Enhanced Corrections in Higgs-to-Higgs Decays of the 2HDM and N2HDM

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[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

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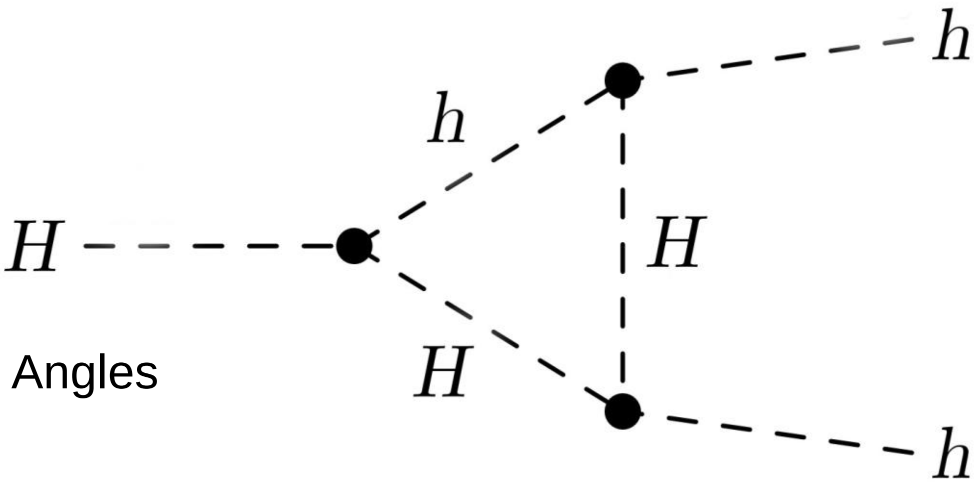
- Motivation
- Introduction to the 2HDM
- Special Limits of the 2HDM
- Renormalization of the Scalar Mixing Angles
- Numerical Analyses



Motivation

- we consider the Two-Higgs-Doublet Model (**2HDM**)
  - no constraints due to SUSY relations
  - provides a dark matter candidate (*Inert Doublet Model*)
  - **extended scalar sector**
    - interesting phenomenology
    - Higgs-to-Higgs decays as interesting signatures

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  - **extended scalar sector**
    - interesting phenomenology
    - Higgs-to-Higgs decays as interesting signatures
- **investigation** of the one-loop electroweak corrections to $H \to h\,h$:
  - size and relevance of the electroweak corrections
  - renormalization scheme dependence of the electroweak corrections

    ⇒ **estimate of theoretical uncertainty** due to missing higher orders
  - size of the electroweak corrections relative to the decay width at tree level

    ⇒ "**numerical stability**" of renormalization schemes

Introduction to the 2HDM: Scalar Potential

- **two** complex $SU(2)_L$ Higgs **doublets**

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix} \;, \qquad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- non-vanishing **vacuum expectation values** (VEVs) v_1, v_2 with

$$v^2 := v_1^2 + v_2^2 \approx (246\ \text{GeV})^2$$

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- scalar Lagrangian with **CP- and** $\mathbb{Z}_2$**-conserving** 2HDM potential:

$$\begin{aligned} V_{\text{2HDM}}(\Phi_1,\Phi_2) = {} & m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) - m_{12}^2\left[\left(\Phi_1^\dagger\Phi_2\right) + \left(\Phi_2^\dagger\Phi_1\right)\right] \\ & + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\ & + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right] \end{aligned}$$

Introduction to the 2HDM: Parameters (I)

- **eight** real-valued potential parameters:
  - dimensionless $\lambda_i \ (i = 1, ..., 5)$
  - squared mass parameters $m_{11}^2,\ m_{22}^2$ and m_{12}^2

- difference w.r.t. MSSM: constants **not fixed through** **SUSY relations**

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- difference w.r.t. MSSM: constants **not fixed through SUSY relations**
- transformation to the Higgs mass basis via **scalar mixing angles**
  - α for the CP-even sector
  - β for the CP-odd **and** charged sectors

⇒ physical Higgs bosons and Goldstones $(H, h, G^0, A, G^\pm, H^\pm)$

CP-even (H, h) CP-odd (G^0, A) charged ($G^\pm, H^\pm$)

- **SM limit** for our analyses: $\sin(\beta - \alpha) \to 1$

Introduction to the 2HDM: Parameters (II)

- masses of **heavier** Higgs bosons $\phi_{\text{heavy}} \in \{H, A, H^{\pm}\}$ take the form

$$m^2_{\phi_{\text{heavy}}} \approx c^2_{\phi_{\text{heavy}}} \frac{m^2_{12}}{\sin\beta\cos\beta} + f(\lambda_i)v^2$$

[S. Kanemura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Rev. D* **70** (2004) 115002]

where $f(\lambda_i)$ is a linear combination of the λ_i and

$$c_{\phi_{\text{heavy}}} = \begin{cases} 1 & \text{for } \phi_{\text{heavy}} \in \{A, H^{\pm}\} \\ \sin(\beta - \alpha) & \text{for } \phi_{\text{heavy}} = H \end{cases}$$

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- two interesting limits in case that $m^2_{\phi_{\text{heavy}}}$ becomes **large**:
  - **decoupling**: $\frac{m^2_{12}}{\sin\beta\cos\beta} \gg f(\lambda_i) v^2$ **for all** heavier Higgs bosons
    - $m^2_{\phi_{\text{heavy}}}$ dominated by large $m^2_{12}/\sin\beta\cos\beta$, independent of the λ_i
    - λ_i are small while the $m^2_{\phi_{\text{heavy}}}$ are still large
  - **strong coupling**: $\frac{m^2_{12}}{\sin\beta\cos\beta} \lesssim f(\lambda_i) v^2$ for **at least one** heavier Higgs boson
    - large λ_i required for large $m^2_{\phi_{\text{heavy}}}$

Decoupling Limit of the 2HDM

- **decoupling**: $\frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2$ **for all** $\phi_{\text{heavy}} \in \{H, A, H^\pm\}$
  - $m_{\phi_{\text{heavy}}}^2$ dominated by large $m_{12}^2/\sin\beta\cos\beta$, independent of the λ_i
  - λ_i **are small** while the $m_{\phi_{\text{heavy}}}^2$ are still large
- **trilinear** and **quartic** Higgs couplings can **become small**

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- **decoupling theorem**: loop effects due to heavy Higgs bosons vanish in the limit $m_{\phi_{\rm heavy}} \to \infty$ [T. Appelquist, J. Carazzone, *Phys. Rev. D* **11** (1975) 2856]

- reflects the **decoupling limit in the MSSM** where the Higgs couplings are given in terms of gauge couplings g and g' due to SUSY relations

Wrong-Sign Limit of the 2HDM

- even with large $\frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2$, decoupling is **not always guaranteed**

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- **wrong-sign limit** of the type II (and flipped) 2HDM:
  - **relative minus sign** of the down-type Yukawa couplings compared to the up-type and Higgs-vector-vector couplings
  - reached for $\sin(\alpha+\beta) \to 1$
  - large $\tan\beta$ required in order to simultaneously achieve the SM limit

[P. M. Ferreira, R. Guedes, M. O. P. Sampaio, R. Santos, *JHEP* **12** (2014) 067;
D. Fontes, J. C. Romao, J. P. Silva, *Phys. Rev. D* **90** (2014) 015021 *and references therein*]

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- in the wrong-sign regime of the 2HDM, decoupling is strongly **disfavored** and **strong coupling easily arises**

[P. M. Ferreira, J. F. Gunion, H. E. Haber, R. Santos, *Phys. Rev. D* **89** (2014) 115003]

⇒ for the analyses, **distinguish wrong-sign** and **correct-sign** regimes **within** the “decoupling limit”

Decoupling and Correct-/Wrong-Sign Limit

- consider e.g. the ratio λ_{HHh}/m_H^2 appearing in the **NLO corrections**
- apply the **SM limit** $\sin(\beta - \alpha) \to 1$ and the **decoupling limit**

$$m_H^2 \approx \frac{m_{12}^2}{\sin\beta\cos\beta} \gg f(\lambda_i)v^2 \quad \text{and} \quad m_H^2 \gg m_h^2$$

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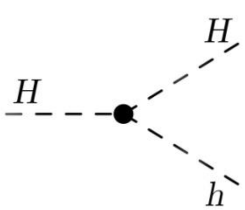
- in these limits, we find:



$$\frac{\lambda_{HHh}}{m_H^2} = -\frac{1}{m_H^2 v}\frac{\sin(\beta-\alpha)}{\sin(2\beta)}\left[\sin(2\alpha)\left(2m_H^2+m_h^2\right) - \frac{m_{12}^2}{\sin\beta\cos\beta}\left(3\sin(2\alpha)+\sin(2\beta)\right)\right]$$

$$\begin{cases} \approx 0 & (\text{correct-sign limit},\ \sin(\beta-\alpha)\to 1) \\ \approx \frac{2}{v} & (\text{wrong-sign limit},\ \sin(\beta-\alpha)\to 1,\ \sin(\alpha+\beta)\to 1,\ \tan\beta \gg 1) \end{cases}$$

[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

➡ **decoupling** in the **correct-sign** regime (decoupling theorem)

➡ **no decoupling** in the **wrong-sign** regime (non-decoupling effects)

Strong Coupling Limit of the 2HDM

- **strong coupling**: $\frac{m_{12}^2}{\sin\beta\cos\beta} \lesssim f(\lambda_i)v^2$ **for at least one** $\phi_{\text{heavy}} \in \{H, A, H^\pm\}$
  - large λ_i required for large $m^2_{\phi_{\text{heavy}}}$
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  - large λ_i required for large $m^2_{\phi_{\text{heavy}}}$
- **trilinear** and **quartic** Higgs couplings **become large**
- decoupling theorem **does not apply**: loop effects due to heavy Higgs bosons do not vanish in the limit $m_{\phi_{\text{heavy}}} \to \infty$
- **reason**: radiative corrections due to heavy Higgs bosons develop a **power-law**-like behavior in $m_{\phi_{\text{heavy}}}$

➡ **large NLO corrections** due to **non-decoupling effects**

➡ for $H \to h\,h$: corrections grow with $m^4_{\phi_{\text{heavy}}}$

[S. Kanemura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Rev. D* **70** (2004) 115002;
S. Kanemura, S. Kiyoura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Lett.* **B558** (2003) 157]

Renormalization of the 2HDM

- set of free parameters of the 2HDM (excluding CKM elements, …)

$$\left\{ T_{h/H},\ \alpha_{\text{em}},\ m_W,\ m_Z,\ m_f,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},\ \alpha,\ \beta,\ m_{12}^2\ ,\cdots \right\}$$

- renormalization program for the 2HDM:
  - tadpole terms ➡ standard / **alternative** tadpole scheme
  - mass counterterms ➡ on-shell
  - fine-structure constant ➡ at Z mass
  - soft-$\mathbb{Z}_2$-breaking scale m_{12}^2 ➡ $\overline{\text{MS}}$ or process-dependent
  - **scalar mixing angles** ➡ **several different schemes**

[MK, *Master's thesis* (2016), KIT;
MK, R. Lorenz, M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

Renormalization: Scalar Mixing Angles (I)

- renormalization of mixing angles α and β is **non-trivial** in the 2HDM
- simplest approach: $\overline{\mathrm{MS}}$ conditions for α and β (alternatively: λ_3)
  - can be **numerically unstable** in one-to-two-body decays
  - **divergences for degenerate masses / "dead corners"** of parameter space

[MK, R. Lorenz, M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
A. Denner, S. Dittmaier, J.-N. Lang, *JHEP* **2018** (2018) 104 *and references therein*]

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- analyze renormalization schemes **for the 2HDM** w.r.t.**"three desirable criteria":** [A. Freitas, D. Stöckinger, *Phys. Rev.* D**66** (2002) 095014]
  - gauge independence
  - process independence
  - **numerical stability** (*i.e.* leads to **moderate NLO corrections**)

Types of Numerical Instabilities

- measure for the **relative size** of the **NLO corrections**:

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  - if the LO width becomes **small** such that $\Delta\Gamma$ becomes very sensitive on $\Gamma^{\rm NLO}$
  - if vertex corrections, CTs and/or WFRCs are **parametrically enhanced**
  - due to **unsuitable renormalization schemes** for some parameters
    - *e.g.* $\overline{\rm MS}$: finite parts of $\delta\alpha, \delta\beta$ missing for cancellation of large contributions
    - *e.g.* **proc.-dep.**: additional potentially large finite parts included in $\delta\alpha,\ \delta\beta,\ \delta m_{12}^2$

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- in Higgs-to-Higgs decays in the (N)2HDM, $\Delta\Gamma$ may **additionally** become large due to certain **limits in the parameter space**

  ➡ **wrong-sign** limit, **strong coupling** limit

Renormalization: Scalar Mixing Angles (II)

- approach by S. Kanemura *et al.*: connect the definition of α and β with the **inverse propagator matrix** ("KOSY scheme")

[S. Kanemura, Y. Okada, E. Senaha, C. P. Yuan, *Phys. Rev. D* **70** (2004) 115002]

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = R^T_{\theta,0}\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix}_0 \approx R^T_{\delta\theta}R^T_\theta\sqrt{Z_{\widetilde{\phi}}}R_\theta R^T_\theta\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix} \approx \begin{pmatrix}1+\frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2}+\delta\theta\\ \delta C_{\phi_2}-\delta\theta & 1+\frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

- mixing angle counterterms **within the standard tadpole scheme**:

$$\delta\alpha = \frac{1}{2\,(m_H^2-m_h^2)}\mathrm{Re}\Big[\Sigma_{Hh}(m_H^2)+\Sigma_{Hh}(m_h^2)-2\delta T_{Hh}\Big]$$

$$\delta\beta = -\frac{1}{2m_{H^\pm}^2}\mathrm{Re}\Big[\Sigma_{G^\pm H^\pm}(m_{H^\pm}^2)+\Sigma_{G^\pm H^\pm}(0)-2\delta T_{G^\pm H^\pm}\Big]$$

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- the KOSY scheme as described above leads to the inclusion of gauge-dependent contributions in the partial decay widths [MK, *Master‘s thesis* (2016), KIT]

➡ gauge dependences need to be removed

[*cf.* S. Kanemura, M. Kikuchi, K. Sakurai, K. Yagyu, *Phys. Rev.* D **96** (2017) 035014]

Renormalization: Scalar Mixing Angles (III)

- gauge-independent **"OS approach"**: use the **pinch technique** (PT)

- **PT-based definition** of the scalar mixing angle counterterms:
  use the pinched scalar self-energies instead of the usual ones

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  use the pinched scalar self-energies instead of the usual ones

- properties of the pinched scheme:
  - **process-independent**, symmetric in the fields
  - manifestly **gauge-independent** per construction
    ➡ gauge-independent NLO **amplitudes**
  - **numerically stable** (depending on the point in parameter space)

    ➡ proposed solution for renormalizing $\delta\alpha$ and $\delta\beta$ in the 2HDM

[MK, R. Lorenz, M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143]

Renormalization: Scalar Mixing Angles (IV)

- gauge-independent approach: **process-dependent schemes**

[A. Freitas, D. Stöckinger, *Phys. Rev.* D**66** (2002) 095014;
R. Santos, A. Barroso, L. Brucher, *Phys. Lett.* B **391** (1997) 429-433]

- idea: impose the gauge-invariant condition

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for **different decays** in order to define $\delta\alpha, \, \delta\beta$ and also δm_{12}^2

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- we consider $H \to AA$ for δm_{12}^2 and the following combinations for $\delta\alpha,\ \delta\beta$:
  - **proc.-dep. 1**: $A \to \tau\tau$ for $\delta\beta$ and $H \to \tau\tau$ for $\delta\alpha$
  - **proc.-dep. 2**: $A \to \tau\tau$ for $\delta\beta$ and $h \to \tau\tau$ for $\delta\alpha$
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- properties of process-dependent schemes:
  - **process-dependent** per construction
  - **gauge-independent**
  - **potentially numerically unstable** over large parameter ranges

Renormalization: Scalar Mixing Angles (V)

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- gauge-independent **“physical OS approach”**: use S matrix elements through a process [A. Denner, S. Dittmaier, J.-N. Lang, *JHEP* **2018** (2018) 104]
- idea: introduce **two right-handed fermion singlets** ν_{iR} with additional $\mathbb{Z}_2$ symmetries to prevent generation mixing

  ⇒ massive neutrinos with Yukawa couplings y_{ν_i}

Renormalization: Scalar Mixing Angles (V)

- gauge-independent **"physical OS approach"**: use S matrix elements through a process [A. Denner, S. Dittmaier, J.-N. Lang, *JHEP* **2018** (2018) 104]
- idea: introduce **two right-handed fermion singlets** ν_{iR} with additional $\mathbb{Z}_2$ symmetries to prevent generation mixing
  ➡ massive neutrinos with Yukawa couplings y_{ν_i}
- renormalization of $\delta\alpha$ and $\delta\beta$ through requirement that ratios of tree-level and one-loop S matrix elements are equivalent, e.g.:

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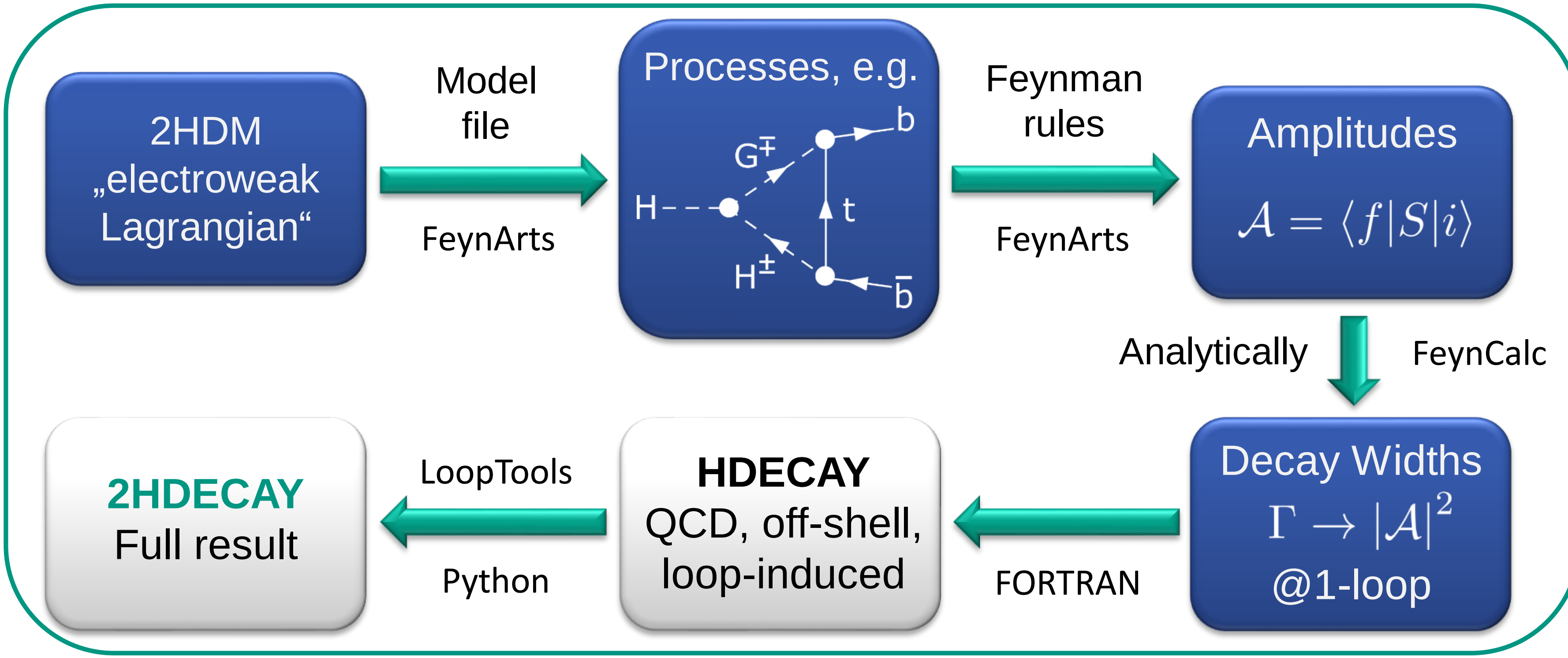
- after renormalization: **recover the 2HDM** by decoupling the singlets
- properties of the "physical OS approach":
  - CTs are defined purely through gauge-independent S matrix elements
    ➡ manifestly **gauge-independent** per construction
  - **numerically stable** (depending on the point in parameter space)

Implementation: 2HDECAY (I)



[FeynArts: T. Hahn, *Comput. Phys. Commun.* **140** (2001) 418; LoopTools: T. Hahn, M. Pérez-Victoria, *Comput. Phys. Commun.* **118** (1999) 153;
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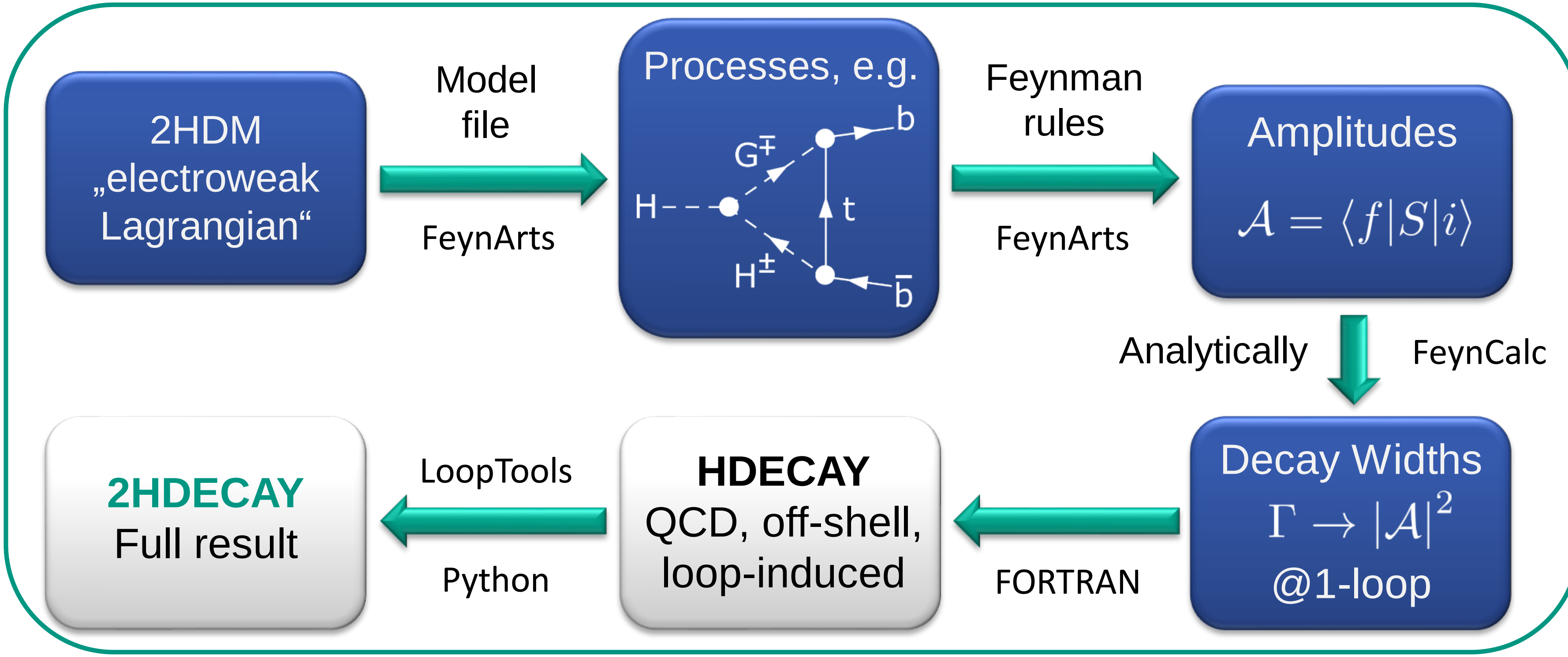

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2HDECAY: “2HDM HDECAY“

A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

[MK, M. Mühlleitner, M. Spira, *Computer Physics Communications* **2019**, *arXiv:1810.00768*] ⇩ **https://github.com/marcel-krause/2HDECAY**

Implementation: 2HDECAY (II)

- 17 renormalization schemes are implemented in **2HDECAY**:

Input ID	Tadpole scheme	$\delta\alpha$	$\delta\beta$	Label
1	standard	KOSY	KOSY (odd)	KOSY^o(std)
2	standard	KOSY	KOSY (charged)	KOSY^c(std)
3	alternative (FJ)	KOSY	KOSY (odd)	KOSY^o
4	alternative (FJ)	KOSY	KOSY (charged)	KOSY^c
5	alternative (FJ)	p_*-pinched	p_*-pinched (odd)	p_*^o
6	alternative (FJ)	p_*-pinched	p_*-pinched (charged)	p_*^c
7	alternative (FJ)	OS-pinched	OS-pinched (odd)	OS^o
8	alternative (FJ)	OS-pinched	OS-pinched (charged)	OS^c
9	alternative (FJ)	proc.-dep. 1	proc.-dep. 1	proc1
10	alternative (FJ)	proc.-dep. 2	proc.-dep. 2	proc2
11	alternative (FJ)	proc.-dep. 3	proc.-dep. 3	proc3
12	alternative (FJ)	physical OS1	physical OS1	OS1
13	alternative (FJ)	physical OS2	physical OS2	OS2
14	alternative (FJ)	physical OS12	physical OS12	OS12
15	alternative (FJ)	rigid symmetry (BFM)	BFMS	BFMS
16	standard	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$(std)
17	alternative (FJ)	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$

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Numerical Analysis: Input Parameters (I)

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- exemplarily, we consider a **type II** **2HDM** in the following
- keep in mind: the 2HDM contains **a lot of free parameters:**

$$\{m_H, m_A, m_{H^\pm}, m_{12}^2, \tan\beta, \alpha\}$$

➡ **scanning** through the parameter space is possible

- chosen parameter points respect **several experimental and theoretical constraints**

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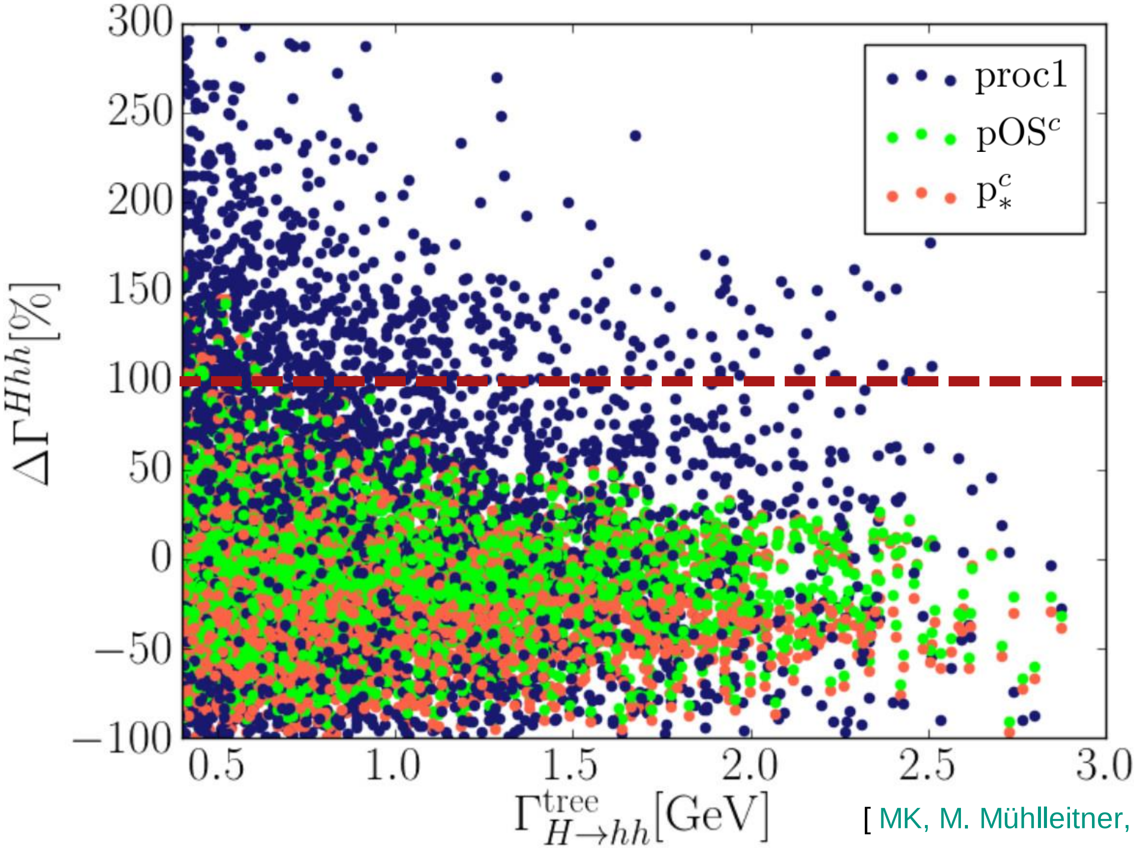
- **aim**: distinguish large NLO corrections due to the strong coupling regime from **numerical instability** due to the chosen renormalization scheme

Numerical Analysis: Condition I



Condition I: $m_H \geq 2m_h$

- $\text{proc}1$: process-dependent 1
- pOS^c : “on-shell pinched”
- p_*^c : “p*-pinched”

relative size of NLO corrections:

$$\Delta\Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

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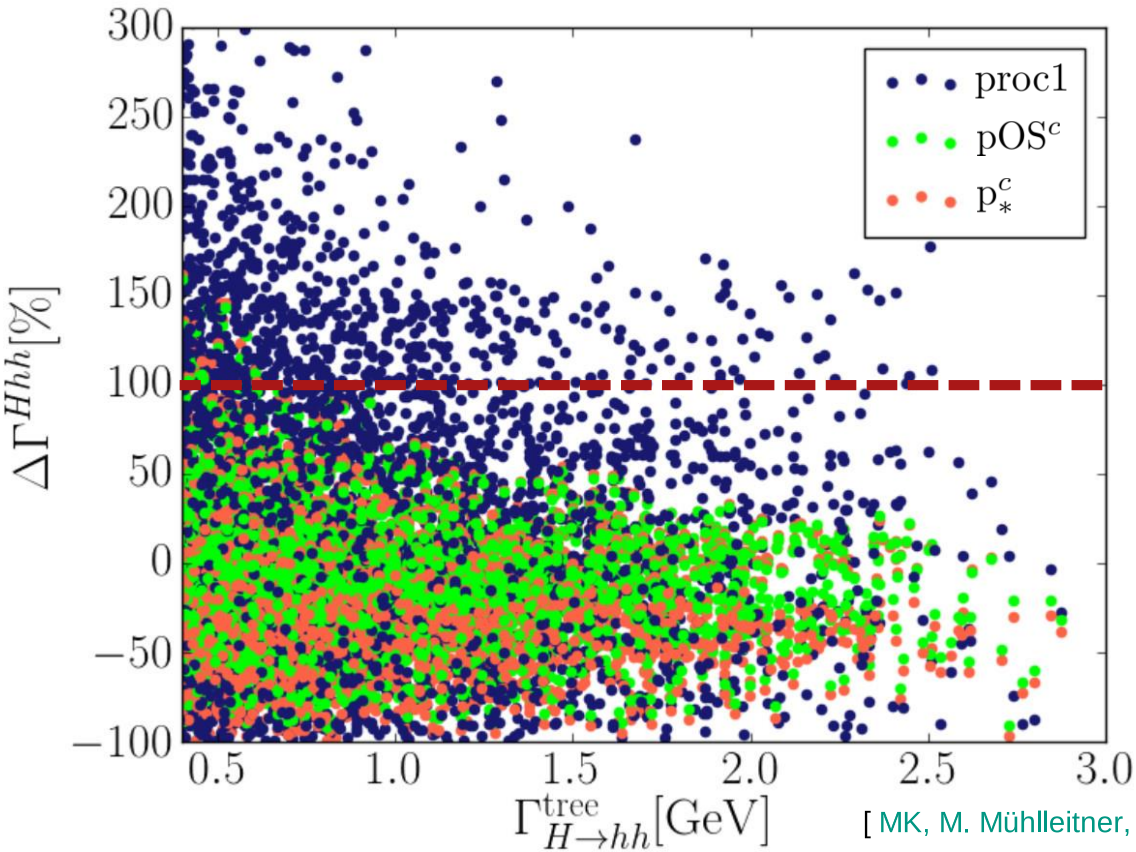

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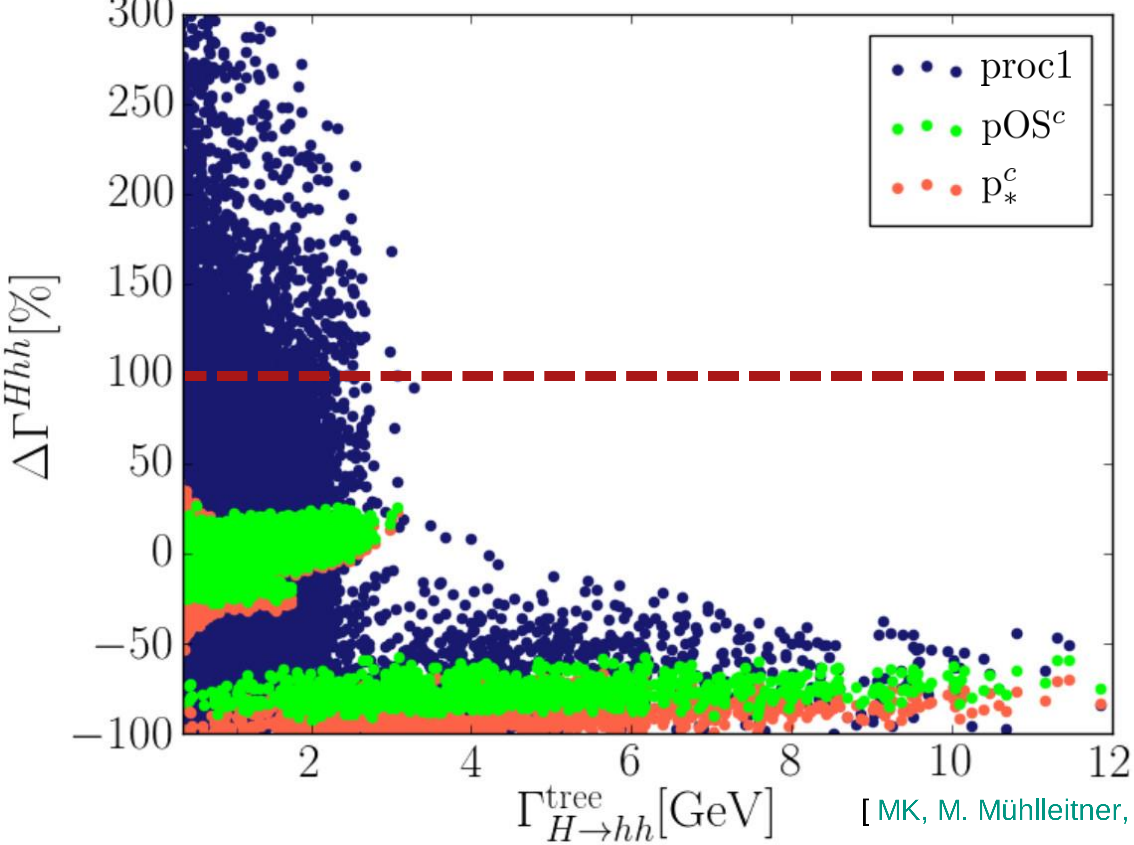
- process-dependent scheme: typically **huge** NLO corrections
- pinched schemes: well-behaving for **large parameter ranges**, but also **large NLO corrections possible** → **numerical instability?**

Numerical Analysis: Condition II



Condition II: $m_H \geq 2m_h$ **and**

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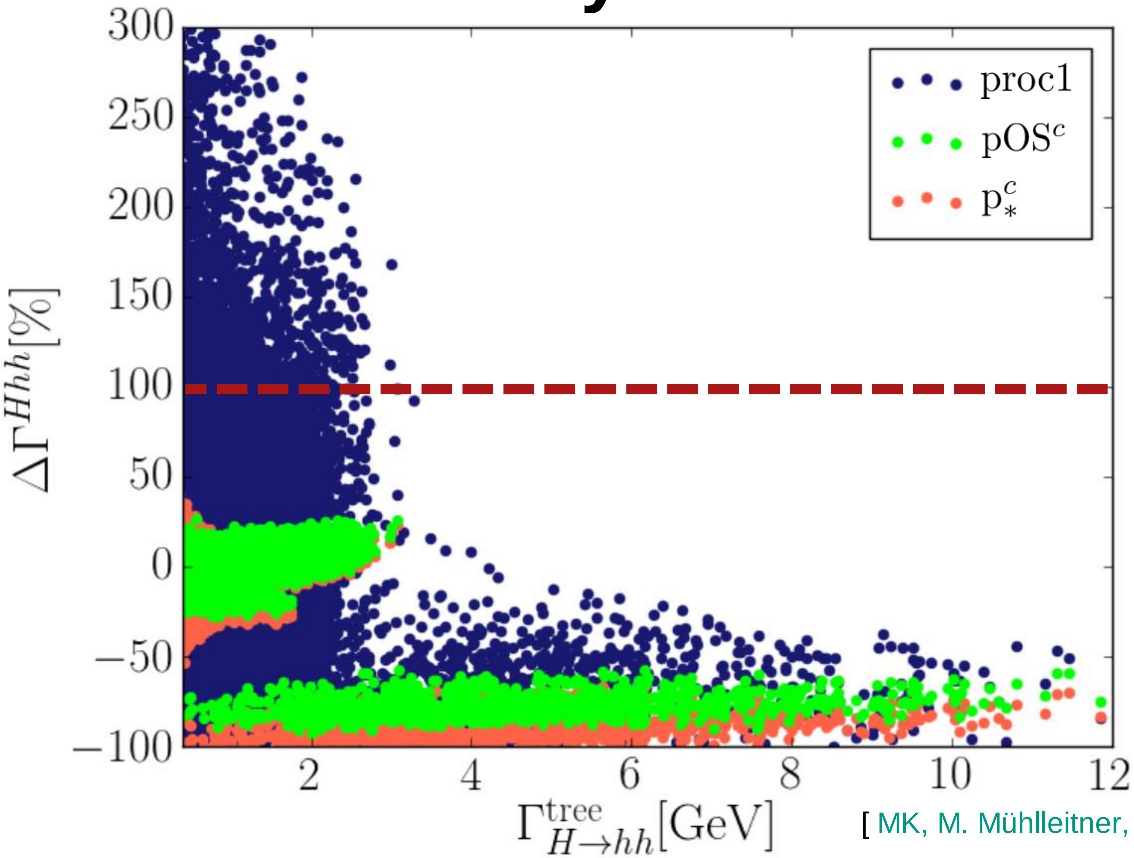

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- process-dependent scheme: **still** typically **huge** NLO corrections
- pinched schemes: one **well-behaving regime** and one regime with **large NLO corrections**

➡ **numerical instability** or still **strong coupling**?

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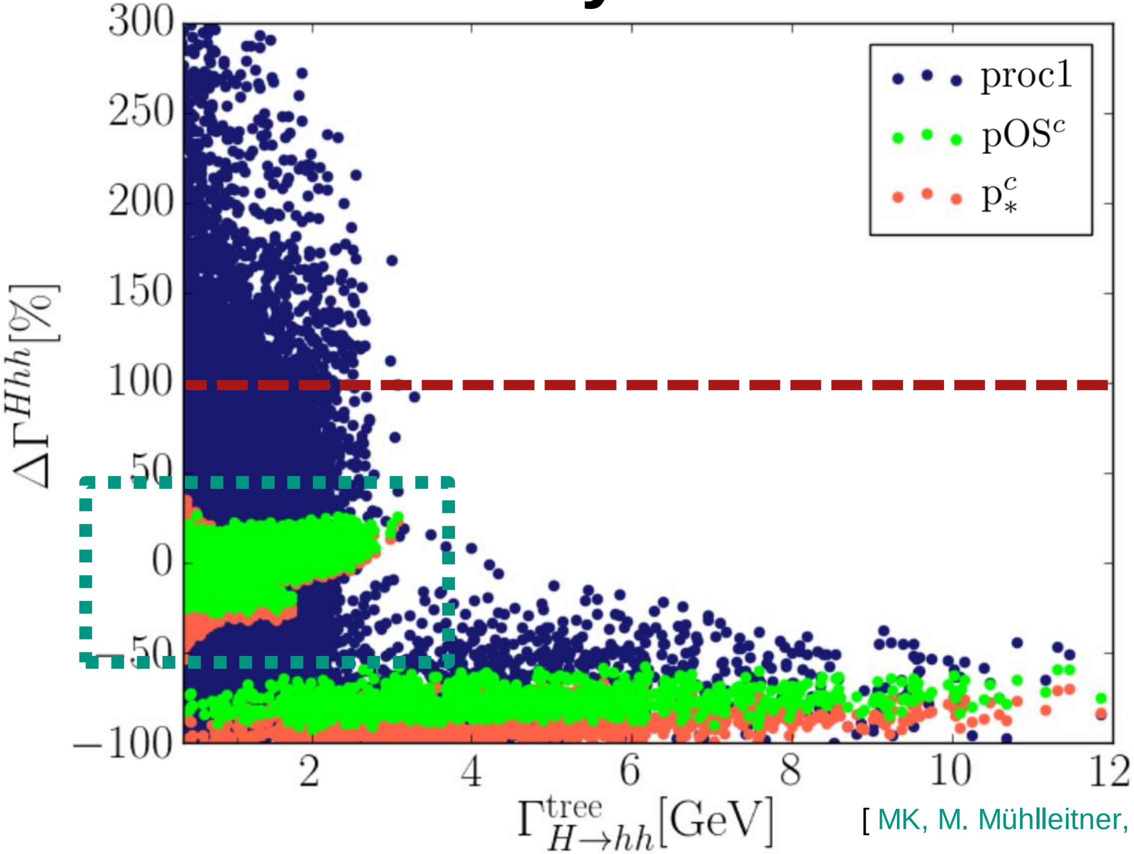

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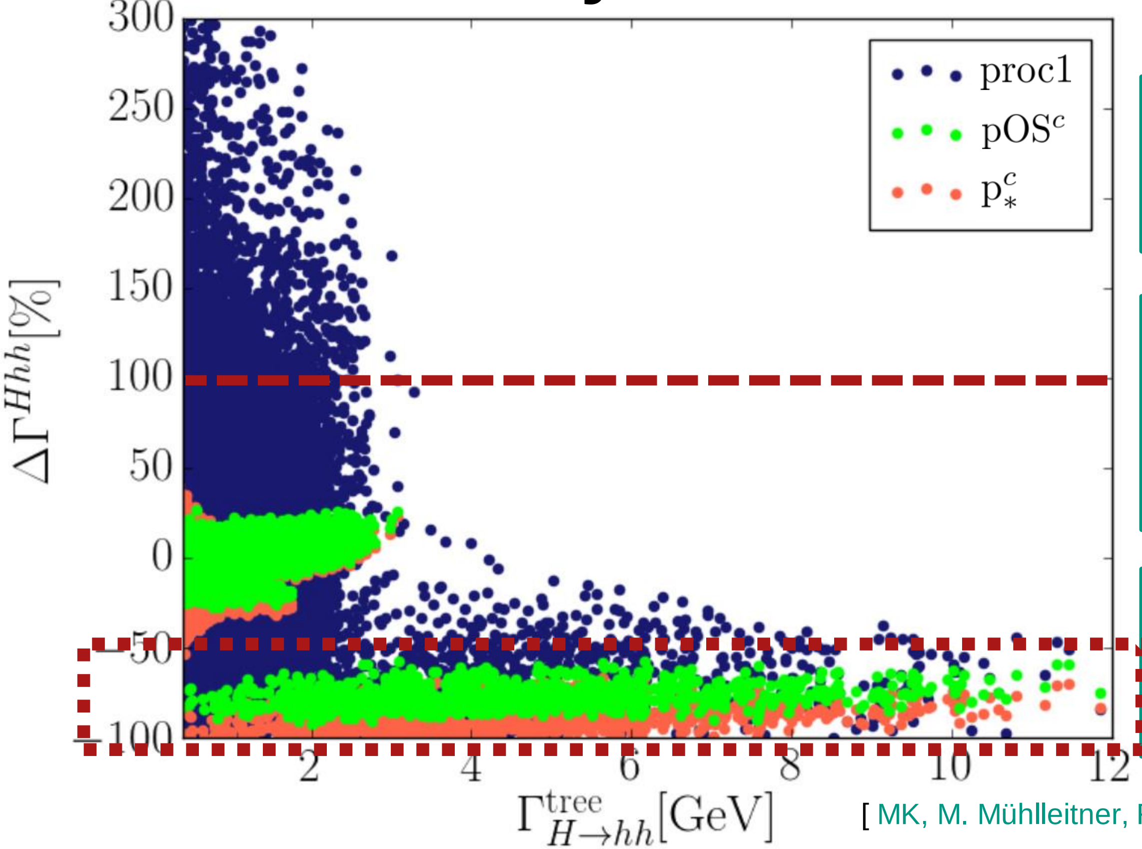

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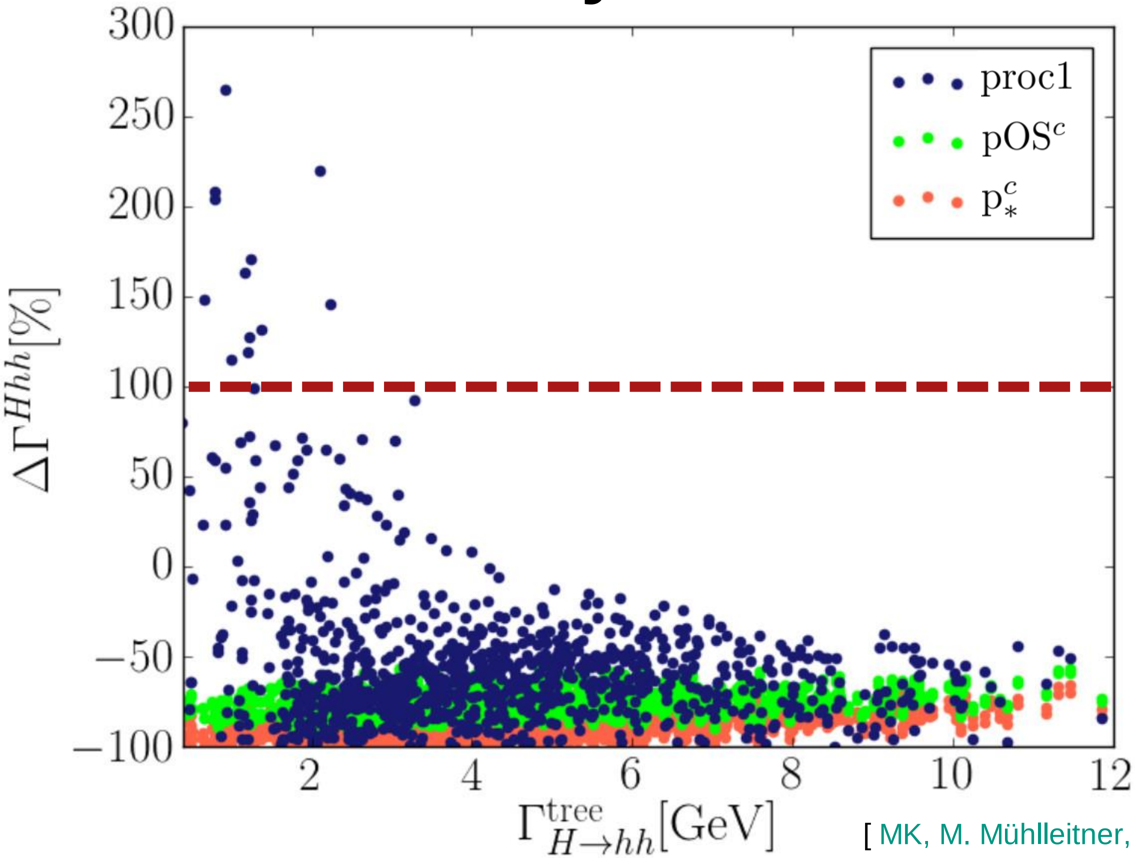

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and wrong-sign only

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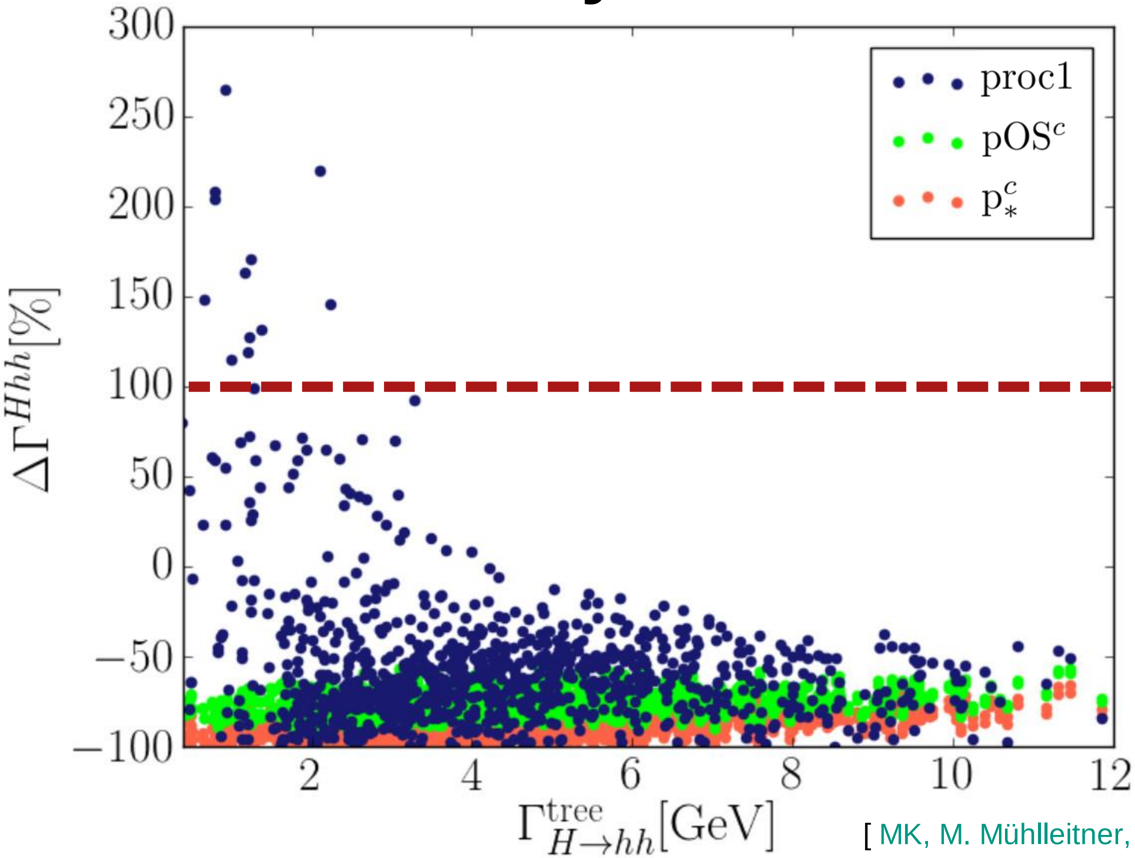

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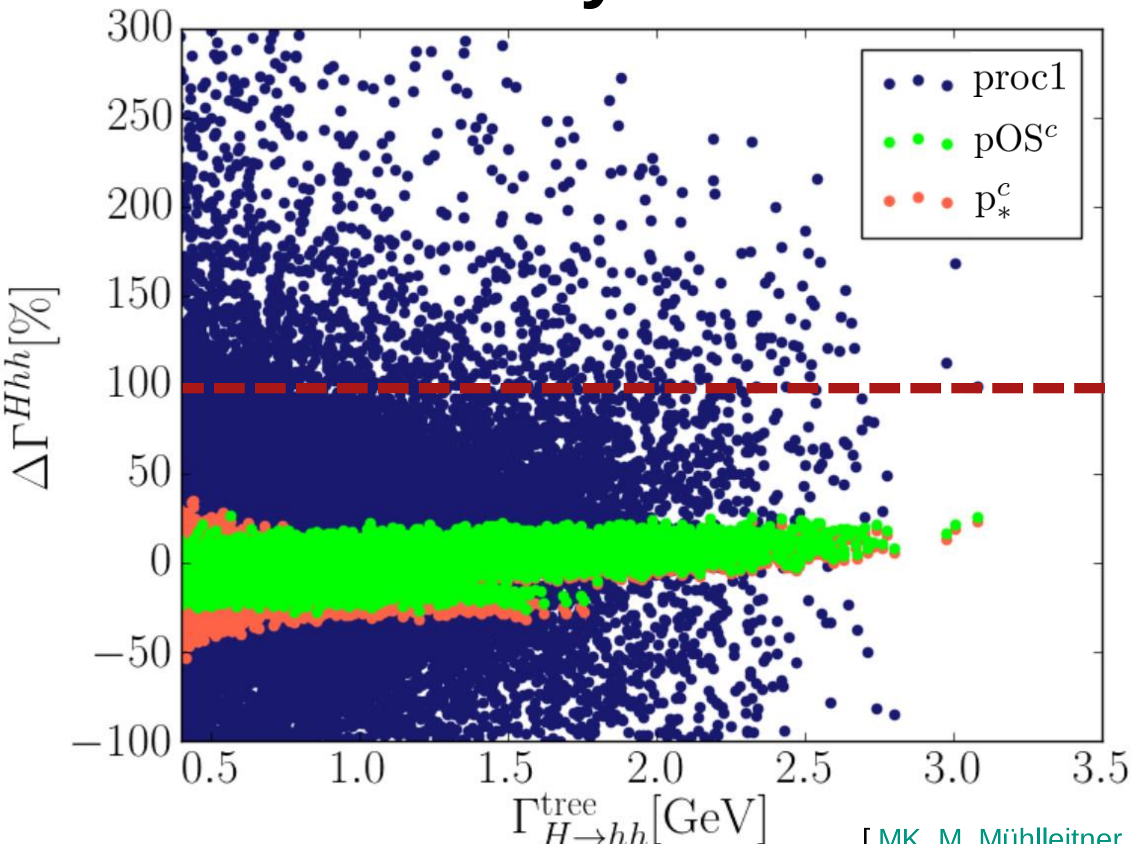
- process-dependent scheme: large variety of NLO corrections
- all schemes: mostly **large NLO corrections**
  - ⇒ decoupling is not possible in wrong-sign type II 2HDM
  - ⇒ **non-decoupling effects** increase NLO corrections

Numerical Analysis: Condition II



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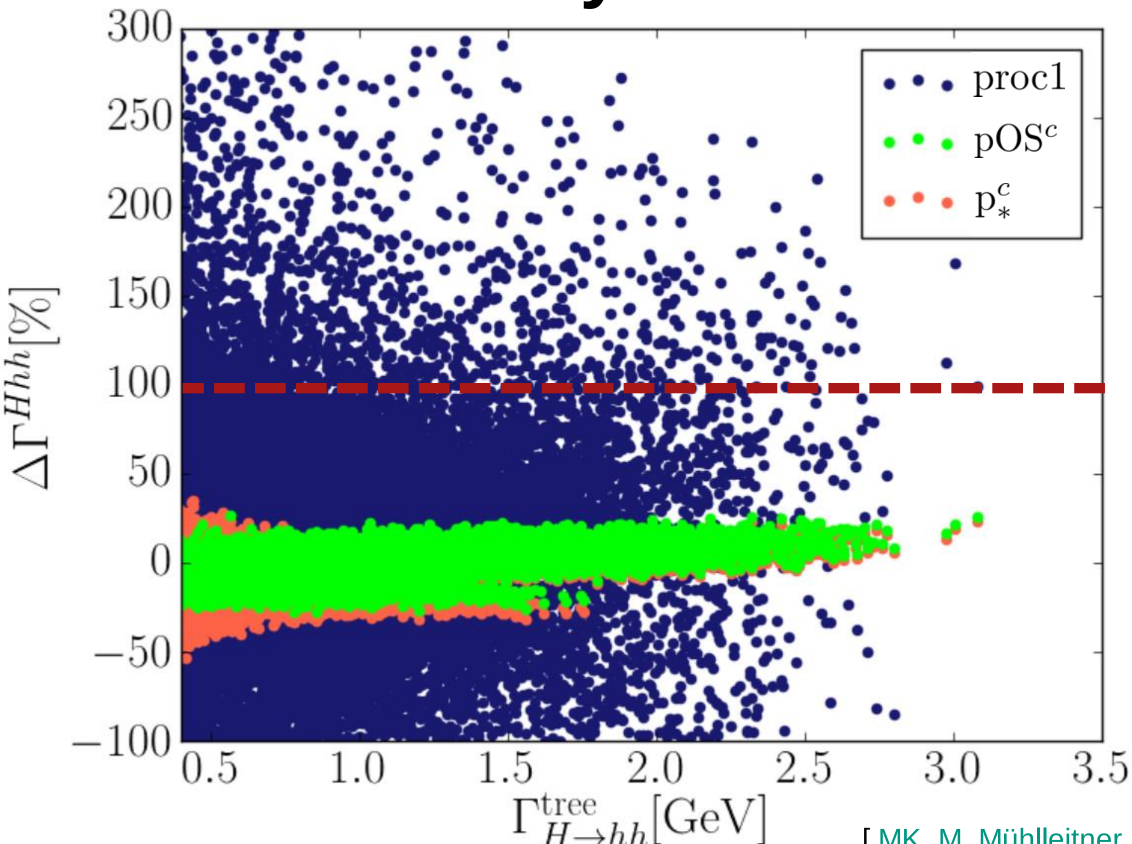

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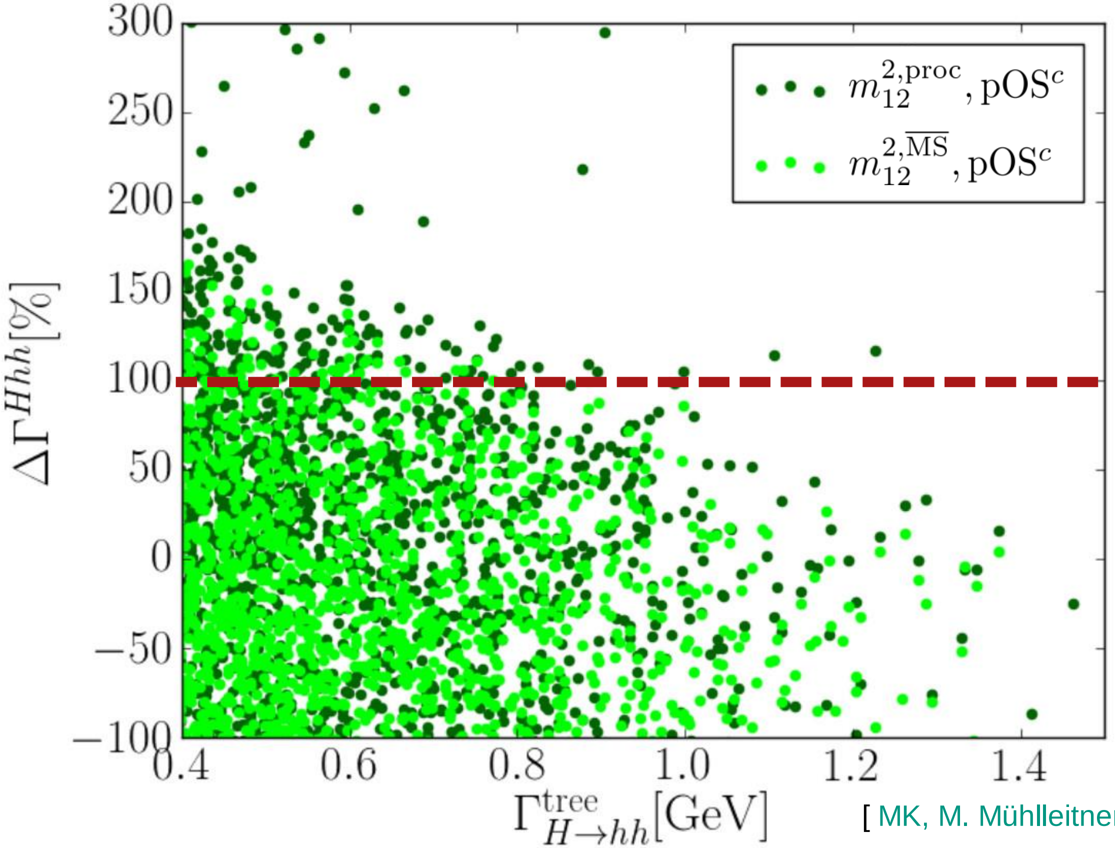
[MK, M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019]

- process-dependent scheme: still **huge NLO corrections**
  - ⇒ **numerical instability** of the scheme
- pinched schemes: mostly **moderate NLO corrections**
  - ⇒ **numerically stable scheme**

Numerical Analysis: Condition III



Condition III: $m_H \geq 2m_h$ **and** $m_H \geq 2m_A$

- pOS^c : "on-shell pinched"
- $\overline{\text{MS}}$: m_{12}^2 ren. via $\overline{\text{MS}}$
- proc: m_{12}^2 ren. via $H \to A\,A$

relative size of NLO corrections:

$$\Delta\Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

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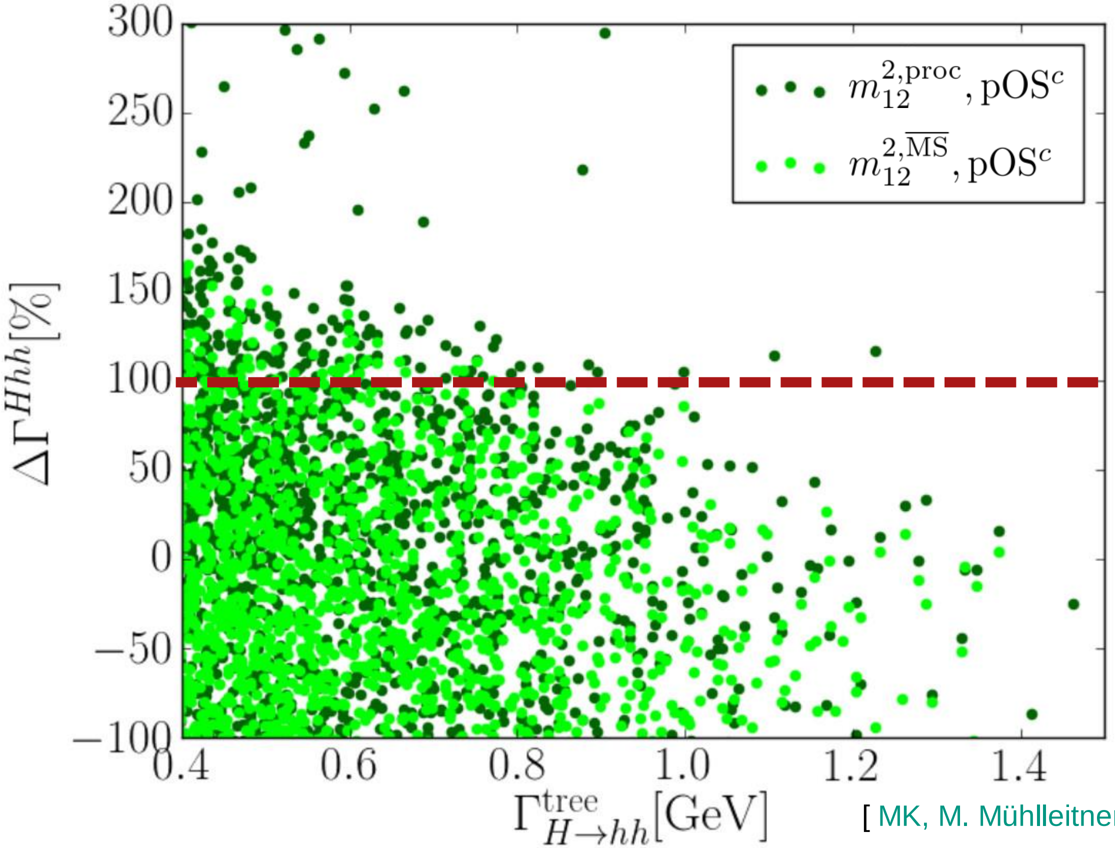

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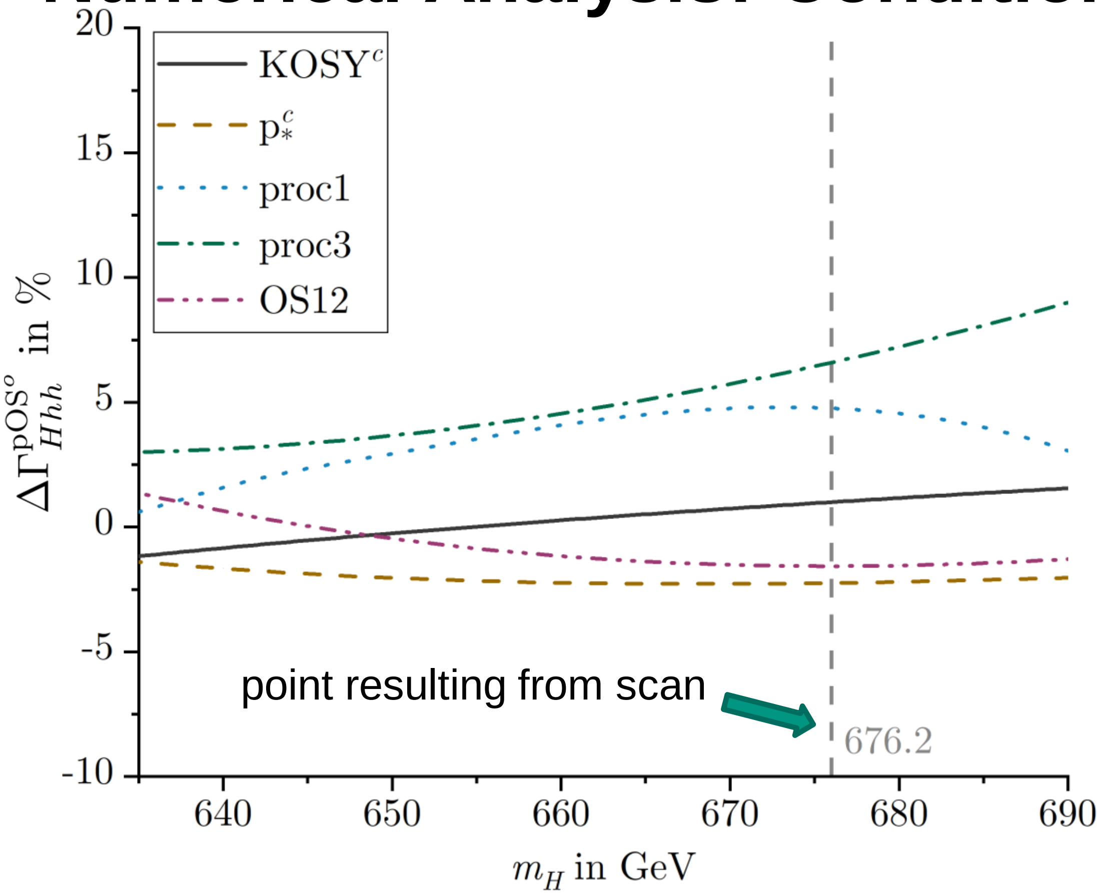
- **no decoupling regime** due to additional OS condition $m_H \geq 2m_A$
- large NLO corrections for both the $\overline{\text{MS}}$ and proc.-dep. scheme for m_{12}^2

➡ both **numerical instability and strong coupling** at work

Numerical Analysis: Condition II



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and correct-sign only; vary m_H

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- p_*^c : “p*-pinched”
- $\mathrm{proc}1/3$: proc.-dep.1/3
- OS12 : physical OS scheme O12

relative difference between pOS^o **and the other schemes:**

$$\Delta\Gamma^{\mathrm{pOS}^o} \equiv \frac{\Gamma^x_{\mathrm{NLO}} - \Gamma^{\mathrm{pOS}^o}_{\mathrm{NLO}}}{\Gamma^{\mathrm{pOS}^o}_{\mathrm{NLO}}}$$

⇓ **https://github.com/marcel-krause/2HDECAY**

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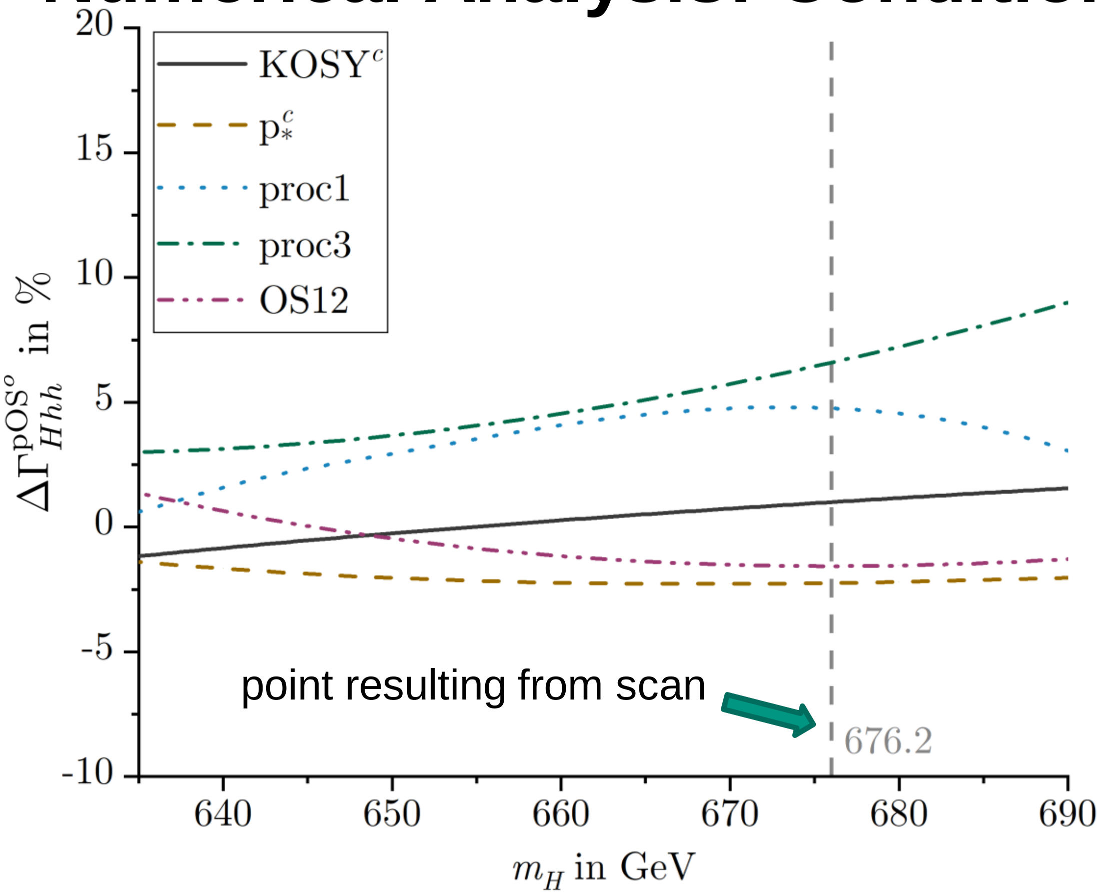

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⇩ **https://github.com/marcel-krause/2HDECAY**

- parameters are **converted** from **reference scheme** pOS^o **to all others**
- relative difference over large range of m_H between **-2% and 6%**

➡ **moderate uncertainty** for considered parameter point and decay

Conclusions and Outlook

- the (N)2HDM features **interesting limits** in parameter space
- NLO corrections to Higgs-to-Higgs decays can become **large**
  - due to chosen renormalization schemes ("**numerical instability**")
  - if the LO width becomes very small
  - due to parametrically enhanced contributions from VCs, CTs and WFRCs
  - in certain limits of the (N)2HDM due to **non-decoupling effects**
- analyses of the NLO corrections performed with **(ewN)2HDECAY**: several different renormalization schemes included
- for correct-sign decoupling: **moderate corrections** for certain schemes

  ⟹ **numerically stable** schemes
- phenomenological studies in the future:
  - dependence of NLO corrections on **(N)2HDM type**
  - extended analysis for **interesting limits** (decoupling, wrong-sign, …)

Backup slides

Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet** $(\phi_1,\ \phi_2)$
- field strength renormalization:

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = \sqrt{Z_\phi}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \approx \left(1_{2\times 2} + \frac{\delta Z_\phi}{2}\right)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \quad , \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix}\frac{\delta Z_{\phi_1\phi_1}}{2} & \frac{\delta Z_{\phi_1\phi_2}}{2}\\ \frac{\delta Z_{\phi_2\phi_1}}{2} & \frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}$$

- two-point correlation function for the doublet with momentum p^2:

$$\widehat{\Gamma}_\phi(p^2) := \begin{pmatrix}\widehat{\Gamma}_{\phi_1\phi_1}(p^2) & \widehat{\Gamma}_{\phi_1\phi_2}(p^2)\\ \widehat{\Gamma}_{\phi_1\phi_2}(p^2) & \widehat{\Gamma}_{\phi_2\phi_2}(p^2)\end{pmatrix}$$

$$= i\sqrt{Z_\phi}^\dagger\left[p^2 1_{2\times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2\right]\sqrt{Z_\phi} \approx i\left[p^2 1_{2\times 2} - D_\phi^2 + \widehat{\Sigma}_\phi(p^2)\right]$$

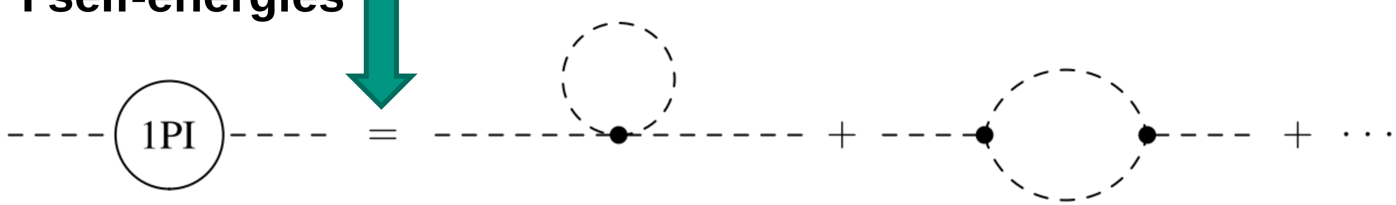
mass matrices

mass CTs

renormalized self-energies

1PI self-energies

$i\Sigma(p^2) :=$



Renormalization: On-Shell Conditions (II)

- **on-shell conditions**:
  - mixing of fields vanishes for $p^2 = m^2_{\phi_i}$
  - masses $m^2_{\phi_i}$ are the real parts of the poles of the propagator
  - normalization: residue of the propagator at its pole equals i

- fixation of **diagonal** mass counterterms:

$$\mathrm{Re}\Big[\delta D^2_{\phi_1\phi_1}\Big] = \mathrm{Re}\Big[\Sigma_{\phi_1\phi_1}(m^2_{\phi_1})\Big] \quad , \quad \mathrm{Re}\Big[\delta D^2_{\phi_2\phi_2}\Big] = \mathrm{Re}\Big[\Sigma_{\phi_2\phi_2}(m^2_{\phi_2})\Big]$$

- fixation of field strength renormalization constants:

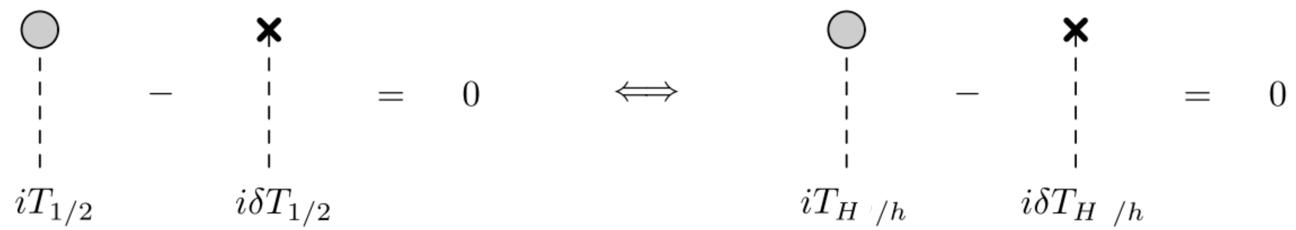
$$\delta Z_{\phi_1\phi_1} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_1\phi_1}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_1}} \quad , \quad \delta Z_{\phi_2\phi_2} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_2\phi_2}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_2}}$$

$$\delta Z_{\phi_1\phi_2} = \frac{2}{m^2_{\phi_1}-m^2_{\phi_2}}\mathrm{Re}\Big[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta D^2_{\phi_1\phi_2}\Big] \quad , \quad \delta Z_{\phi_2\phi_1} = \frac{2}{m^2_{\phi_2}-m^2_{\phi_1}}\mathrm{Re}\Big[\Sigma_{\phi_1\phi_2}(m^2_{\phi_1}) - \delta D^2_{\phi_1\phi_2}\Big]$$

- the **specific form** of the $\delta D^2_{\phi_i\phi_j}$ **depends on the tadpole scheme**

Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:



- conversion from gauge to **mass basis**:

$$\begin{pmatrix} \delta T_1 \\ \delta T_2 \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\alpha & -\mathrm{s}_\alpha \\ \mathrm{s}_\alpha & \mathrm{c}_\alpha \end{pmatrix} \begin{pmatrix} \delta T_H \\ \delta T_h \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\alpha \delta T_H - \mathrm{s}_\alpha \delta T_h \\ \mathrm{s}_\alpha \delta T_H + \mathrm{c}_\alpha \delta T_h \end{pmatrix}$$

- **purpose**: restoring the minimum conditions of the potential at NLO
- **practical effect**: **no tadpole diagrams** in NLO calculations

Renormalization: Standard Tadpole Scheme

- **standard scheme**: vevs are derived from the **loop-corrected potential**

- vevs in the mass relations produce correct one-loop OS masses, e.g.

$$m_W^2 = g^2 \frac{v^2}{4} \quad , \qquad m_A^2 = v^2 \left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right)$$

- tadpole terms appear explicitly in the bare mass matrices
  ➔ mass matrix counterterms contain the **tadpole counterterms**:

$$\delta D_\phi^2 \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \begin{pmatrix} \delta T_{\phi_1 \phi_1} & \delta T_{\phi_1 \phi_2} \\ \delta T_{\phi_1 \phi_2} & \delta T_{\phi_2 \phi_2} \end{pmatrix}$$

- one-loop corrected potential is gauge-dependent
  ➔ **vevs** are gauge-dependent
  ➔ **mass counterterms** become **gauge-dependent**

Renormalization: Alternative Tadpole Scheme (I)

- **alternative scheme**: vevs represent the same minimum as at **tree level**
  [based on: J. Fleischer, F. Jegerlehner, *Phys. Rev.* D **23** (1981) 2001-2026]

- bare masses are expressed through gauge-independent **tree-level vevs**
  ➔ **mass CTs become gauge-independent**

- correct minimum conditions @NLO require a **shift in the vevs**

$$v_1 \rightarrow v_1 + \delta v_1 \;, \qquad v_2 \rightarrow v_2 + \delta v_2$$

- fixation of the shifts by **applying the tadpole conditions**:

$$\begin{pmatrix} \delta v_1 \\ \delta v_2 \end{pmatrix} = \begin{pmatrix} \frac{\delta T_H}{m_H^2} \mathrm{c}_\alpha - \frac{\delta T_h}{m_h^2} \mathrm{s}_\alpha \\ \frac{\delta T_H}{m_H^2} \mathrm{s}_\alpha + \frac{\delta T_h}{m_h^2} \mathrm{c}_\alpha \end{pmatrix}$$

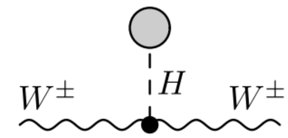
- the shifts translate into **every CT, wave function renormalization constants** and **Feynman rules**

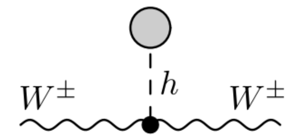
- alternative tadpole scheme **worked out for the 2HDM at one-loop**

Renormalization: Alternative Tadpole Scheme (II)

- example: **W boson mass**

$$m_W^2 = g^2 \frac{v^2}{4} \quad \longrightarrow \quad m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} \quad = \quad m_W^2 + i \left(\vphantom{\frac{a}{b}} \right) + i \left(\vphantom{\frac{a}{b}} \right)$$

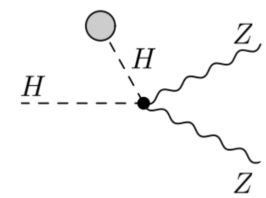




- example: **coupling between Higgs and Z bosons**

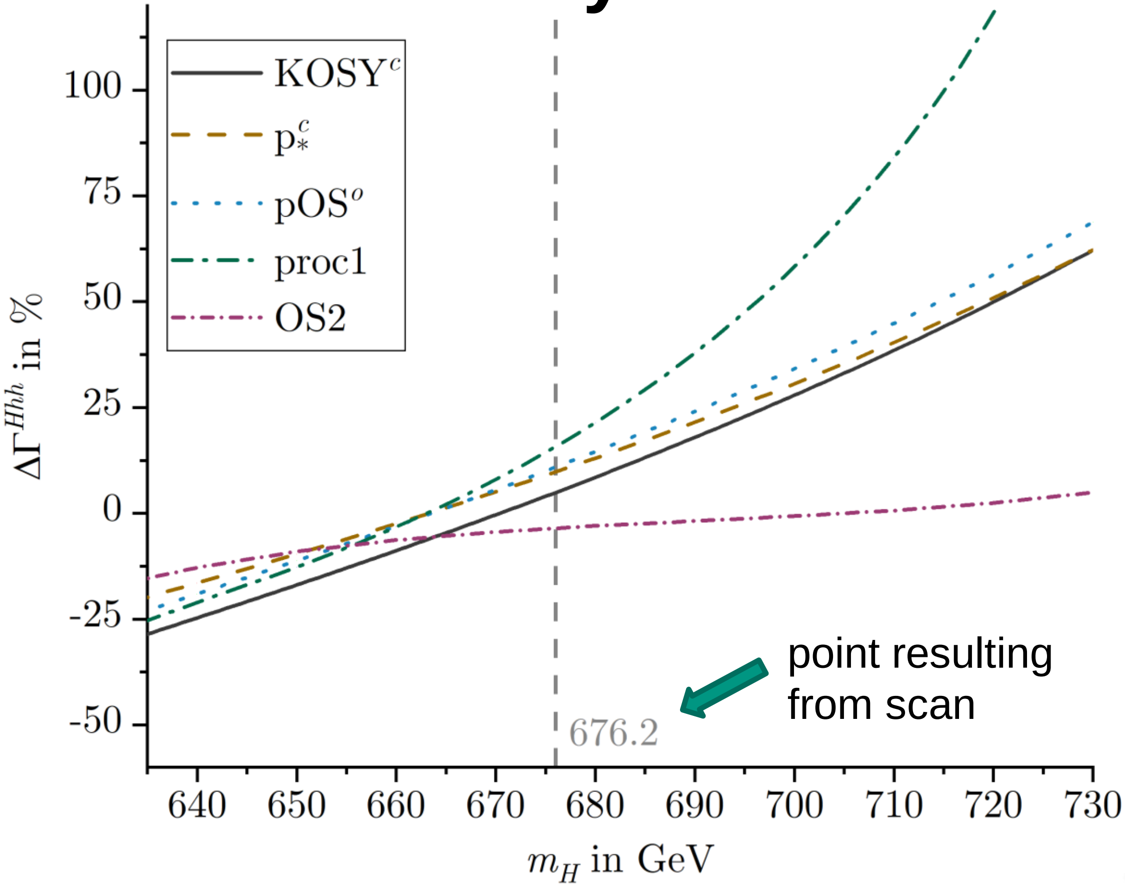
$$i g_{HZZ} = \frac{i g^2}{2 c_W^2} \left(c_\alpha v_1 + s_\alpha v_2 \right) \quad , \qquad i g_{HHZZ} = \frac{i g^2}{2 c_W^2}$$

$$i g_{HZZ} \;\; \rightarrow \;\; i g_{HZZ} + \frac{i g^2}{2 c_W^2} \left(c_\alpha \delta v_1 + s_\alpha \delta v_2 \right) = i g_{HZZ} + \left(\vphantom{\frac{a}{b}} \right)_{\text{trunc}}$$



- **effects** of the alternative tadpole scheme:
  - **tadpole diagrams are added everywhere** where they exist in the 2HDM
  - mass counterterms become **manifestly gauge-independent**
  - tadpole counterterms in the scalar sector are **removed**

Numerical Analysis: Condition II



Condition II: $m_H \geq 2m_h$ **and**

$$m_{\phi_{\text{heavy}}} \approx \frac{m_{12}}{\sqrt{\sin\beta\cos\beta}} \pm 5\,\%$$

and no wrong-sign; vary m_H

- KOSY^c : Kanemura *et al.*
- p$_*^c$: "p*-pinched"
- pOS^o : "on-shell pinched"
- proc1 : process-dependent 1
- OS2 : physical OS scheme OS2

relative size of NLO corrections:

$$\Delta\Gamma \equiv \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

⇩ https://github.com/marcel-krause/2HDECAY

- analyze the sensitivity of the NLO corrections w.r.t. a variation of m_H
- parameters are **converted** from the pOS^o scheme **to all others**

➡ allows for an **estimate** of **remaining theoretical uncertainty**