Corrections to 2HDM Higgs Decays with 2HDECAY

Marcel Krause
Institute for Theoretical Physics (ITP), Karlsruhe Institute of Technology (KIT)


- Motivation
- Principle of Gauge Invariance
- Introduction to the 2HDM
- Automated 1-Loop Calculations with 2HDECAY
- Numerical Results
2HDM: one of the simplest extensions of the SM
- dark matter candidate (*Inert Doublet Model*)
- source of CP violation
- extended scalar sector
- renormalizable
Motivation (I): Two-Higgs-Doublet Model

- 2HDM: one of the simplest extensions of the SM
  - dark matter candidate (Inert Doublet Model)
  - source of CP violation
  - extended scalar sector
  - renormalizable

- renormalization of the two scalar mixing angles in the 2HDM is non-trivial

- previously existing schemes are either numerically unstable, process-dependent or gauge-dependent

- search for a suitable renormalization scheme of the scalar mixing angles
  - full electroweak NLO corrections to all decays within the 2HDM
Motivation (II): Electroweak @1-Loop

- high-precision predictions for branching ratios in BSM models
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- **state-of-the-art** code for branching ratios of Higgs decays in the 2HDM: **HDECAY**
  - off-shell decay modes for final-state massive vector bosons / heavy quarks
  - loop-induced decays to final-state gluon/photon pairs and $Z\gamma$
  - QCD corrections to final-state quark pairs
- electroweak corrections at one-loop are **still missing**
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- **interesting theoretical studies** with one-loop electroweak corrections:
  - differences w.r.t. MSSM one-loop corrections (integrate out SUSY masses)
    - 2HDM as effective theory for the MSSM with heavy sparticles
  - studies on renormalization scheme dependence (estimate of theoretical errors due to missing higher orders)
  - phenomenologically interesting limits (decoupling, alignment, wrong-sign, …)
Gauge Invariance in Electrodynamics / QFTs

- consider classical electrodynamics ("Theo C"): $\vec{E}$ and $\vec{B}$ fields

\[ \vec{E} = -\nabla \Phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A} \]

$\Phi$: scalar potential

$\vec{A}$: vector potential
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  $\vec{A}$ : vector potential

- fields are invariant under simultaneous gauge transformations
  \[ \Phi \rightarrow \Phi - \frac{\partial \Lambda}{\partial t}, \quad \vec{A} \rightarrow \vec{A} + \nabla \Lambda \]
  $\Lambda$ : arbitrary field

  Maxwell’s equations are invariant as well
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- a gauge fixing sets conditions on \( \Lambda \) (and hence, on the potentials)

  - Coulomb gauge: \( \nabla \cdot \vec{A} = 0 \)

  - Lorenz gauge: \( \nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \Phi}{\partial t} = 0 \)

  can be used to simplify Maxwell’s equations
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- in QFTs: dependence on $\xi_V$ introduced through gauge-fixing Lagrangian

  individual Feynman diagrams dependent on $\xi_V$
Cancellation of Gauge Dependences

- $\xi_V$ encodes **redundant** (unphysical) degrees of freedom
  - observables, decay amplitudes, etc. **must not depend** on $\xi_V$
  - cancellation is ensured by BRST symmetry

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- for LO OS processes, cancellation of $\xi_V$ dependences is straightforward

- at higher orders, the cancellation becomes **very intricate**

- possible **violation** of the cancellation: renormalization conditions for mixing angles

  - SM: CKM matrix solved
    

  - 2HDM: **scalar mixing angles**

  ?
Introduction to the 2HDM (I): Potential

- **two complex SU(2)_L Higgs doublets**

\[ \Phi_1 = \left( \begin{array}{c} \omega_1^+ \\ \frac{v_1 + \rho_1 + i \eta_1}{\sqrt{2}} \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} \omega_2^+ \\ \frac{v_2 + \rho_2 + i \eta_2}{\sqrt{2}} \end{array} \right) \]

- **non-vanishing vacuum expectation values (VEVs)** \( v_1, v_2 \) with

\[ v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2 \]
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  \[
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- scalar Lagrangian with **CP- and $\mathbb{Z}_2$-conserving** 2HDM potential:
  \[
  V_{\text{2HDM}} (\Phi_1, \Phi_2) = m_{11}^2 \left( \Phi_1^\dagger \Phi_1 \right) + m_{22}^2 \left( \Phi_2^\dagger \Phi_2 \right) - m_{12}^2 \left[ \left( \Phi_1^\dagger \Phi_2 \right) + \left( \Phi_2^\dagger \Phi_1 \right) \right]
  + \frac{\lambda_1}{2} \left( \Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left( \Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left( \Phi_1^\dagger \Phi_1 \right) \left( \Phi_2^\dagger \Phi_2 \right)
  + \lambda_4 \left( \Phi_1^\dagger \Phi_2 \right) \left( \Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left[ \left( \Phi_1^\dagger \Phi_2 \right)^2 + \left( \Phi_2^\dagger \Phi_1 \right)^2 \right]
  \]
Introduction to the 2HDM (II): Parameters

- **eight** real-valued potential parameters:
  - dimensionless $\lambda_i$ ($i = 1, ..., 5$)
  - mass-squared parameters $m_{11}^2$, $m_{22}^2$ and $m_{12}^2$

- difference w.r.t. MSSM: constants are **fixed through SUSY relations**

- transformation to the Higgs mass basis via **scalar mixing angles**
  - $\alpha$ for the CP-even sector
  - $\beta$ for the CP-odd and charged sector

\[ (H, h, G^0, A, G^\pm, H^\pm) \]
Electroweak Corrections @1-Loop (I)

- **aim:** calculate all 2HDM Higgs boson decays \( @1\text{-loop (electroweak)} \)
- use **perturbation theory** to solve the scattering matrix \( S \) at 1-loop level
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2HDM "electroweak Lagrangian“

Model file

FeynArts

Processes, e.g.

\[ H \rightarrow G^+ b \]

\[ H^\pm \rightarrow H^\pm t \]

---

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Amplitudes

$A = \langle f | S | i \rangle$

Analytically

FeynCalc

Decay Widths

$\Gamma \rightarrow |A|^2$

@1-loop

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![Diagram](image)

- **2HDM „electroweak Lagrangian“**
  - Model file: FeynArts
- **Processes, e.g.**
  - Feynman rules: FeynArts
  - Amplitudes $A = \langle f | S | i \rangle$
    - Analytically: FeynCalc
- **2HDECAY**
  - Full result
  - Python
- **HDECAY**
  - QCD, off-shell, loop-induced
  - FORTRAN
- **Decay Widths**
  - $\Gamma \rightarrow |A|^2$
    - @1-loop

Electroweak Corrections @1-Loop (II)

- all decay topologies are considered \[\rightarrow\text{large number of diagrams}\]
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- decay channels that are considered:

  - $h/H/A \rightarrow f \bar{f}$ ($f = c, s, t, b, \mu, \tau$)
  - $h/H \rightarrow VV$ ($V = W^\pm, Z$)
  - $h/H \rightarrow VS$ ($V = Z, W^\pm$, $S = A, H^\pm$)
  - $H^\pm \rightarrow f \bar{f}$ ($f = c, t, \nu_\mu, \nu_\tau$, $\bar{f} = \bar{s}, \bar{b}, \mu^+, \tau^+$)
  - $h/H \rightarrow SS$ ($S = A, H^\pm$)
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- (semi-)Automated calculation of the decays

- Many diagrams contain UV divergences → Renormalization
Renormalization of the 2HDM (I)

- set of free parameters of the 2HDM (excluding CKM elements, …)

\[ \{ T_{h/H}, \alpha_{em}, m_W, m_Z, m_f, m_h, m_H, m_A, m_{H^\pm}, \alpha, \beta, m_{12}^2, \cdots \} \]
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- renormalization program for the 2HDM:
  - tadpole terms \(\rightarrow\) standard / alternative tadpole scheme
  - mass counterterms \(\rightarrow\) on-shell
  - fine-structure constant \(\rightarrow\) at Z mass
  - soft-\(\mathbb{Z}_2\)-breaking scale \(m_{12}^2 \rightarrow \overline{\text{MS}}\)
  - scalar mixing angles \(\rightarrow\) ?

[full details: MK, Master's thesis (2016), KIT;
Renormalization of the 2HDM (II)

- renormalization of mixing angles $\alpha$ and $\beta$ is non-trivial in the 2HDM
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- renormalization of mixing angles $\alpha$ and $\beta$ is **non-trivial** in the 2HDM

- simplest approach: $\overline{\text{MS}}$ conditions for $\alpha$ and $\beta$
  - can be **numerically unstable**
  - **unsuitable scheme**

Renormalization of the 2HDM (II)

- renormalization of mixing angles $\alpha$ and $\beta$ is non-trivial in the 2HDM

- simplest approach: $\overline{\text{MS}}$ conditions for $\alpha$ and $\beta$
  - can be numerically unstable
  - unsuitable scheme

- other schemes used in literature yield gauge-dependent results
  

- is there a renormalization scheme for the 2HDM satisfying the three criteria
  
  - gauge independence,
  - process independence (i.e. not fixed over a decay width),
  - numerical stability (i.e. leads to moderate NLO corrections)?

Renormalization of the 2HDM (III)

- gauge-independent approach: use the **pinch technique** (PT)

- the PT was worked out
Renormalization of the 2HDM (III)

- **gauge-independent approach**: use the **pinch technique** (PT)

- **the PT was worked out**
  - to all orders in the SM  
  - at one-loop for the 2HDM  

- **PT-based definition** of the scalar mixing angle counterterms:
  - use the pinched scalar self-energies instead of the usual ones

- **properties of the pinched scheme**:
  - process-independent
  - manifestly **gauge-independent** by construction
    - gauge-independent NLO amplitudes
  - numerically stable (depending on the point in parameter space)
    - proposed solution for renormalizing $\delta \alpha$ and $\delta \beta$ in the 2HDM
Numerical Analysis (I)

- we consider the **exemplary process** $H^\pm \rightarrow W^\pm h$

- exemplary parameter points (all other parameters: SM-like):

  \[
  m_h = 125.09 \text{ GeV}, \quad m_H = 742.84 \text{ GeV}, \quad m_A = 700.13 \text{ GeV}, \quad m_{12} = 440.57 \text{ GeV}
  \]

  \[
  \tan \beta = 1.46, \quad \alpha = -0.57, \quad m_{H^\pm} = (654 \cdots 804) \text{ GeV}
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  \end{align*} \]

- keep in mind: the 2HDM contains **a lot of free parameters**
  
  - scanning through the parameter space is possible

- chosen parameter points respect **several constraints**:
  
  - theoretical (boundedness from below, tree-level unitarity, global minimum)
  - experimental (S/T/U parameters, lower bound on $m_{H^\pm}, \ldots$)
Numerical Analysis (II)

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- pOS: “on-shell pinched”
- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

Superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

Relative size of NLO corrections:

\[ \Delta \Gamma = \frac{\Gamma_{NLO} - \Gamma_{LO}}{\Gamma_{LO}} \]
Numerical Analysis (II)

- for pinched schemes: NLO corrections are moderate (up to 20%)
- relatively large difference in finite parts → missing higher orders
  (for full analysis: rescale the parameters → future work)

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scan over large parameter ranges

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Numerical Analysis (III)

- process-dependent scheme: **huge** NLO corrections (unsuitable)
- pinched schemes: well-behaving for **large parameter ranges**

**Relative size of NLO corrections:**

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scan over large parameter ranges
Conclusions and Outlook

- gauge parameter independence: **key principle** for observables in QFTs
- certain renormalization schemes **spoil this independence** in the 2HDM
- a gauge-independent, process-independent and numerically stable scheme for $\delta \alpha$ and $\delta \beta$ was worked out **for the first time for the 2HDM**
- **full** electroweak one-loop corrections to 2HDM Higgs decays calculated
- combination with **state-of-the-art corrections from HDECAY**:
  development of new tool **2HDECAY**
- phenomenological studies (planned):
  - dependence of NLO corrections on **2HDM type**
  - analysis for certain **interesting limits** (decoupling, alignment, …)
  - effect of NLO electroweak corrections on **parameter space restriction**
Thanks!
Corrections to 2HDM Higgs Decays with 2HDECAY

M. Krause:

ITP, KIT
Backup slides
Motivation (III): Gauge Parameter Independence

- many phenomenologically interesting models are based on gauge theories
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- gauge theories imply the need for fixing a gauge, e.g. general $R_\xi$ gauge necessary for removal of redundant degrees of freedom

- the class of $R_\xi$ gauges form an equivalence class of the gauge theory equations of motions, observables, ... must not depend on $\xi$
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- the class of $R_\xi$ gauges form an equivalence class of the gauge theory equations of motions, observables, ... **must not depend** on $\xi$

- higher-order calculations: cancellation of gauge dependences becomes very **intricate**

- in the 2HDM: unsuitable renormalization of mixing angles **spoils gauge parameter independence**
Gauge Invariance in QED

- consider Quantum Electrodynamics with spinors $\Psi(x)$, photon $A_\mu(x)$

- we demand invariance under local $U(1)$ gauge transformations

\[ \Psi(x) \rightarrow e^{i\alpha(x)} \Psi(x) \]
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- we demand invariance under local U(1) gauge transformations
  \[ \Psi(x) \to e^{i\alpha(x)} \Psi(x) \]

- proper inclusion of the transformation: covariant derivative
  \[ D_\mu = \partial_\mu + ieA_\mu(x) \quad \Rightarrow \quad D_\mu \Psi(x) \to e^{i\alpha(x)} D_\mu \Psi(x) \]

- renormalizability: QED Lagrangian up to dim-4 operators
  \[ \mathcal{L}_{\text{QED}} = \overline{\Psi} \left( i\gamma^\mu \partial_\mu - m \right) \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

  ($m$: fermion mass, $F_{\mu\nu}$: photon field strength tensor)
Gauge Dependences in QED

- quantization e.g. through the Faddeev-Popov method:

\[ \mathcal{L}_{\text{QED}} = \bar{\Psi} \left( i \not{D} - m \right) \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{\xi} (\partial_{\mu} A_{\mu})(\partial_{\nu} A^{\nu}) \]

- introduction of gauge-fixing and Lagrangian

  ➔ preservation of unitarity
  ➔ cancellation of unphysical polarization degrees of freedom
Gauge Dependences in QED

- **quantization** e.g. through the *Faddeev-Popov* method:

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- introduction of **gauge-fixing** and Lagrangian

  ➔ preservation of **unitarity**

  ➔ cancellation of **unphysical polarization** degrees of freedom

- Feynman rules depend on **gauge-fixing parameter** \( \xi_V \):

\[ \mu \quad \underbrace{\vphantom{\mu} \vphantom{\nu}}_{\text{dependence}} \nu \quad = \quad \frac{-i}{k^2 - m_V^2} \left[ g_{\mu\nu} - (1 - \xi_V) \frac{k^\mu k_\nu}{k^2 - \xi_V m_V^2} \right] \]

  ➔ introduction of \( \xi_V \) dependence in (loop) calculations
many diagrams contain **UV divergences**, i.e. formally, we have

\[ p^2 \rightarrow \infty \]
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\[ p^2 \rightarrow \infty \rightarrow \infty \]

- use **dimensional regularization** \( (d = 4 - 2\epsilon) \), isolate the divergences:

\[ d = 4 - 2\epsilon \rightarrow \frac{1}{\epsilon} + \text{finite} \]
many diagrams contain UV divergences, i.e. formally, we have

\[ p^2 \to \infty \rightarrow \infty \]

use dimensional regularization \((d = 4 - 2\epsilon)\), isolate the divergences:

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remove the divergences via renormalization

idea: split ‘bare’ parameters into renormalized values and counterterms

\[ m_i^2 \rightarrow m_i^2 + \delta m_i^2 \]

counterterms need to be fixed via renormalization conditions
Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet** \((\phi_1, \phi_2)\)

- field strength renormalization:

\[
\left(\begin{array}{c}
\phi_1 \\
\phi_2
\end{array}\right)_0 = \sqrt{Z_\phi} \left(\begin{array}{c}
\phi_1 \\
\phi_2
\end{array}\right) \approx \left(\begin{array}{c}
1_{2\times 2} + \frac{\delta Z_\phi}{2}
\end{array}\right) \left(\begin{array}{c}
\phi_1 \\
\phi_2
\end{array}\right), \quad \frac{\delta Z_\phi}{2} = \left(\begin{array}{cc}
\frac{\delta Z_{\phi_1 \phi_1}}{2} & \frac{\delta Z_{\phi_2 \phi_1}}{2} \\
\frac{\delta Z_{\phi_1 \phi_2}}{2} & \frac{\delta Z_{\phi_2 \phi_2}}{2}
\end{array}\right)
\]

- two-point correlation function for the doublet with momentum \(p^2\):

\[
\hat{\Gamma}_\phi(p^2) := \left(\begin{array}{cc}
\hat{\Gamma}_{\phi_1 \phi_1}(p^2) & \hat{\Gamma}_{\phi_1 \phi_2}(p^2) \\
\hat{\Gamma}_{\phi_2 \phi_1}(p^2) & \hat{\Gamma}_{\phi_2 \phi_2}(p^2)
\end{array}\right)
\]

\[
= i \sqrt{Z_\phi} \left[ p^2 1_{2\times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2 \right] \sqrt{Z_\phi} \approx i \left[ p^2 1_{2\times 2} - D_\phi^2 + \hat{\Sigma}_\phi(p^2) \right]
\]

- mass matrices \(\leftrightarrow\) mass CTs \(\leftrightarrow\) renormalized self-energies

**1PI self-energies**

\[
i\Sigma(p^2) := \begin{array}{c}
\text{1PI}
\end{array} = \begin{array}{c}
\text{1PI}
\end{array} + \cdots
\]
Renormalization: On-Shell Conditions (II)

- **on-shell conditions:**
  - mixing of fields vanishes for \( p^2 = m_{\phi_i}^2 \)
  - masses \( m_{\phi_i}^2 \) are the real parts of the pole of the propagator
  - normalization: residue of the propagator at its pole equals \( i \)

- **fixation of diagonal mass counterterms:**
  \[
  \text{Re} \left[ \delta D_{\phi_1 \phi_1}^2 \right] = \text{Re} \left[ \Sigma_{\phi_1 \phi_1} (m_{\phi_1}^2) \right], \quad \text{Re} \left[ \delta D_{\phi_2 \phi_2}^2 \right] = \text{Re} \left[ \Sigma_{\phi_2 \phi_2} (m_{\phi_2}^2) \right]
  \]

- **fixation of field strength renormalization constants:**
  \[
  \delta Z_{\phi_1 \phi_1} = -\text{Re} \left[ \frac{\partial \Sigma_{\phi_1 \phi_1} (p^2)}{\partial p^2} \right]_{p^2 = m_{\phi_1}^2}, \quad \delta Z_{\phi_2 \phi_2} = -\text{Re} \left[ \frac{\partial \Sigma_{\phi_2 \phi_2} (p^2)}{\partial p^2} \right]_{p^2 = m_{\phi_2}^2}
  \]
  \[
  \delta Z_{\phi_1 \phi_2} = \frac{2}{m_{\phi_1}^2 - m_{\phi_2}^2} \text{Re} \left[ \Sigma_{\phi_1 \phi_2} (m_{\phi_2}^2) - \delta D_{\phi_1 \phi_2}^2 \right], \quad \delta Z_{\phi_2 \phi_1} = \frac{2}{m_{\phi_2}^2 - m_{\phi_1}^2} \text{Re} \left[ \Sigma_{\phi_1 \phi_2} (m_{\phi_1}^2) - \delta D_{\phi_1 \phi_2}^2 \right]
  \]

- **the specific form of the \( \delta D_{\phi_i \phi_j}^2 \) depends on the tadpole scheme**
Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:

\[
\begin{align*}
    i T_{1/2} & - i \delta T_{1/2} = 0 \\
    i T_{H^0/h^0} & - i \delta T_{H^0/h^0} = 0
\end{align*}
\]

- conversion from gauge to mass basis:

\[
\begin{pmatrix}
    \delta T_1 \\
    \delta T_2
\end{pmatrix} =
\begin{pmatrix}
    c_\alpha & -s_\alpha \\
    s_\alpha & c_\alpha
\end{pmatrix}
\begin{pmatrix}
    \delta T_{H^0} \\
    \delta T_{h^0}
\end{pmatrix} =
\begin{pmatrix}
    c_\alpha \delta T_{H^0} - s_\alpha \delta T_{h^0} \\
    s_\alpha \delta T_{H^0} + c_\alpha \delta T_{h^0}
\end{pmatrix}
\]

- **purpose**: restoring the minimum conditions of the potential at NLO

- **practical effect**: no tadpole diagrams in NLO calculations
Renormalization: Standard Tadpole Scheme

- **standard scheme**: vevs are derived from the loop-corrected potential
  (e.g. in A. Denner: arXiv:0709.1075)

- vevs in the mass relations produce correct one-loop OS masses, e.g.
  \[ m_W^2 = g^2 \frac{v^2}{4}, \quad m_{A_0}^2 = v^2 \left( \frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right) \]

- tadpole terms appear explicitly in the bare mass matrices
  \[ \Rightarrow \text{mass matrix counterterms contain the tadpole counterterms:} \]
  \[ \delta D^2_{\phi} \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \begin{pmatrix} \delta T_{\phi_1 \phi_1} & \delta T_{\phi_1 \phi_2} \\ \delta T_{\phi_1 \phi_2} & \delta T_{\phi_2 \phi_2} \end{pmatrix} \]

- one-loop corrected potential is gauge-dependent
  \[ \Rightarrow \text{vevs are gauge-dependent} \]
  \[ \Rightarrow \text{mass counterterms become gauge-dependent} \]
alternative scheme: vevs represent the same minimum as at tree level

bare masses are expressed through gauge-independent tree-level vevs ➔ mass CTs become gauge-independent

correct minimum conditions @NLO require a shift in the vevs

\[ v_1 \rightarrow v_1 + \delta v_1, \quad v_2 \rightarrow v_2 + \delta v_2 \]

fixation of the shifts by applying the tadpole conditions:

\[
\begin{pmatrix}
\delta v_1 \\
\delta v_2
\end{pmatrix} = 
\begin{pmatrix}
\frac{\delta T_{H^0}}{m_{H^0}^2} c_\alpha - \frac{\delta T_{h_0}}{m_{h_0}^2} s_\alpha \\
\frac{\delta T_{h_0}}{m_{H^0}^2} s_\alpha + \frac{\delta T_{h_0}}{m_{h_0}^2} c_\alpha
\end{pmatrix}
\]

the shifts translate into every CT, wave function renormalization constants and Feynman rules

alternative tadpole scheme worked out for the 2HDM
Renormalization: Alternative Tadpole Scheme

- **example:** *W* boson mass

\[ m_W^2 = g^2 \frac{v^2}{4} \rightarrow m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} = m_W^2 + i \left( W^\pm \right)_H^0 W^\pm + i \left( W^\pm \right)_h^0 W^\pm \]

- **example:** coupling between Higgs and *Z* bosons

\[ ig_{H^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2} (c_\alpha v_1 + s_\alpha v_2) \quad , \quad ig_{H^0 h^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2} \]

\[ ig_{H^0 Z^0 Z^0} \rightarrow ig_{H^0 Z^0 Z^0} + \frac{ig^2}{2c_W^2} (c_\alpha \delta v_1 + s_\alpha \delta v_2) = ig_{H^0 Z^0 Z^0} + \left( \begin{array}{c} H^0 \\ \hline \\ \hline \\ Z^0 \end{array} \right)_{\text{trunc}} \]

- **effects** of the alternative tadpole scheme:
  - tadpole diagrams are added everywhere where they exist in the 2HDM
  - mass counterterms become *manifestly gauge-independent*
  - tadpole counterterms in the scalar sector are removed
Renormalization of the 2HDM (II)

- **no-go theorem** for the MSSM: a renormalization scheme for $\tan \beta$
  - may not be simultaneously
    - gauge-independent
    - process-independent
    - numerically stable (i.e. leads to moderate NLO corrections)

Renormalization: Scalar Mixing Angles

- approach by S. Kanemura et al.: connect the definition of $\alpha$ and $\beta$ with the inverse propagator matrix

$$\left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right) \sim R_{\theta,0}^T \left( \begin{array}{c} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{array} \right) \approx R_{\delta \theta}^T R_{\theta}^T \sqrt{Z_\phi} R_{\theta} R_{\theta}^T \left( \begin{array}{c} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{array} \right) \approx \left( \begin{array}{c} 1 + \frac{\delta Z_{\phi_1 \phi_1}}{2} \\ \delta C_{\phi_2} + \delta \theta \end{array} \right) \left( \begin{array}{c} \delta^C_{\phi_2} - \delta \theta \\ 1 + \frac{\delta Z_{\phi_2 \phi_2}}{2} \end{array} \right) \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right)$$

- mixing angle counterterms within the standard tadpole scheme:

$$\delta \alpha = \frac{1}{2 \left( m_{H^0}^2 - m_{h^0}^2 \right)} \text{Re} \left[ \Sigma_{H^0 h^0} (m_{H^0}^2) + \Sigma_{H^0 h^0} (m_{h^0}^2) - 2 \delta T_{H^0 h^0} \right]$$

$$\delta \beta = - \frac{1}{2 m_{H^\pm}^2} \text{Re} \left[ \Sigma_{G^\pm H^\pm} (m_{H^\pm}^2) + \Sigma_{G^\pm H^\mp} (0) - 2 \delta T_{G^\pm H^\mp} \right]$$


- it was shown analytically that Kanemura’s scheme introduces an intricate gauge-dependence in $\delta \alpha$ and $\delta \beta$

(M. Krause, Master’s thesis, Karlsruhe Institute of Technology, 2016)
we consider a fermion scattering process at one-loop QCD:

\[ A_{\text{full}}(s, t, m_1, m_2) = A_{\text{box}}(s, t, m_1, m_2; \xi) + A_{\text{tri}}(t, m_1, m_2; \xi) + A_{\text{self}}(t; \xi) \]

the gauge dependences have to cancel within the individual topologies

\[ s = (r_1 + p_1)^2 = (r_2 + p_2)^2 \]
\[ t = (r_1 - r_2)^2 = (p_1 - p_2)^2 \]

\[ \Rightarrow \text{rearrangement of the contributions is always possible} \]

\[ \Rightarrow \text{rearrangement shows that all gauge dependences have self-energy-like or triangle-like form} \]

\[ A_{\text{full}}(s, t, m_1, m_2) = \tilde{A}_{\text{box}}(s, t, m_1, m_2) + \tilde{A}_{\text{tri}}(t, m_1, m_2) + \tilde{A}_{\text{self}}(t) \]

\[ A_{\text{tri}}(t, m_1, m_2; \xi) \rightarrow \tilde{A}_{\text{tri}}(t, m_1, m_2) + f_{\text{self}}(t; \xi) \]

etc.
Pinch Technique: Introduction (II)

- determination of the gauge-dependent contributions: “pinching”

- main idea: trigger the **elementary Ward identity** for the loop momentum

\[
\bar{k} = (\bar{k} + \bar{\phi} - \bar{m}) - (\bar{\phi} - \bar{m}) = S^{-1}(k + p) - S^{-1}(p)
\]

- right expression: vanishes OS between spinors

- left expression: **cancels** (“pinches out”) an **internal fermion** propagator

\[
k \equiv S^{-1}(p_2 + k) - S^{-1}(p_2)
\]
Pinch Technique: Results (I)

- (almost) all pinch contributions are proportional to $(1 - \xi)$

- the non-pinched contributions are equivalent to diagrams calculated in Feynman-‘t Hooft gauge, i.e. for $\xi \equiv 1$

- the pinch contributions are self-energy like, i.e. functions of only $t$ 
  ➔ reallocation of pinch contributions to the gluon self-energy possible
Pinch Technique: Results (II)

- sum of all pinch contributions ➔ cancellation of gauge dependences

\[
\begin{align*}
& g_s^2 t (1 - \xi)^2 \int k \frac{k^\mu k'^\nu}{k^4 (k+q)^4} & & g_s^2 t (1 - \xi) \int k \frac{k^\mu k'^\nu}{k^4 (k+q)^2} & & g_s^2 t (1 - \xi) \int k \frac{q^\mu}{k^4} & & g_s^2 t (1 - \xi) \int k \frac{q^\mu}{k^2} \\
& i \Sigma_{\text{box}}^\mu & & t \frac{C_A}{2} & & 0 & & -t C_A & & 0 \\
& i \Sigma_{\text{tri1}}^\mu & & 0 & & 0 & & 0 & & C_A - 2 C_f \\
& i \Sigma_{\text{tri2}}^\mu & & -t C_A & & 2 C_A & & 2 t C_A & & -2 C_A \\
& i \Sigma_{\text{self,q}}^\mu & & 0 & & 0 & & 0 & & 2 C_f \\
& i \Sigma_{\text{self,g}}^\mu & & t \frac{C_A}{2} & & -2 C_A & & -t C_A & & C_A \\
& \text{Sum} & & 0 & & 0 & & 0 & & 0 \\
\end{align*}
\]  

\( q^2 \equiv t \)

- main results from the application of the pinch technique:
  - demonstration of intricate cancellation of gauge dependences
  - cancellation is not accidental, but follows from Ward identities

\( C_A, C_f \) : Casimir operators
Gauge-Independent Self-Energies via PT

- all pinch contributions are self-energy-like
  ➔ **reallocate** pinch contributions to the gluon self-energy

- the pinched self-energy is equivalent to the one evaluated for $\xi \equiv 1$
after the cancellation of all gauge dependences
  ➔ Feynman-‘t Hooft-gauge is a **special gauge choice**

**Interesting properties** of the pinched gluon self-energy:

- analogy to the gluon self-energy given by the **Background Field Method**
- **uniquely defined** by the pinch technique framework
- manifestly **gauge-independent** ➔ allows for gauge-independent **counterterms**
- obeys **QED-like Ward identities** instead of complicated Slavnov-Taylor identities

*[for more details cf. e.g. D. Binosi, J. Papavassiliou, Phys. Rep. 479 (2009) 1]*
Applications of the Pinch Technique

- the pinch technique can be applied to e.g. the SM, MSSM, (N)2HDM, …

- for consistency: tadpole diagrams have to be taken into account → “alternative tadpole scheme” is needed (cf. part II of the talk)

- applications of the pinched self-energies:
  - definition of gauge-independent counterterms (cf. part III of the talk)
  - construction of QED-like Ward identities for e.g. QCD
  - gauge-independent definition of electroweak parameters
  - consistent resummation for resonant transition amplitudes
  - extraction of gauge-independent part of BFM self-energies

Implementation: 2HDECAY (I)

2HDM „electroweak Lagrangian“

Model file
FeynArts

Processes, e.g.
Feynman rules

Amplitudes
\[ A = \langle f | S | i \rangle \]

Analytically
FeynCalc

Decay Widths
\[ \Gamma \rightarrow |A|^2 \]

@1-loop

2HDECAY
Full result

HDECAY
QCD, off-shell, loop-induced

Python

FORTRAN
Implementation: 2HDECAY (I)

2HDECAY: “2HDM HDECAY“
A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

Implementation: 2HDECAY (II)

List of input files

$2HDECAY/\text{Input}$

2HDECAY.py

$2HDECAY$

HDECAY (minimal run)

$2HDECAY/\text{HDECAY}$

electroweakCorrections

$2HDECAY$

HDECAY

$2HDECAY/\text{HDECAY}$

Iterate over all input files

List of output files

$2HDECAY/\text{Results}$

$m_c(\text{OS})$, $m_b(\text{OS})$

Numerical Analysis (II)

\[ m_h = 125.09 \text{ GeV}, \quad m_H = 742.84 \text{ GeV} \]
\[ m_A = 700.13 \text{ GeV}, \quad m_{12} = 440.57 \text{ GeV} \]
\[ \tan \beta = 1.46, \quad \alpha = -0.57 \]

- proc: process-dependent
- pOS: “on-shell pinched”
- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

\[ \Delta \Gamma = \frac{\Gamma_{NLO} - \Gamma_{LO}}{\Gamma_{LO}} \]
kinks: **thresholds** for certain mass configurations

process-dependent scheme is often **unsuitable** (large NLO corrections)

\[
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**p***: “p*-pinched”

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Numerical Analysis (IV)

- for LO approaching zero, $\Delta \Gamma$ may become large (**numerical instability**)
- numerical instability is “artificial” (**no** problem of renormalization scheme)

\[
\Delta \Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}
\]

- proc: process-dependent
- pOS: “on-shell pinched”
- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

superscript “c”: definition over charged sector

scan over large parameter ranges