits in case that $m^2_{\phi_{\text{heavy}}}$ becomes large:

  - **Decoupling**: $\frac{m^2_{12}}{\sin \beta \cos \beta} \gg f(\lambda_i)v^2$ for all heavier Higgs bosons
  - $m^2_{\phi_{\text{heavy}}}$ dominated by large $m^2_{12}/\sin \beta \cos \beta$, independent of the $\lambda_i$
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- **strong coupling:** $\frac{m^2_{12}}{\sin \beta \cos \beta} \lesssim f(\lambda_i)v^2$ for at least one heavier Higgs boson
  - Large $\lambda_i$ required for large $m^2_{\phi_{\text{heavy}}}$
Decoupling Limit of the 2HDM

- **decoupling:** \[ \frac{m_{12}^2}{\sin \beta \cos \beta} \gg f(\lambda_i) v^2 \] for all \( \phi_{\text{heavy}} \in \{H, A, H^\pm\} \)

- \( m_{\phi_{\text{heavy}}}^2 \) dominated by large \( m_{12}^2 / \sin \beta \cos \beta \), independent of the \( \lambda_i \)

- \( \lambda_i \) are small while the \( m_{\phi_{\text{heavy}}}^2 \) are still large

- **trilinear** and **quartic** Higgs couplings can become small
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- **decoupling theorem**: loop effects due to heavy Higgs bosons vanish in the limit \( m_{\phi_{\text{heavy}}} \to \infty \)


- reflects the **decoupling limit in the MSSM** where the Higgs couplings are given in terms of gauge couplings \( g \) and \( g' \) due to SUSY relations
Wrong-Sign Limit of the 2HDM

- even with large \( \frac{m_{12}^2}{\sin \beta \cos \beta} \gg f(\lambda_i)v^2 \), decoupling is not always guaranteed
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- even with large \(\frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2\), decoupling is **not always guaranteed**

- **wrong-sign limit** of the type II (and flipped) 2HDM:
  - relative minus sign of the down-type Yukawa couplings compared to the up-type and Higgs-vector-vector couplings
  - reached for \(\sin(\alpha + \beta) \to 1\)
  - large \(\tan\beta\) required in order to simultaneously achieve the SM limit

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  - relative minus sign of the down-type Yukawa couplings compared to the up-type and Higgs-vector-vector couplings
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- in the wrong-sign regime of the 2HDM, decoupling is strongly disfavored and strong coupling easily arises


  for the analyses, distinguish wrong-sign and correct-sign regimes within the “decoupling limit”
Decoupling and Correct-/Wrong-Sign Limit

- consider e.g. the ratio $\frac{\lambda_{HHh}}{m_H^2}$ appearing in the NLO corrections

- apply the SM limit $\sin(\beta - \alpha) \to 1$ and the decoupling limit

$$m_H^2 \approx \frac{m_{12}^2}{\sin \beta \cos \beta} \gg f(\lambda_i) v^2 \quad \text{and} \quad m_H^2 \gg m_h^2$$
Decoupling and Correct-/Wrong-Sign Limit

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- in these limits, we find:

\[
\frac{\lambda_{HHh}}{m_H^2} = -\frac{1}{m_H^2 v} \frac{\sin(\beta - \alpha)}{\sin(2\beta)} \left[ \sin(2\alpha) \left( 2m_H^2 + m_h^2 \right) - \frac{m_{12}^2}{\sin \beta \cos \beta} \left( 3 \sin(2\alpha) + \sin(2\beta) \right) \right]
\]

\[
\approx 0 \quad \text{(correct-sign limit, } \sin(\beta - \alpha) \to 1) \\
\approx \frac{2}{v} \quad \text{(wrong-sign limit, } \sin(\beta - \alpha) \to 1, \sin(\alpha + \beta) \to 1, \tan \beta \gg 1) 
\]

\[ \text{[ MK, M. Mühleitner, R. Santos, H. Ziesche, } \text{Phys. Rev. D} \text{ 95 (2017) 075019] } \]

- **decoupling** in the correct-sign regime (decoupling theorem)

- **no decoupling** in the wrong-sign regime (non-decoupling effects)
Strong Coupling Limit of the 2HDM

- **strong coupling:** \( \frac{m_{12}^2}{\sin \beta \cos \beta} \lesssim f(\lambda_i) v^2 \) for at least one \( \phi_{\text{heavy}} \in \{H, A, H^\pm\} \)

  - large \( \lambda_i \) required for large \( m_{\phi_{\text{heavy}}}^2 \)

- **trilinear and quartic** Higgs couplings **become large**
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- **trilinear** and **quartic** Higgs couplings become large

- decoupling theorem **does not apply**: loop effects due to heavy Higgs bosons do not vanish in the limit \( m_{\phi_{\text{heavy}}} \to \infty \)

- **reason**: radiative corrections due to heavy Higgs bosons develop a power-law-like behavior in \( m_{\phi_{\text{heavy}}} \)

- **large NLO corrections** due to **non-decoupling effects**

  - for \( H \to h h \): corrections grow with \( m_{\phi_{\text{heavy}}}^4 \)

Electroweak Corrections at One-Loop Level (I)

- **aim:** calculate all (N)2HDM Higgs decays at one-loop (electroweak)
Electroweak Corrections at One-Loop Level (I)

- **aim:** calculate all (N)2HDM Higgs decays at one-loop (electroweak)
- **method:** full diagrammatic calculation

![Diagram of electroweak processes](image)

Model file

(N)2HDM „electroweak Lagrangian“

Processes, e.g.

FeynArts

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\[ \langle f \mid S \mid i \rangle \]

\[ \Gamma \rightarrow |A|^2 \]

Model file

(N)2HDM "electroweak Lagrangian"

FeynArts

Processes, *e.g.*, \( H \rightarrow t \rightarrow H^\pm \rightarrow b \)

Feynman rules

Amplitudes

\[ A = \langle f \mid S \mid i \rangle \]

Analytically

FeynCalc

Decay Widths

\[ \Gamma \rightarrow |A|^2 \]

at one-loop

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- **method**: full diagrammatic calculation

**(N)2HDM „electroweak Lagrangian“**

- Model file
- FeynArts

**(N2)HDECAY**

- Processes, *e.g.*
- Feynman rules
- Amplitudes
  \[ A = \langle f | S | i \rangle \]
- Analytically
  - FeynCalc

**(ewN)2HDECAY**

- Full result
- LoopTools
- Python

**(N)2HDECAY**

- QCD, off-shell, loop-induced
- Decay Widths
  \[ \Gamma \rightarrow |A|^2 \]
- at one-loop

- FORTRAN

Electroweak Corrections at One-Loop Level (II)

contributing topologies at one-loop level:
Electroweak Corrections at One-Loop Level (II)

- contributing topologies at one-loop level:

- 2HDM decay channels that are considered (OS, non-loop-induced):
  - $h/H/A \rightarrow f \bar{f} \ (f = c, s, t, b, \mu, \tau)$
  - $h/H \rightarrow VV \ (V = W^\pm, Z)$
  - $h/H \rightarrow VS \ (V = Z, W^\pm, S = A, H^\pm)$
  - $H^\pm \rightarrow f \bar{f} \ (f = c, t, \nu_\mu, \nu_\tau, \bar{f} = \bar{s}, \bar{b}, \mu^+, \tau^+)$
  - $h/H \rightarrow SS \ (S = A, H^\pm)$
  - $H^\pm \rightarrow VS \ (V = W^\pm, S = h, H, A)$
  - $A \rightarrow VS \ (V = Z, W^\pm, S = h, H, H^\pm)$
  - $H \rightarrow hh$
Electroweak Corrections at One-Loop Level (II)

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- 2HDM decay channels that are considered (OS, non-loop-induced):
  - $h/H/A \rightarrow f \bar{f}$ ($f = c, s, t, b, \mu, \tau$)
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  - $A \rightarrow VS$ ($V = Z, W^\pm$, $S = h, H, H^\pm$)
  - $H \rightarrow hh$

- semi-automated calculation of the decays with 2HDMCalc
  - https://github.com/marcel-krause/2HDMCalc
Electroweak Corrections at One-Loop Level (III)

- contributing topologies at one-loop level:

- N2HDM decay channels that are considered (OS, non-loop-induced):
  - $H_i/A \to f \bar{f} \quad (f \bar{f} = s \bar{s}, c \bar{c}, b \bar{b}, t \bar{t}, \mu^- \mu^+, \tau^- \tau^+)$
  - $A \to V S \quad (V S = Z H_i, W^\pm H^\mp)$
  - $H_i \to V V \quad (V V = W^\pm W^\mp, Z Z)$
  - $H_i \to V S \quad (V S = Z A, W^\pm H^\mp)$
  - $H_i \to S S \quad (S S = A A, H^\pm H^\mp)$
  - $H^\pm \to V S \quad (V S = W^\pm H_i, W^\pm A)$
  - $H^\pm \to f \bar{f} \quad (f = u \bar{s}, u \bar{b}, c \bar{d}, c \bar{s}, c \bar{b}, t \bar{d}, t \bar{s}, t \bar{b}, \nu_\mu \mu^+, \nu_\tau \tau^+)$
Electroweak Corrections at One-Loop Level (III)

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- N2HDM decay channels that are considered (OS, non-loop-induced):
  - $H_i/A \rightarrow f \bar{f}$ ($f \bar{f} = s \bar{s}, c \bar{c}, b \bar{b}, t \bar{t}, \mu^-, \mu^+, \tau^- \tau^+$)
  - $A \rightarrow V S$ ($V S = Z H_i, W^\pm H^\mp$)
  - $H_2 \rightarrow H_1 H_1$
  - $H_3 \rightarrow H_1 H_1$
  - $H_i \rightarrow V V$ ($V V = W^\pm W^\mp, Z Z$)
  - $H_2 \rightarrow H_1 H_1$
  - $H_3 \rightarrow H_1 H_2$
  - $H_i \rightarrow V S$ ($V S = Z A, W^\pm H^\mp$)
  - $H_3 \rightarrow H_1 H_2$
  - $H_i \rightarrow S S$ ($S S = A A, H^\pm H^\mp$)
  - $H_3 \rightarrow H_2 H_2$
  - $H^\pm \rightarrow V S$ ($V S = W^\pm H_i, W^\pm A$)
  - $H^\pm \rightarrow f \bar{f}$ ($f = u \bar{u}, u \bar{b}, c \bar{d}, c \bar{s}, c \bar{b}, t \bar{d}, t \bar{s}, t \bar{b}, \nu_\mu, \mu^+, \nu_\tau, \tau^+$)

semi-automated calculation of the decays with N2HDMCalc

https://github.com/marcel-krause/N2HDMCalc
Electroweak Corrections at One-Loop Level (IV)

- many diagrams contain **UV divergences**, *i.e.* formally, we have

\[ l^2 \rightarrow \infty \]
Electroweak Corrections at One-Loop Level (IV)

- many diagrams contain **UV divergences**, *i.e.* formally, we have

  \[ l^2 \to \infty \quad \Rightarrow \quad \infty \]

- use **dimensional regularization** \((d = 4 - 2\epsilon)\) to isolate the divergences:

  \[ d = 4 - 2\epsilon \quad \Rightarrow \quad \frac{1}{\epsilon} + \text{finite} \]
many diagrams contain **UV divergences**, \(i.e.\) formally, we have

\[ l^2 \to \infty \to \infty \]

use **dimensional regularization** \((d = 4 - 2\epsilon)\) to isolate the divergences:

\[ d = 4 - 2\epsilon \to \frac{1}{\epsilon} + \text{finite} \]

consistently remove the divergences via **renormalization**

idea: split ‘bare’ parameters into **renormalized** values and **counterterms**

\[ m_i^2 \to m_i^2 + \delta m_i^2 \]

counterterms need to be fixed via **renormalization conditions**
Renormalization of the (N)2HDM

- set of free parameters of the 2HDM (excluding CKM elements, …)
  \[ \{ T_h, T_H, \alpha_{\text{em}}, m_W, m_Z, m_f, m_h, m_H, m_A, m_{H\pm}, \alpha, \beta, m_{12}^2, \cdots \} \]

- set of free parameters of the N2HDM (excluding CKM elements, …)
  \[ \{ T_{H1}, T_{H2}, T_{H3}, \alpha_{\text{em}}, m_W, m_Z, m_f, m_{H1}, m_{H2}, m_{H3}, m_A, m_{H\pm}, \alpha_1, \alpha_2, \alpha_3, \beta, m_{12}^2, v_s, \cdots \} \]
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Renormalization of the (N)2HDM

- set of free parameters of the 2HDM (excluding CKM elements, …)
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- renormalization program for the (N)2HDM:
  - tadpole terms \( \rightarrow \) standard / alternative tadpole scheme
  - mass counterterms \( \rightarrow \) on-shell
  - fine-structure constant \( \rightarrow \) at Z mass
  - soft-\( \mathbb{Z}_2 \)-breaking scale \( m_{12}^2 \) \( \rightarrow \) \( \overline{\text{MS}} \)
  - singlet VEV \( v_s \) \( \rightarrow \) \( \overline{\text{MS}} \)
  - scalar mixing angles \( \rightarrow \) ?

[ MK, Master’s thesis (2016), KIT;
Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:

\[ iT_{1/2} - i\delta T_{1/2} = 0 \quad \iff \quad iT_{H/\hbar} - i\delta T_{H/\hbar} = 0 \]

- conversion from gauge to \textbf{mass basis}:

\[
\begin{pmatrix}
\delta T_1 \\
\delta T_2
\end{pmatrix} =
\begin{pmatrix}
c_\alpha & -s_\alpha \\
s_\alpha & c_\alpha
\end{pmatrix}
\begin{pmatrix}
\delta T_H
\end{pmatrix} =
\begin{pmatrix}
c_\alpha \delta T_H - s_\alpha \delta T_h \\
s_\alpha \delta T_H + c_\alpha \delta T_h
\end{pmatrix}
\]
Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:

\[
\begin{align*}
T_{1/2} & - \delta T_{1/2} = 0 \\
\quad & \quad \\
\Leftrightarrow \quad & \\
T_{H/2} & - \delta T_{H/2} = 0
\end{align*}
\]

- conversion from gauge to \textbf{mass basis}:

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s_\alpha & c_\alpha
\end{pmatrix}
\begin{pmatrix}
\delta T_H
\end{pmatrix} =
\begin{pmatrix}
c_\alpha \delta T_H - s_\alpha \delta T_h \\
s_\alpha \delta T_H + c_\alpha \delta T_h
\end{pmatrix}
\]

- \textbf{purpose}: restoring the minimum conditions of the potential at NLO

- \textbf{textbook explanation}: \textbf{no tadpole diagrams} in NLO calculations

\[\rightarrow \text{ really?}\]
Renormalization: Standard Tadpole Scheme

- **standard scheme**: VEVs are derived from the loop-corrected potential.

- VEVs in the mass relations produce correct one-loop OS masses, *e.g.*

\[
m_W^2 = g^2 \frac{v^2}{4}, \quad m_A^2 = \frac{m_{12}^2}{\sin \beta \cos \beta} - \lambda_5 v^2
\]
Renormalization: Standard Tadpole Scheme

- **standard scheme**: VEVs are derived from the **loop-corrected** potential

- VEVs in the mass relations produce correct one-loop OS masses, *e.g.*
  \[ m_W^2 = g^2 \frac{v^2}{4}, \quad m_A^2 = \frac{m_{12}^2}{\sin \beta \cos \beta} - \lambda_5 v^2 \]

- tadpole terms **appear explicitly** in the bare mass matrices

  Mass matrix counterterms **contain** the **tadpole counterterms**:

  \[
  \delta D^2_\phi \approx \begin{pmatrix}
  \delta m^2_{\phi_1} & 0 \\
  0 & \delta m^2_{\phi_2}
  \end{pmatrix} + \begin{pmatrix}
  \delta T_{\phi_1\phi_1} & \delta T_{\phi_1\phi_2} \\
  \delta T_{\phi_1\phi_2} & \delta T_{\phi_2\phi_2}
  \end{pmatrix}
  \]
Renormalization: Standard Tadpole Scheme

- **standard scheme**: VEVs are derived from the *loop-corrected* potential

- VEVs in the mass relations produce correct one-loop OS masses, *e.g.*
  \[
  m_W^2 = \frac{g^2 v^2}{4}, \quad m_A^2 = \frac{m_{12}^2}{\sin \beta \cos \beta} - \lambda_5 v^2
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- Tadpole terms **appear explicitly** in the bare mass matrices
  - Mass matrix counterterms **contain** the tadpole counterterms:
  \[
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  \delta m_{\phi_1}^2 & 0 \\
  0 & \delta m_{\phi_2}^2
  \end{pmatrix} + \begin{pmatrix}
  \delta T_{\phi_\ell\phi_\ell} & \delta T_{\phi_1\phi_2} \\
  \delta T_{\phi_1\phi_2} & \delta T_{\phi_2\phi_2}
  \end{pmatrix}
  \]

- One-loop corrected potential is **gauge-dependent**
  - VEVs are gauge-dependent
  - Mass counterterms become **gauge-dependent**
alternative scheme: VEVs represent the same minimum as at tree level

Renormalization: Alternative Tadpole Scheme (I)

- **alternative scheme**: VEVs represent the same minimum as at tree level

- bare masses are expressed through gauge-independent tree-level VEVs
  mass CTs become **gauge-independent**
Renormalization: Alternative Tadpole Scheme (I)

- **alternative scheme**: VEVs represent the same minimum as at **tree level**
  

- bare masses are expressed through gauge-independent **tree-level VEVs**
  
  - **mass CTs become gauge-independent**

- correct minimum conditions at NLO require a **shift in the VEVs**

\[
v_1 \to v_1 + \delta v_1, \quad v_2 \to v_2 + \delta v_2
\]

- fixation of the shifts by **applying the minimum conditions**:

\[
(\delta v_1) = \left( \frac{T_H}{m_H^2} c_\alpha - \frac{T_h}{m_h^2} s_\alpha \right)
\]

\[
(\delta v_2) = \left( \frac{T_H}{m_H^2} s_\alpha + \frac{T_h}{m_h^2} c_\alpha \right)
\]
Renormalization: Alternative Tadpole Scheme (I)

alternative scheme: VEVs represent the same minimum as at tree level

bare masses are expressed through gauge-independent tree-level VEVs
mass CTs become gauge-independent

correct minimum conditions at NLO require a shift in the VEVs
\[ v_1 \rightarrow v_1 + \delta v_1 , \quad v_2 \rightarrow v_2 + \delta v_2 \]

fixation of the shifts by applying the minimum conditions:
\[ \begin{pmatrix} \delta v_1 \\ \delta v_2 \end{pmatrix} = \begin{pmatrix} \frac{T_H}{m_H^2} c_\alpha - \frac{T_h}{m_h^2} s_\alpha \\ \frac{T_H}{m_H^2} s_\alpha + \frac{T_h}{m_h^2} c_\alpha \end{pmatrix} \]

the shifts translate into every CT, wave function renormalization constants and Feynman rules

alternative tadpole scheme worked out for the (N)2HDM at one-loop
Renormalization: Alternative Tadpole Scheme (II)

- example: \textbf{W boson mass}

\[ m_W^2 = g^2 \frac{v^2}{4} \]
Renormalization: Alternative Tadpole Scheme (II)

example: W boson mass

\[ m_W^2 = g^2 \frac{v^2}{4} \rightarrow m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} \]
Renormalization: Alternative Tadpole Scheme (II)

- example: W boson mass

\[ m_W^2 = \frac{g^2 v^2}{4} \rightarrow m_W^2 + g^2 \left( \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} \right) = m_W^2 + i \left( \begin{array}{c} \text{example} \\ \text{example} \end{array} \right) \]
Renormalization: Alternative Tadpole Scheme (II)

- **example: W boson mass**

\[
m_W^2 = g^2 \frac{v^2}{4} \quad \rightarrow \quad m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} = m_W^2 + i \left( \begin{array}{c} W^\pm \\ H \\ W^\pm \end{array} \right) + i \left( \begin{array}{c} W^\pm \\ h \\ W^\pm \end{array} \right)
\]

- **example: coupling between Higgs and Z bosons**

\[
ig_{HZZ} = \frac{ig^2}{2c_W^2} (c_\alpha v_1 + s_\alpha v_2) , \quad ig_{HZZ} = \frac{ig^2}{2c_W^2}
\]

\[
ig_{HZZ} \rightarrow ig_{HZZ} + \frac{ig^2}{2c_W^2} (c_\alpha \delta v_1 + s_\alpha \delta v_2) = ig_{HZZ} + \left( \begin{array}{c} H \\ Z \\ Z \end{array} \right)_{\text{trunc}}
\]
Renormalization: Alternative Tadpole Scheme (II)

- **example: W boson mass**

\[ m_W^2 = g^2 \frac{v^2}{4} \rightarrow m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} = m_W^2 + i \begin{pmatrix} W^+ \ H \ W^- \end{pmatrix} + i \begin{pmatrix} W^+ \ h \ W^- \end{pmatrix} \]

- **example: coupling between Higgs and Z bosons**

\[ i g_{HZZ} = \frac{i g^2}{2 c_w^2} \left( c_\alpha v_1 + s_\alpha v_2 \right) \quad , \quad i g_{HZZ} = \frac{i g^2}{2 c_w^2} \]

\[ i g_{HZZ} \rightarrow i g_{HZZ} + \frac{i g^2}{2 c_w^2} \left( c_\alpha \delta v_1 + s_\alpha \delta v_2 \right) = i g_{HZZ} + \]

- **effects** of the alternative tadpole scheme:
  - tadpole diagrams are added everywhere where they exist in the 2HDM
  - mass counterterms become **manifestly gauge-independent**
  - tadpole counterterms in the scalar sector are **removed**
Renormalization: Scalar Mixing Angles (I)

- renormalization of mixing angles $\alpha$ and $\beta$ is non-trivial in the 2HDM

- simplest approach: $\overline{\text{MS}}$ conditions for $\alpha$ and $\beta$ (alternatively: $\lambda_3$)

  - can be numerically unstable in one-to-two-body decays
  - divergences for degenerate masses / “dead corners” of parameter space

Renormalization: Scalar Mixing Angles (I)

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- simplest approach: $\overline{\text{MS}}$ conditions for $\alpha$ and $\beta$ (alternatively: $\lambda_3$)
  - can be numerically unstable in one-to-two-body decays
  - divergences for degenerate masses / “dead corners” of parameter space

analyze renormalization schemes for the 2HDM w.r.t.
"three desirable criteria":
  - gauge independence
  - process independence
  - numerical stability (i.e. leads to moderate NLO corrections)

Intermezzo: Types of Numerical Instabilities

- measure for the relative size of the NLO corrections:

\[ \Delta \Gamma \equiv \frac{\Gamma^{\text{NLO}} - \Gamma^{\text{LO}}}{\Gamma^{\text{LO}}} \]
Intermezzo: Types of Numerical Instabilities

- measure for the **relative size** of the NLO corrections:

\[
\Delta \Gamma \equiv \frac{\Gamma^{NLO} - \Gamma^{LO}}{\Gamma^{LO}}
\]

- the relative corrections \( \Delta \Gamma \) may **become large**
  - if the LO width becomes **small** such that \( \Delta \Gamma \) becomes very sensitive on \( \Gamma^{NLO} \)
  - if vertex corrections, CTs and/or WFRCs are **parametrically enhanced**
  - due to **unsuitable renormalization schemes** for some parameters
    - *e.g.* \( \overline{MS} \): finite parts of \( \delta \alpha, \delta \beta \) missing for cancellation of large contributions
    - *e.g.* **proc.-dep.**: additional potentially large finite parts included in \( \delta \alpha, \delta \beta, \delta m_{12}^2 \)
Intermezzo: Types of Numerical Instabilities

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\[ \Delta \Gamma \equiv \frac{\Gamma^{\text{NLO}} - \Gamma^{\text{LO}}}{\Gamma^{\text{LO}}} \]

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  - e.g. proc.-dep.: additional potentially large finite parts included in \( \delta \alpha, \delta \beta, \delta m_{12}^2 \)

  in this talk: "**numerical instability**" of the renormalization scheme
Intermezzo: Types of Numerical Instabilities

- measure for the **relative size** of the NLO corrections:
  $$\Delta \Gamma \equiv \frac{\Gamma^{\text{NLO}} - \Gamma^{\text{LO}}}{\Gamma^{\text{LO}}}$$

- the relative corrections $\Delta \Gamma$ may **become large**
  - if the LO width becomes **small** such that $\Delta \Gamma$ becomes very sensitive on $\Gamma^{\text{NLO}}$
  - if vertex corrections, CTs and/or WFRCs are **parametrically enhanced**

- due to **unsuitable renormalization schemes** for some parameters
  - *e.g.* $\overline{\text{MS}}$: finite parts of $\delta \alpha, \delta \beta$ missing for cancellation of large contributions
  - *e.g.* proc.-dep.: additional potentially large finite parts included in $\delta \alpha, \delta \beta, \delta m_{12}^2$

  - in this talk: “**numerical instability**” of the renormalization scheme

- in Higgs-to-Higgs decays in the (N)2HDM, $\Delta \Gamma$ may **additionally** become large due to certain **limits in the parameter space**
  - wrong-sign limit, strong coupling limit
Renormalization: Scalar Mixing Angles (II)

- approach by S. Kanemura et al.: connect the definition of $\alpha$ and $\beta$ with the inverse propagator matrix (“KOSY scheme”)

\[
\begin{pmatrix}
\tilde{\phi}_1 \\
\tilde{\phi}_2
\end{pmatrix}
= R_{\theta,0}^T \begin{pmatrix}
\tilde{\phi}_1 \\
\tilde{\phi}_2
\end{pmatrix} 
\approx R_{\delta\theta}^T R_{\theta}^T \sqrt{Z_{\phi}} R_{\theta} R_{\theta}^T \begin{pmatrix}
\tilde{\phi}_1 \\
\tilde{\phi}_2
\end{pmatrix} 
\approx \begin{pmatrix}
1 + \frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2} + \delta \theta \\
\delta C_{\phi_2} - \delta \theta & 1 + \frac{\delta Z_{\phi_2\phi_2}}{2}
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
\]

- mixing angle counterterms within the standard tadpole scheme:

\[
\delta \alpha = \frac{1}{2 (m_H^2 - m_h^2)} \Re \left[ \Sigma_{Hh}(m_H^2) + \Sigma_{Hh}(m_h^2) - 2 \delta T_{Hh} \right]
\]

\[
\delta \beta = -\frac{1}{2 m_{H^\pm}^2} \Re \left[ \Sigma_{G^\pm H^\pm}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}(0) - 2 \delta T_{G^\pm H^\pm} \right]
\]
Renormalization: Scalar Mixing Angles (II)

- Approach by S. Kanemura et al.: connect the definition of $\alpha$ and $\beta$ with the inverse propagator matrix ("KOSY scheme")

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
= R_{\theta,0}^T \begin{pmatrix}
\tilde{\phi}_1 \\
\tilde{\phi}_2
\end{pmatrix}
\approx R_{\delta \theta}^T R_{\theta}^T \sqrt{Z^2_{\phi}} R_{\theta} R_{\theta}^T \begin{pmatrix}
\tilde{\phi}_1 \\
\tilde{\phi}_2
\end{pmatrix}
\approx \begin{pmatrix}
1 + \frac{\delta Z_{\phi_1} \phi_1}{2} & \delta C_{\phi_2} + \delta \theta \\
\delta C_{\phi_2} - \delta \theta & 1 + \frac{\delta Z_{\phi_2} \phi_2}{2}
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
\]

- Mixing angle counterterms within the standard tadpole scheme:

\[
\delta \alpha = \frac{1}{2 (m_H^2 - m_h^2)} \text{Re} \left[ \Sigma_{Hh} (m_H^2) + \Sigma_{Hh} (m_h^2) - 2 \delta T_{Hh} \right]
\]

\[
\delta \beta = -\frac{1}{2 m_{H+}^2} \text{Re} \left[ \Sigma_{G+H+} (m_{H+}^2) + \Sigma_{G+H+} (0) - 2 \delta T_{G+H+} \right]
\]

- The KOSY scheme as described above leads to the inclusion of gauge-dependent contributions in the partial decay widths [MK, Master's thesis (2016), KIT]

Renormalization: Scalar Mixing Angles (III)

- gauge-independent "OS approach": use the \textit{pinch technique} (PT)

- \textbf{PT-based definition} of the scalar mixing angle counterterms: use the pinched scalar self-energies instead of the usual ones

- necessary requirement: use the \textit{alternative tadpole scheme}
Renormalization: Scalar Mixing Angles (III)

- gauge-independent "OS approach": use the pinch technique (PT)

- **PT-based definition** of the scalar mixing angle counterterms:
  use the pinched scalar self-energies instead of the usual ones

- necessary requirement: use the alternative tadpole scheme

- properties of the pinched scheme:
  - **process-independent**, symmetric in the fields
  - manifestly **gauge-independent** per construction
    - gauge-independent NLO amplitudes
  - **numerically stable** (depending on the point in parameter space)
    - proposed solution for renormalizing $\delta\alpha$ and $\delta\beta$ in the 2HDM

Renormalization: Scalar Mixing Angles (IV)

- gauge-independent approach: **process-dependent schemes**
  

- idea: impose the gauge-invariant condition
  
  \[
  \Gamma_{\phi ff}^{\text{LO}} \equiv \Gamma_{\phi ff}^{\text{NLO}}
  \]

  for **different decays** in order to define \( \delta \alpha \) and \( \delta \beta \)
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  for **different decays** in order to define \( \delta \alpha \) and \( \delta \beta \)

- we consider the following combinations for \( \delta \alpha, \delta \beta \):
  - **proc.-dep. 1:** \( A \to \tau \tau \) for \( \delta \beta \) and \( H \to \tau \tau \) for \( \delta \alpha \)
  - **proc.-dep. 2:** \( A \to \tau \tau \) for \( \delta \beta \) and \( h \to \tau \tau \) for \( \delta \alpha \)
  - **proc.-dep. 3:** \( H \to \tau \tau \) and \( h \to \tau \tau \) for both \( \delta \alpha, \delta \beta \)
Renormalization: Scalar Mixing Angles (IV)

- gauge-independent approach: **process-dependent schemes**

- idea: impose the gauge-invariant condition
  \[ \Gamma_{\phi f f}^{LO} \equiv \Gamma_{\phi f f}^{NLO} \]
  for **different decays** in order to define \( \delta \alpha \) and \( \delta \beta \)

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  - **proc.-dep. 3:** \( H \rightarrow \tau\tau \) and \( h \rightarrow \tau\tau \) for both \( \delta \alpha, \delta \beta \)

- properties of process-dependent schemes:
  - **process-dependent** per construction
  - **gauge-independent**
  - potentially **numerically unstable** over large parameter ranges
Renormalization: Scalar Mixing Angles (V)

- generalization to the N2HDM is straightforward
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- the CP-odd and charged mixing angle $\beta$ is analogously renormalized
Renormalization: Scalar Mixing Angles (V)

- generalization to the N2HDM is straightforward
- the CP-odd and charged mixing angle $\beta$ is analogously renormalized
- for the three CP-even mixing angles $\alpha_1, \alpha_2, \alpha_3$, we consider several different schemes:
  - \( \overline{\text{MS}} \) scheme
  - adapted KOSY schemes
  - PT-based schemes
Implementation: 2HDECA Y (I)

2HDM „electroweak Lagrangian“

Model file

FeynArt s

Processes, e.g.

Feynman rules

Amplitudes

FeynArts

Analytically

FeynCalc

2HDECA Y

Full result

LoopTools

Python

HDECA Y

QCD, off-shell, loop-induced

FORTRAN

Decay Widths

\[ \Gamma \rightarrow |A|^2 \]

at one-loop
Implementation: 2HDECAY (I)

**2HDECAY**: “2HDM HDECAY“
A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

[ MK, M. Mühlleitner, M. Spira, Computer Physics Communications 246 (2020) 106852 ]

[https://github.com/marcel-krause/2HDECAY](https://github.com/marcel-krause/2HDECAY)
### Implementation: 2HDECAY (II)

- 17 renormalization schemes are implemented in **2HDECAY**:

<table>
<thead>
<tr>
<th>Input ID</th>
<th>Tadpole scheme</th>
<th>$\delta\alpha$</th>
<th>$\delta\beta$</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>standard</td>
<td>KOSY</td>
<td>KOSY (odd)</td>
<td>KOSY$^o$(std)</td>
</tr>
<tr>
<td>2</td>
<td>standard</td>
<td>KOSY</td>
<td>KOSY (charged)</td>
<td>KOSY$^c$(std)</td>
</tr>
<tr>
<td>3</td>
<td>alternative (FJ)</td>
<td>KOSY</td>
<td>KOSY (odd)</td>
<td>KOSY$^o$</td>
</tr>
<tr>
<td>4</td>
<td>alternative (FJ)</td>
<td>KOSY</td>
<td>KOSY (charged)</td>
<td>KOSY$^c$</td>
</tr>
<tr>
<td>5</td>
<td>alternative (FJ)</td>
<td>$p_\star$-pinched</td>
<td>$p_\star$-pinched (odd)</td>
<td>$p_\star^o$</td>
</tr>
<tr>
<td>6</td>
<td>alternative (FJ)</td>
<td>$p_\star$-pinched</td>
<td>$p_\star$-pinched (charged)</td>
<td>$p_\star^c$</td>
</tr>
<tr>
<td>7</td>
<td>alternative (FJ)</td>
<td>OS-pinched</td>
<td>OS-pinched (odd)</td>
<td>OS$^o$</td>
</tr>
<tr>
<td>8</td>
<td>alternative (FJ)</td>
<td>OS-pinched</td>
<td>OS-pinched (charged)</td>
<td>OS$^c$</td>
</tr>
<tr>
<td>9</td>
<td>alternative (FJ)</td>
<td>proc.-dep. 1</td>
<td>proc.-dep. 1</td>
<td>proc1</td>
</tr>
<tr>
<td>10</td>
<td>alternative (FJ)</td>
<td>proc.-dep. 2</td>
<td>proc.-dep. 2</td>
<td>proc2</td>
</tr>
<tr>
<td>11</td>
<td>alternative (FJ)</td>
<td>proc.-dep. 3</td>
<td>proc.-dep. 3</td>
<td>proc3</td>
</tr>
<tr>
<td>12</td>
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<td>physical OS1</td>
<td>physical OS1</td>
<td>OS1</td>
</tr>
<tr>
<td>13</td>
<td>alternative (FJ)</td>
<td>physical OS2</td>
<td>physical OS2</td>
<td>OS2</td>
</tr>
<tr>
<td>14</td>
<td>alternative (FJ)</td>
<td>physical OS12</td>
<td>physical OS12</td>
<td>OS12</td>
</tr>
<tr>
<td>15</td>
<td>alternative (FJ)</td>
<td>rigid symmetry (BFM)</td>
<td>BFMS</td>
<td>BFMS</td>
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<tr>
<td>16</td>
<td>standard</td>
<td>$\overline{\text{MS}}$</td>
<td>$\overline{\text{MS}}$</td>
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[https://github.com/marcel-krause/2HDECAY](https://github.com/marcel-krause/2HDECAY)
Implementation: ewN2HDECAY (I)

- N2HDM „electroweak Lagrangian“
- Processes, e.g. $H^{-} \rightarrow t\bar{b}$
- Feynman rules
- Amplitudes $\mathcal{A} = \langle f \mid S \mid i \rangle$
- Analytically
- ewN2HDECAY Full result
- N2HDECAY QCD, off-shell, loop-induced
- Decay Widths $\Gamma \rightarrow |\mathcal{A}|^2$
- at one-loop

Model file
FeynArts
Processes, e.g.
FeynArts
Amplitudes
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LoopTools
Python
N2HDECAY
FORTRAN

ewN2HDECAY
QCD, off-shell, loop-induced

21.11.2019 - ITP Research Seminar
M. Krause: (N)2HDM - Renormalization, Extended Higgs Decays and All That
ITP, KIT
Implementation: ewN2HDECAY (I)

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https://github.com/marcel-krause/ewN2HDECAY
10 renormalization schemes are implemented in **ewN2HDECAY**:

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[ ](https://github.com/marcel-krause/ewN2HDECAY)
Numerical Analysis: Input Parameters (I)

- consider ew. one-loop corrections to the 2HDM process $H \to h\, h$
Numerical Analysis: Input Parameters (I)

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- the SM input parameters are fixed; $h$ corresponds to the SM-like Higgs:

  $$m_h = 125.09 \text{ GeV}$$

- exemplarily, we consider a type II 2HDM in the following
Numerical Analysis: Input Parameters (I)

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- the SM input parameters are fixed; $h$ corresponds to the SM-like Higgs:

$$m_h = 125.09 \text{ GeV}$$

- exemplarily, we consider a type II 2HDM in the following

- keep in mind: the 2HDM contains a lot of free parameters:

$$\{m_H, m_A, m_{H^\pm}, m_{12}^2, \tan \beta, \alpha\}$$

  → scanning through the parameter space is possible

- chosen parameter points respect several experimental and theoretical constraints
we consider two separate conditions for the analysis:
we consider **two separate conditions** for the analysis:

- **Condition I**: OS threshold for $H \rightarrow hh$

\[ m_H \geq 2m_h \]

most general case for the OS decay
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- **Condition II**: masses of heavy Higgs bosons are dominated by $m_{12}^2$
  \[ m_H \geq 2m_h \quad \text{and large} \quad m_{\text{heavy}} \approx \frac{m_{12}}{\sqrt{\sin \beta \cos \beta}} \pm 5\% \]
  decoupling possible, since the couplings $\lambda_i$ can be small
**Numerical Analysis: Input Parameters (II)**

- We consider **two separate conditions** for the analysis:
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    **decoupling** possible, since the couplings $\lambda_i$ can be small

- **aim**: distinguish large NLO corrections due to the strong coupling regime from **numerical instability** due to the chosen renormalization scheme
Numerical Analysis: Condition I

**Condition I:** $m_H \geq 2m_h$

- **proc1:** process-dependent 1
- **pOS$^c$:** “on-shell pinched”
- **$p_*^c$:** “$p^*$-pinched”

Relative size of NLO corrections:

$$\Delta \Gamma \equiv \frac{\Gamma_{\text{NLO}}-\Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

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**relative size** of NLO corrections:

$$\Delta \Gamma = \frac{\Gamma_{NLO} - \Gamma_{LO}}{\Gamma_{LO}}$$

- **process-dependent scheme:** typically **huge** NLO corrections
- **pinched schemes:** well-behaving for **large parameter ranges**, but also **large NLO corrections possible** → numerical instability?

Numerical Analysis: Condition II

Condition II: \( m_H \geq 2m_h \) and

\[
m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin \beta \cos \beta}} \pm 5\% \]

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Relative size of NLO corrections:

$$\Delta \Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- process-dependent scheme: still typically huge NLO corrections
- pinched schemes: one well-behaving regime and one regime with large NLO corrections
  - numerical instability or still strong coupling?

Numerical Analysis: Condition II

\[ \Gamma_{\text{tree}}^{H \rightarrow hh}[\text{GeV}] \]

**Condition II:** \( m_H \geq 2m_h \) and

\[ m_{\phi_{\text{heavy}}} \approx \frac{m_{212}}{\sqrt{\sin \beta \cos \beta}} \pm 5\% \]

and wrong-sign only

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relative size of NLO corrections:

$$\Delta \Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- all schemes: mostly large NLO corrections

- decoupling is not possible in wrong-sign type II 2HDM

- non-decoupling effects increase NLO corrections

Numerical Analysis: Condition II

**Condition II**: \( m_H \geq 2m_h \) and

\[
m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin \beta \cos \beta}} \pm 5\%
\]

and correct-sign only

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**relative size of NLO corrections:**

\[
\Delta \Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}
\]

Numerical Analysis: Condition II

- process-dependent scheme: still huge NLO corrections
  - numerical instability of the scheme
- PT-based schemes: mostly moderate NLO corrections
  - numerically stable scheme

Condition II: $m_H \geq 2m_h$ and

$$m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin \beta \cos \beta}} \pm 5\%$$

and correct-sign only

- proc1: process-dependent 1
- pOS$^c$: “on-shell pinched”
- $p^c_*$: “$p^*$-pinched”

Relative size of NLO corrections:

$$\Delta \Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

Conclusions and Outlook

- renormalization can spoil gauge independence in the (N)2HDM
- a gauge-independent, process-independent and numerically stable scheme for $\delta \alpha$ and $\delta \beta$ worked out for the first time for the (N)2HDM
- NLO corrections to Higgs-to-Higgs decays can become large
  - due to chosen renormalization schemes (“numerical instability”)
  - if the LO width becomes very small
  - due to parametrically enhanced contributions from VCs, CTs and WFRCs
  - in certain limits of the (N)2HDM due to non-decoupling effects
- analyses of the NLO corrections performed with (ewN)2HDECAY:
  - several different renormalization schemes included
  - for correct-sign decoupling: moderate corrections for certain schemes
    - numerically stable schemes
- dedicated phenomenological studies in the very near future: stay tuned!
So long, and **thanks** for all the fish!

[ source: https://xkcd.com/1437 ]
ITP Research Seminar
ITP, KIT

M. Krause: (N)2HDM - Renormalization, Extended Higgs Decays and All That
Backup slides
Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet** \((\phi_1, \phi_2)\)

- wave-function renormalization constants (WFRCs):

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
_0 = \sqrt{Z_\phi}
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
\approx \left(1_{2 \times 2} + \frac{\delta Z_\phi}{2}\right)
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}
, \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix}
\frac{\delta Z_{\phi_1 \phi_1}}{2} \\
\frac{\delta Z_{\phi_2 \phi_2}}{2}
\end{pmatrix}
\]

- two-point correlation function for the doublet with momentum \(p^2\):

\[
\hat{\Gamma}_\phi(p^2) := \begin{pmatrix}
\hat{\Gamma}_{\phi_1 \phi_1}(p^2) & \hat{\Gamma}_{\phi_1 \phi_2}(p^2) \\
\hat{\Gamma}_{\phi_2 \phi_1}(p^2) & \hat{\Gamma}_{\phi_2 \phi_2}(p^2)
\end{pmatrix}
\]

\[
= i\sqrt{Z_\phi} \left[p^2 1_{2 \times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2\right] \sqrt{Z_\phi} \approx i \left[p^2 1_{2 \times 2} - D_\phi^2 + \hat{\Sigma}_\phi(p^2)\right]
\]

- mass matrices ↔ mass CTs
- renormalized self-energies

\[
i\Sigma(p^2) := \quad \text{1PI} \quad = \quad + \quad \cdots
\]
Renormalization: On-Shell Conditions (II)

- **on-shell conditions:**
  - mixing of fields **vanishes** for \( p^2 = m_{\phi_i}^2 \)
  - squared **masses** \( m_{\phi_i}^2 \) are the real parts of the **poles** of the propagator
  - **field normalization:** residue of the propagator at its pole equals \( i \)

- fixation of **diagonal** mass counterterms:
  \[
  \text{Re} \left[ \delta D_{\phi_1 \phi_1}^2 \right] = \text{Re} \left[ \Sigma_{\phi_1 \phi_1} (m_{\phi_1}^2) \right] , \quad \text{Re} \left[ \delta D_{\phi_2 \phi_2}^2 \right] = \text{Re} \left[ \Sigma_{\phi_2 \phi_2} (m_{\phi_2}^2) \right]
  \]

- fixation of WFRCs:
  \[
  \delta Z_{\phi_1 \phi_1} = -\text{Re} \left[ \frac{\partial \Sigma_{\phi_1 \phi_1} (p^2)}{\partial p^2} \right]_{p^2=m_{\phi_1}^2} , \quad \delta Z_{\phi_2 \phi_2} = -\text{Re} \left[ \frac{\partial \Sigma_{\phi_2 \phi_2} (p^2)}{\partial p^2} \right]_{p^2=m_{\phi_2}^2}
  \]
  \[
  \delta Z_{\phi_1 \phi_2} = \frac{2}{m_{\phi_1}^2 - m_{\phi_2}^2} \text{Re} \left[ \Sigma_{\phi_1 \phi_2} (m_{\phi_2}^2) - \delta D_{\phi_1 \phi_2}^2 \right] , \quad \delta Z_{\phi_2 \phi_1} = \frac{2}{m_{\phi_2}^2 - m_{\phi_1}^2} \text{Re} \left[ \Sigma_{\phi_1 \phi_2} (m_{\phi_1}^2) - \delta D_{\phi_1 \phi_2}^2 \right]
  \]

- the **specific form** of the \( \delta D_{\phi_i \phi_j}^2 \) **depends on the tadpole scheme**
Renormalization: Alternative Tadpole Scheme (III)

- technical note: distinguish between tadpole renormalization and renormalization of the other physical parameters

- at one-loop, the proper renormalized VEV is given by the tree-level VEV:

\[ v^{\text{ren}}_{\text{FJ}} = v^{\text{tree}}_{\text{FJ}} = \left( \frac{2m_W}{g} \right)_{\text{tree}} \]

- the effect of the shifts \( \delta v_i \) were already applied

- at NLO, the other tree-level parameters \( m_W \) and \( g \) still have to be renormalized:

\[ \left( \frac{2m_W}{g} \right)_{\text{tree}} \rightarrow \left( \frac{2m_W}{g} \right)_{\text{FJ}}^{\text{ren}} + \left( \frac{2m_W}{g} \right)_{\text{FJ}} \left( \frac{\delta m_W^2}{2m_W^2} - \frac{\delta g}{g} \right) \equiv \Delta v \]

- the quantity \( \Delta v \) combines the effect of the CTs of \( m_W \) and \( g \)
Renormalization: Alternative Tadpole Scheme (IV)

- **generalization** to more complicated Higgs models, e.g. the singlet extensions of the SM (“HSM”) or N2HDM is straightforward

- the shifts $\delta v_i$ (including $\delta v_S$) are connected to the tadpole diagrams

- after performing the shifts, $\Delta v_S$ still has to be renormalized

- in the standard tadpole scheme: $\Delta v_S$ is protected from UV divergences


  freedom of choice: set $\Delta v_S = 0$

- in the alternative tadpole scheme: $\Delta v_S$ becomes UV-divergent


  renormalization through $\overline{MS}$, process-dependent scheme, …
Renormalization: Scalar Mixing Angles (V)

- gauge-independent "physical OS approach": use S matrix elements through a process

- idea: introduce two right-handed fermion singlets $\nu_{iR}$ with additional $\mathbb{Z}_2$ symmetries to prevent generation mixing
  - massive neutrinos with Yukawa couplings $y_{\nu_i}$

- renormalization of $\delta\alpha$ and $\delta\beta$ through requirement that ratios of tree-level and one-loop S matrix elements are equivalent, e.g.:
  \[
  \frac{A_1^{H\nu_i\nu_i}}{A_1^{H\nu_i\nu_i}} \equiv \frac{A_0^{H\nu_i\nu_i}}{A_0^{H\nu_i\nu_i}} \quad (i = 1, 2)
  \]

- after renormalization: recover the 2HDM by decoupling the singlets

- properties of the "physical OS approach":
  - CTs are defined purely through gauge-independent S matrix elements
    - manifestly gauge-independent per construction
  - numerically stable (depending on the point in parameter space)

we consider a **fermion scattering process** at one-loop QCD:

\[
\mathcal{A}_{\text{full}}(s, t, m_1, m_2) = \mathcal{A}_{\text{box}}(s, t, m_1, m_2; \xi) + \mathcal{A}_{\text{tri}}(t, m_1, m_2; \xi) + \mathcal{A}_{\text{self}}(t; \xi)
\]

the gauge dependences **have to cancel** within the individual topologies

- rearrangement of the contributions is **always possible**
- rearrangement shows that all gauge dependences have **self-energy-like** or triangle-like form

\[
\mathcal{A}_{\text{full}}(s, t, m_1, m_2) = \tilde{\mathcal{A}}_{\text{box}}(s, t, m_1, m_2) + \tilde{\mathcal{A}}_{\text{tri}}(t, m_1, m_2) + \tilde{\mathcal{A}}_{\text{self}}(t)
\]

\[
\mathcal{A}_{\text{tri}}(t, m_1, m_2; \xi) \rightarrow \tilde{\mathcal{A}}_{\text{tri}}(t, m_1, m_2) + \tilde{f}_{\text{self}}(t; \xi)
\]
Pinch Technique: Introduction (II)

- determination of the gauge-dependent contributions: “pinching”
- main idea: trigger the elementary Ward identity for the loop momentum

\[ k = (k + p - m) - (p - m) = S^{-1}(k + p) - S^{-1}(p) \]

- right expression: vanishes OS between spinors
- left expression: cancels (“pinches out”) an internal fermion propagator

\[ k \equiv S^{-1}(p_2 + k) - S^{-1}(p_2) \]

![Diagram showing pinch technique](image)
Pinch Technique: Results (I)

- (almost) all pinch contributions are proportional to \((1 - \xi)\)

- the non-pinched contributions are equivalent to diagrams calculated in Feynman-‘t Hooft gauge, i.e. for \(\xi \equiv 1\)

\[
\begin{align*}
\mathcal{A}_{\text{box}} & \quad \mathcal{A}_{\text{box}} \\
\xi & \equiv 1 \\
\text{pinch contributions} & \quad \text{gauge-independent part}
\end{align*}
\]

- the pinch contributions are self-energy like, i.e. functions of only \(t\) ➔ reallocation of pinch contributions to the gluon self-energy possible
Pinch Technique: Results (II)

- sum of all pinch contributions ⇒ **cancelation of gauge dependences**

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i\Sigma_{\text{box}}^{\mu\nu}$</td>
<td>$t \frac{C_A}{2}$</td>
<td>0</td>
</tr>
<tr>
<td>$i\Sigma_{\text{tril}}^{\mu\nu}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$i\Sigma_{\text{tril}}^{\mu\nu}$</td>
<td>$-tC_A$</td>
<td>2$C_A$</td>
</tr>
<tr>
<td>$i\Sigma_{\text{self,q}}^{\mu\nu}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$i\Sigma_{\text{self,g}}^{\mu\nu}$</td>
<td>$t \frac{C_A}{2}$</td>
<td>$-2C_A$</td>
</tr>
<tr>
<td>Sum</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$(q^2 \equiv t)$

$C_A, C_f$ : Casimir operators

- **main results** from the application of the pinch technique:
  - demonstration of **intricate cancelation** of gauge dependences
  - cancelation is **not accidental**, but follows from **Ward identities**
Gauge-Independent Self-Energies via PT

- all pinch contributions are self-energy-like
  - **reallocate** pinch contributions to the gluon self-energy

- the pinched self-energy is equivalent to the one evaluated for $\xi = 1$
after the cancelation of all gauge dependences
  - **Feynman-’t Hooft-gauge** is a **special gauge choice**

**interesting properties** of the pinched gluon self-energy:
- analogy to the gluon self-energy given by the **Background Field Method**
- **uniquely defined** by the pinch technique framework
- manifestly **gauge-independent**
  - **allows for gauge-independent counterterms**
- obeys **QED-like Ward identities** instead of complicated Slavnov-Taylor identities

Applications of the Pinch Technique

- the pinch technique can be applied to e.g. the SM, MSSM, (N)2HDM, …

- for consistency: *tadpole diagrams* have to be taken into account
  → “alternative tadpole scheme” is needed

- applications of the pinched self-energies:
  - definition of *gauge-independent counterterms*
  - construction of *QED-like Ward identities* for e.g. QCD
  - gauge-independent definition of *electroweak parameters*
  - consistent resummation for resonant transition amplitudes
  - extraction of gauge-independent part of *BFM self-energies*  

Condition II: \( m_H \geq 2m_h \) and
\[ m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin\beta\cos\beta}} \pm 5\% \]
and correct-sign only; vary \( m_H \)

- KOSY\(^c\) : Kanemura et al.
- \( p^c_*\) : “p*-pinched”
- proc1/3 : proc.-dep.1/3
- OS12 : physical OS scheme O12

Relative difference between \( p^{OS^o}\) and the other schemes:
\[ \Delta \Gamma^{p^{OS^o}} = \frac{\Gamma^{p^{OS^o}}_{\text{NLO}} - \Gamma^{p^{OS^o}}_{\text{NLO}}}{\Gamma^{p^{OS^o}}_{\text{NLO}}} \]

- parameters are converted from reference scheme \( p^{OS^o}\) to all others
- relative difference over large range of \( m_H \) between -2% and 6%

Moderate uncertainty for considered parameter point and decay

\[ \text{https://github.com/marcel-krause/2HDECAY} \]
Numerical Analysis: Condition III

- **no decoupling regime** due to additional OS condition $m_H \geq 2m_A$
- large NLO corrections for both the $\overline{\text{MS}}$ and proc.-dep. scheme for $m_{12}^2$

**Condition III:** $m_H \geq 2m_h$ and $m_H \geq 2m_A$

- $\text{pOS}^c$: “on-shell pinched”
- $\overline{\text{MS}}: m_{12}^2$ ren. via $\overline{\text{MS}}$
- proc: $m_{12}^2$ ren. via $H \rightarrow AA$

**relative size** of NLO corrections:

$$\Delta \Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$


Both numerical instability and strong coupling at work