Corrections to 2HDM Higgs Decays with 2HDECAY

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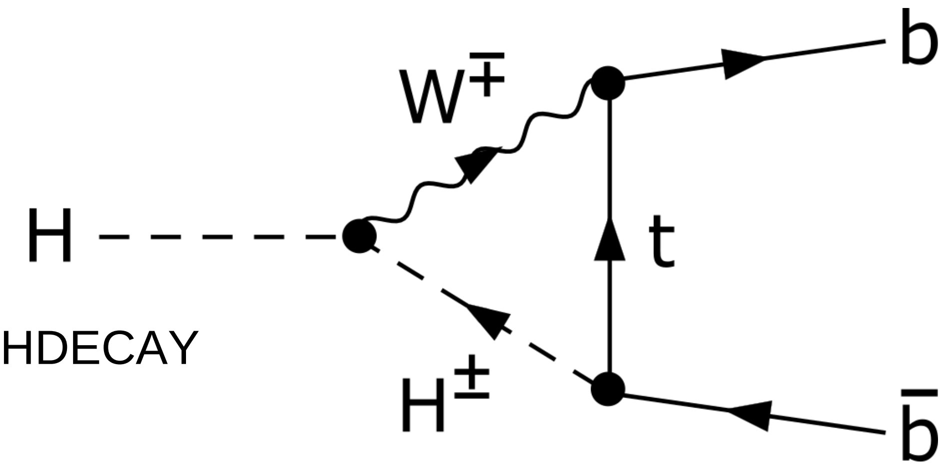
[MK, M. M. Mühlleitner, M. Spira, *in preparation,* arxiv:18MM.XXXXX]

GK-Workshop 2018

September 14, 2018

- Motivation
- Principle of Gauge Invariance
- Introduction to the 2HDM
- Automated 1-Loop Calculations with 2HDECAY
- Numerical Results



Motivation (I): Two-Higgs-Doublet Model

- 2HDM: one of the simplest extensions of the SM
  - dark matter candidate (*Inert Doublet Model*)
  - source of CP violation
  - **extended scalar sector**
  - renormalizable

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  - renormalizable

- renormalization of the two **scalar mixing angles** in the 2HDM is non-trivial

- previously existing schemes are either numerically unstable, process-dependent or **gauge-dependent**

- search for a suitable renormalization scheme of the scalar mixing angles

  ➡ full electroweak NLO corrections to all decays within the 2HDM

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  - off-shell decay modes for final-state massive vector bosons / heavy quarks
  - loop-induced decays to final-state gluon/photon pairs and $Z\gamma$
  - QCD corrections to final-state quark pairs
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- **interesting theoretical studies** with one-loop electroweak corrections:
  - differences w.r.t. MSSM one-loop corrections (integrate out SUSY masses)
    ➡ 2HDM as effective theory for the MSSM with heavy sparticles
  - studies on renormalization scheme dependence (estimate of theoretical errors due to missing higher orders)
  - phenomenologically interesting limits (decoupling, alignment, wrong-sign, …)

Gauge Invariance in Electrodynamics / QFTs

- consider classical electrodynamics ("Theo C"): $\vec{E}$ and $\vec{B}$ fields

$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t} \; , \quad \vec{B} = \nabla \times \vec{A}$$

Φ : scalar potential

$\vec{A}$: vector potential

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$$\Phi \longrightarrow \Phi - \frac{\partial \Lambda}{\partial t} \,, \quad \vec{A} \longrightarrow \vec{A} + \nabla\Lambda$$

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- a **gauge fixing** sets conditions on Λ (and hence, on the potentials)
  - Coulomb gauge: $\nabla\cdot\vec{A} = 0$
  - Lorenz gauge: $\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\Phi}{\partial t} = 0$

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- in QFTs: dependence on ξ_V introduced through gauge-fixing Lagrangian

➡ **individual** Feynman diagrams **dependent** on ξ_V

Cancellation of Gauge Dependences

- ξ_V encodes **redundant** (unphysical) degrees of freedom
  - ⇒ observables, decay amplitudes, etc. **must not depend** on ξ_V
  - ⇒ cancellation is ensured by BRST symmetry

[C. Becchi, A. Rouet, R. Stora, *Ann. Phys.* **98** (1976) 287; M. Z. Iofa, I. V. Tyutin, *Theor. Math. Phys.* **27** (1976) 316]

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- for LO OS processes, cancellation of ξ_V dependences is straightforward

- at higher orders, the cancellation becomes **very intricate**

- possible **violation** of the cancellation: renormalization conditions for mixing angles
  - SM: CKM matrix ⇒ solved

    [B.A. Kniehl, F. Madricardo, M. Steinhauser, *Phys.Rev.* D**62** (2000) 073010; Y. Yamada, *Phys.Rev.* D**64** (2001) 036008; P. Gambino, P.A. Grassi, F. Madricardo, *Phys.Lett.* B**454** (1999) 98-104; A. Barroso, L. Brucher, R. Santos, *Phys.Rev.* D**62** (2000) 096003]

  - 2HDM: **scalar mixing angles** ⇒ **?**

Introduction to the 2HDM (I): Potential

- **two** complex $SU(2)_L$ Higgs **doublets**

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix} \;, \qquad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- non-vanishing **vacuum expectation values** (VEVs) v_1, v_2 with

$$v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$

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- scalar Lagrangian with **CP- and $\mathbb{Z}_2$-conserving** 2HDM potential:

$$\begin{aligned} V_{\text{2HDM}}(\Phi_1,\Phi_2) = {} & m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) - m_{12}^2\left[\left(\Phi_1^\dagger\Phi_2\right)+\left(\Phi_2^\dagger\Phi_1\right)\right] \\ & + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) \\ & + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2+\left(\Phi_2^\dagger\Phi_1\right)^2\right] \end{aligned}$$

Introduction to the 2HDM (II): Parameters

- **eight** real-valued potential parameters:
  - dimensionless $\lambda_i \; (i = 1, ..., 5)$
  - mass-squared parameters m_{11}^2, m_{22}^2 and m_{12}^2
- difference w.r.t. MSSM: constants are **fixed through SUSY relations**
- transformation to the Higgs mass basis via **scalar mixing angles**
  - α for the CP-even sector
  - β for the CP-odd and charged sector

$\Rightarrow$ $(H, h, G^0, A, G^{\pm}, H^{\pm})$

Electroweak Corrections @1-Loop (I)

- **aim:** calculate all 2HDM Higgs boson decays **@1-loop (electroweak)**
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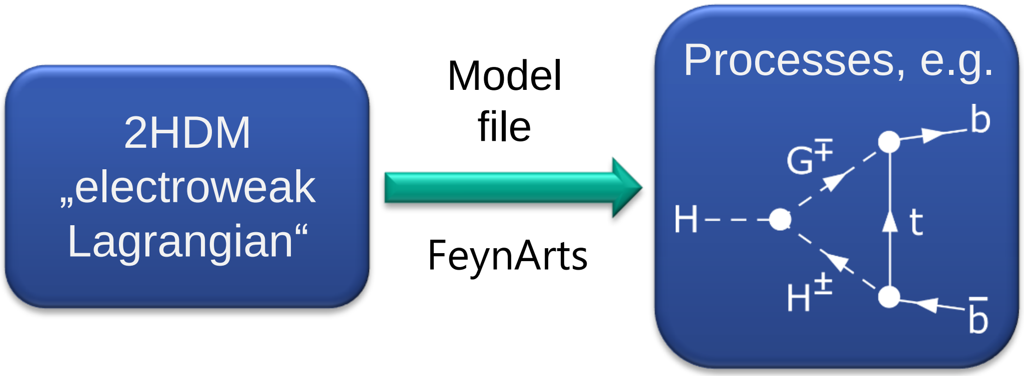

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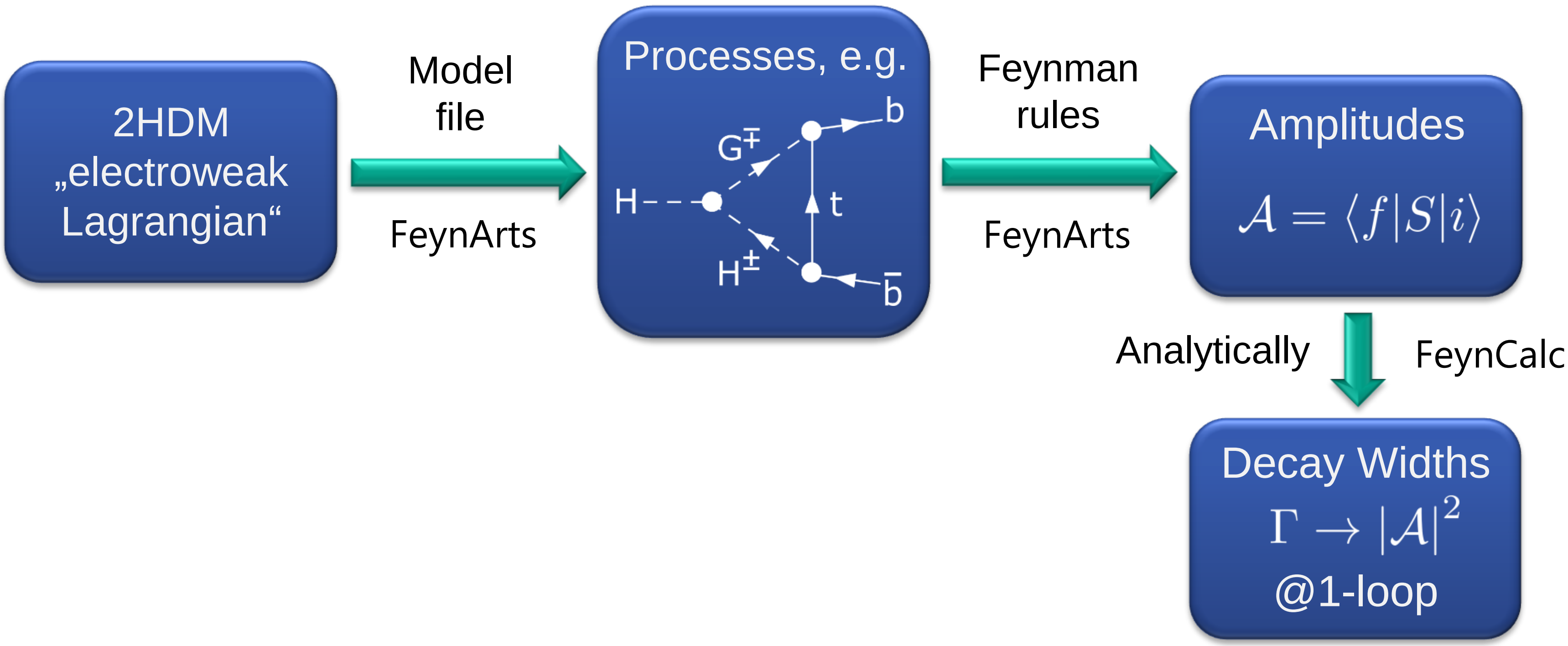

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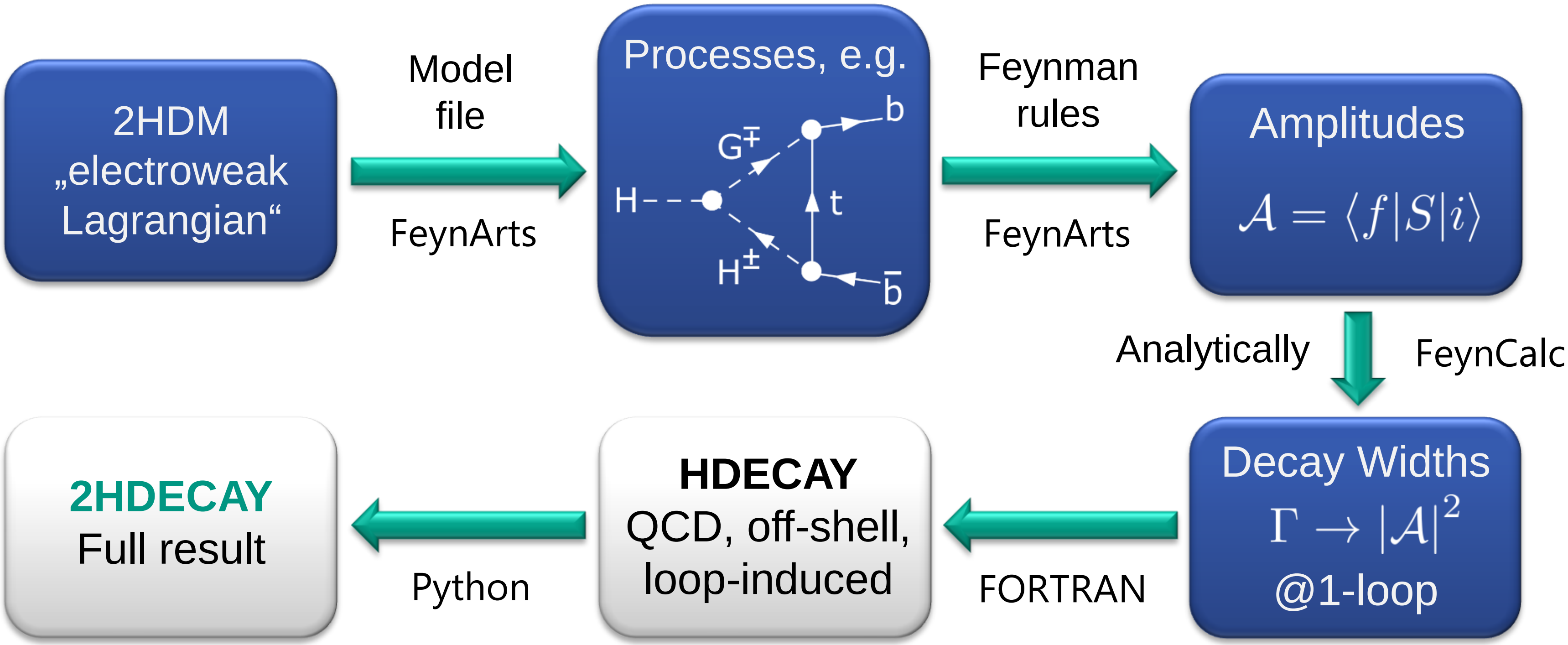

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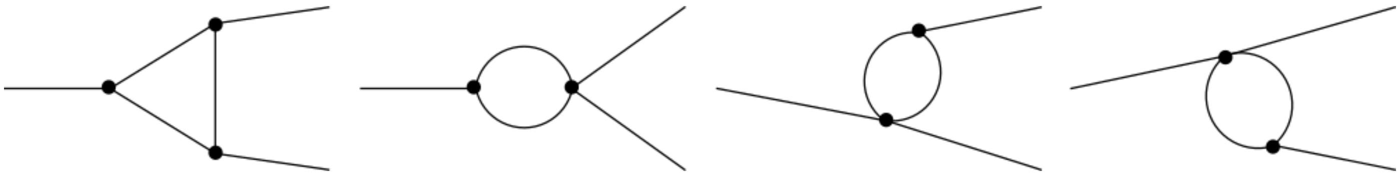

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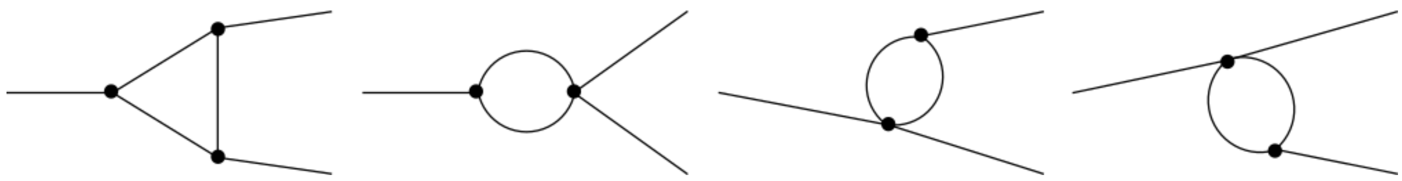

- decay channels that are considered:

- $h/H/A \to f\bar{f}$ $(f = c, s, t, b, \mu, \tau)$
- $h/H \to VV$ $(V = W^{\pm}, Z)$
- $h/H \to VS$ $(V = Z, W^{\pm},\ S = A, H^{\pm})$
- $H^{\pm} \to f\bar{f}$ $(f = c, t, \nu_\mu, \nu_\tau\ ,\ \bar{f} = \bar{s}, \bar{b}, \mu^+, \tau^+)$
- $h/H \to SS$ $(S = A, H^{\pm})$
- $H^{\pm} \to VS$ $(V = W^{\pm},\ S = h, H, A)$
- $A \to VS$ $(V = Z, W^{\pm},\ S = h, H, H^{\pm})$
- $H \to hh$

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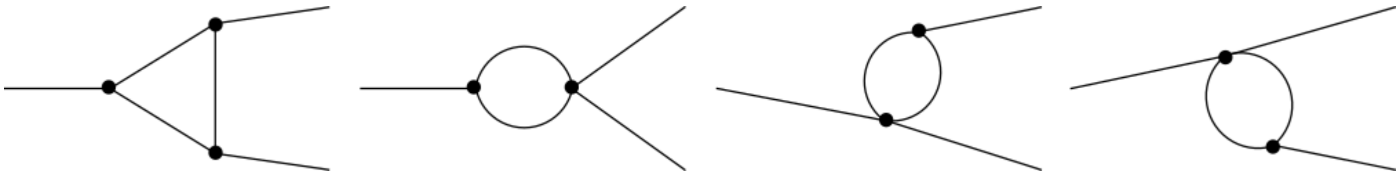

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- (semi-)**automated** calculation of the decays
- many diagrams contain **UV divergences** ⟹ **renormalization**

Renormalization of the 2HDM (I)

- set of free parameters of the 2HDM (excluding CKM elements, …)

$$\left\{ T_{h/H},\ \alpha_{\text{em}},\ m_W,\ m_Z,\ m_f\ ,\ m_h,\ m_H,\ m_A,\ m_{H^\pm},\ \alpha,\ \beta,\ m_{12}^2\ ,\cdots \right\}$$

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- renormalization program for the 2HDM:
  - tadpole terms ⇒ standard / **alternative** tadpole scheme
  - mass counterterms ⇒ on-shell
  - fine-structure constant ⇒ at Z mass
  - soft-$\mathbb{Z}_2$-breaking scale m_{12}^2 ⇒ $\overline{\text{MS}}$
  - **scalar mixing angles** ⇒ **?**

[full details: MK, *Master's thesis* (2016), KIT;
MK, R. Lorenz, M. M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;
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Renormalization of the 2HDM (II)

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- simplest approach: $\overline{\mathrm{MS}}$ conditions for α and β
  ➔ can be **numerically unstable**
  ➔ **unsuitable scheme**

[R. Lorenz, *Master's thesis* (2015), Karlsruhe Institute of Technology; MK, R. Lorenz, M. M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143]

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- other schemes used in literature yield **gauge-dependent** results

  [S. Kanemura *et al.*, *Phys. Rev.* D**70** (2004) 115002]

- is there a renormalization scheme for the 2HDM **satisfying the three criteria**
  - gauge independence,
  - **process independence** (i.e. not fixed over a decay width),
  - numerical stability (i.e. leads to **moderate** NLO corrections)?

Renormalization of the 2HDM (III)

- gauge-independent approach: use the **pinch technique** (PT)
- the PT was worked out
  - to all orders in the SM [D. Binosi, *J. Phys.* **G30** (2004) 1021]
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- **PT-based definition** of the scalar mixing angle counterterms:
  use the pinched scalar self-energies instead of the usual ones
- properties of the pinched scheme:
  - **process-independent**
  - manifestly **gauge-independent** by construction

    ⇒ gauge-independent NLO **amplitudes**
  - **numerically stable** (depending on the point in parameter space)

    ⇒ proposed solution for renormalizing $\delta\alpha$ and $\delta\beta$ in the 2HDM

Numerical Analysis (I)

- we consider the **exemplary process** $H^\pm \to W^\pm h$
- exemplary parameter points (all other parameters: SM-like):

$$m_h = 125.09\,\text{GeV},\; m_H = 742.84\,\text{GeV},\; m_A = 700.13\,\text{GeV},\; m_{12} = 440.57\,\text{GeV}$$
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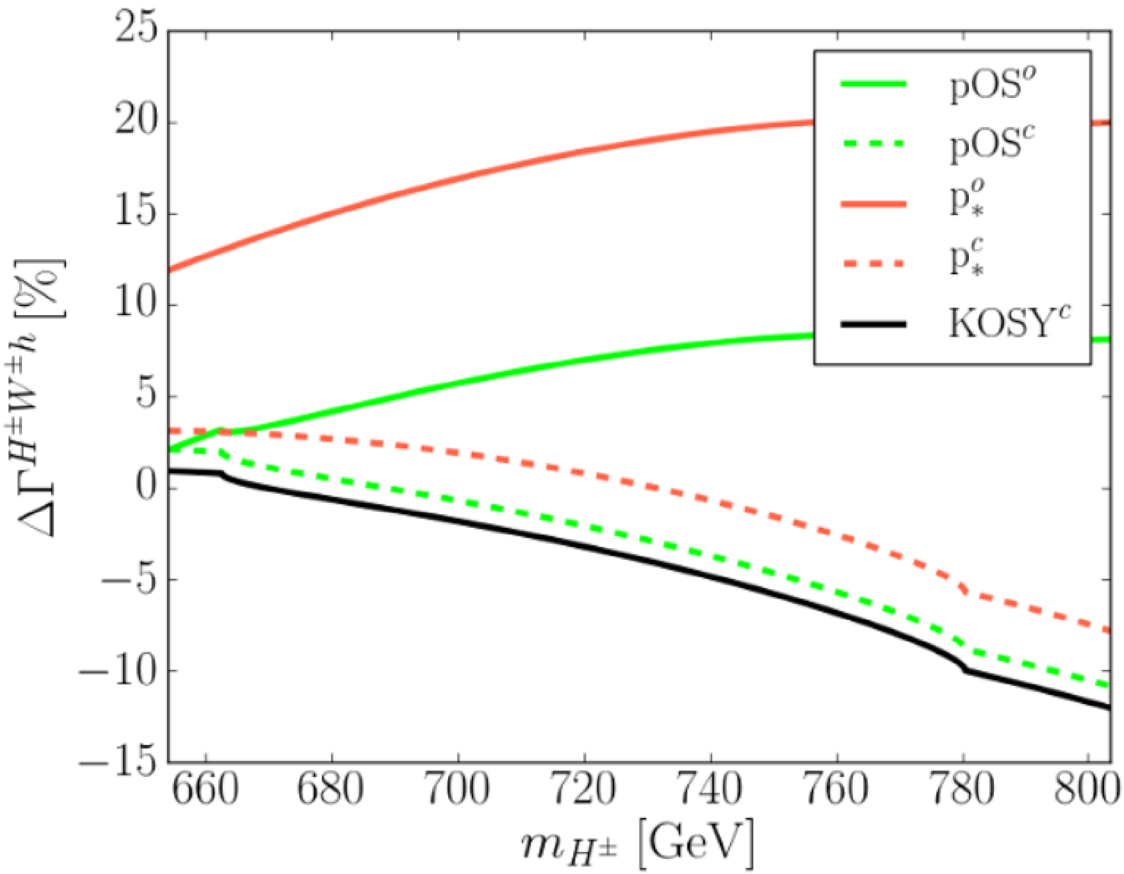
- keep in mind: the 2HDM contains **a lot of free parameters**

  ⇒ scanning through the parameter space is possible

- chosen parameter points respect **several constraints**:
  - theoretical (boundedness from below, tree-level unitarity, global minimum)
  - experimental (S/T/U parameters, lower bound on $m_{H^\pm}$, …)

Numerical Analysis (II)



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- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

relative size of NLO corrections:

$$\Delta\Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

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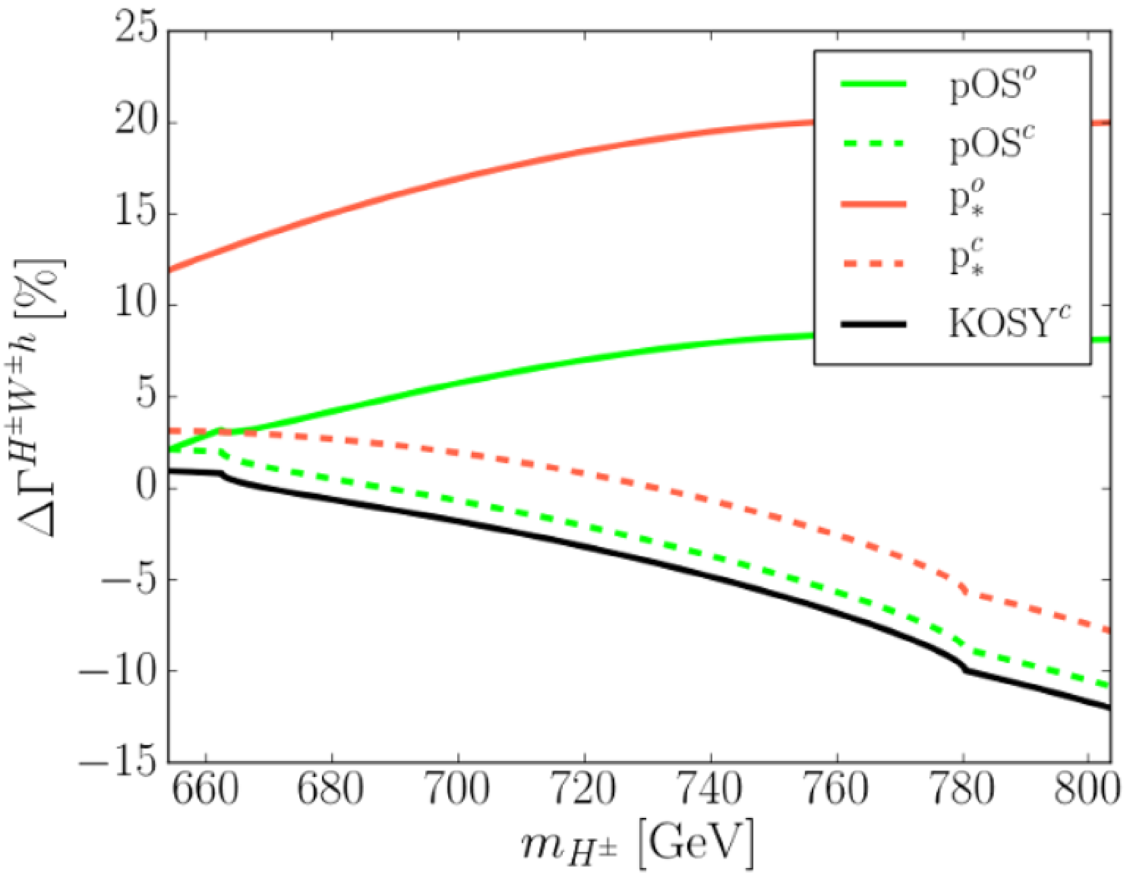
Karlsruhe Institute of Technology



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- for pinched schemes: NLO corrections are **moderate** (up to 20%)
- relatively large difference in finite parts ➡ missing higher orders (for full analysis: **rescale the parameters** ➡ future work)

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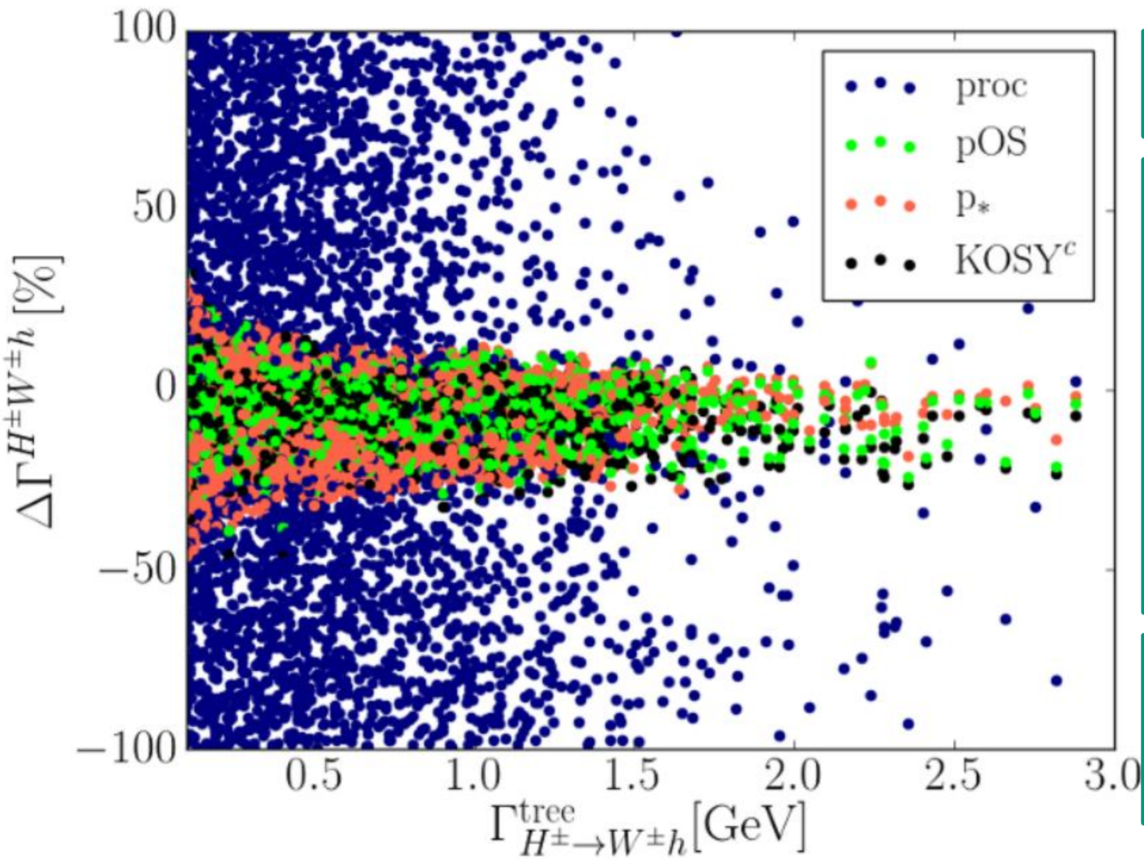

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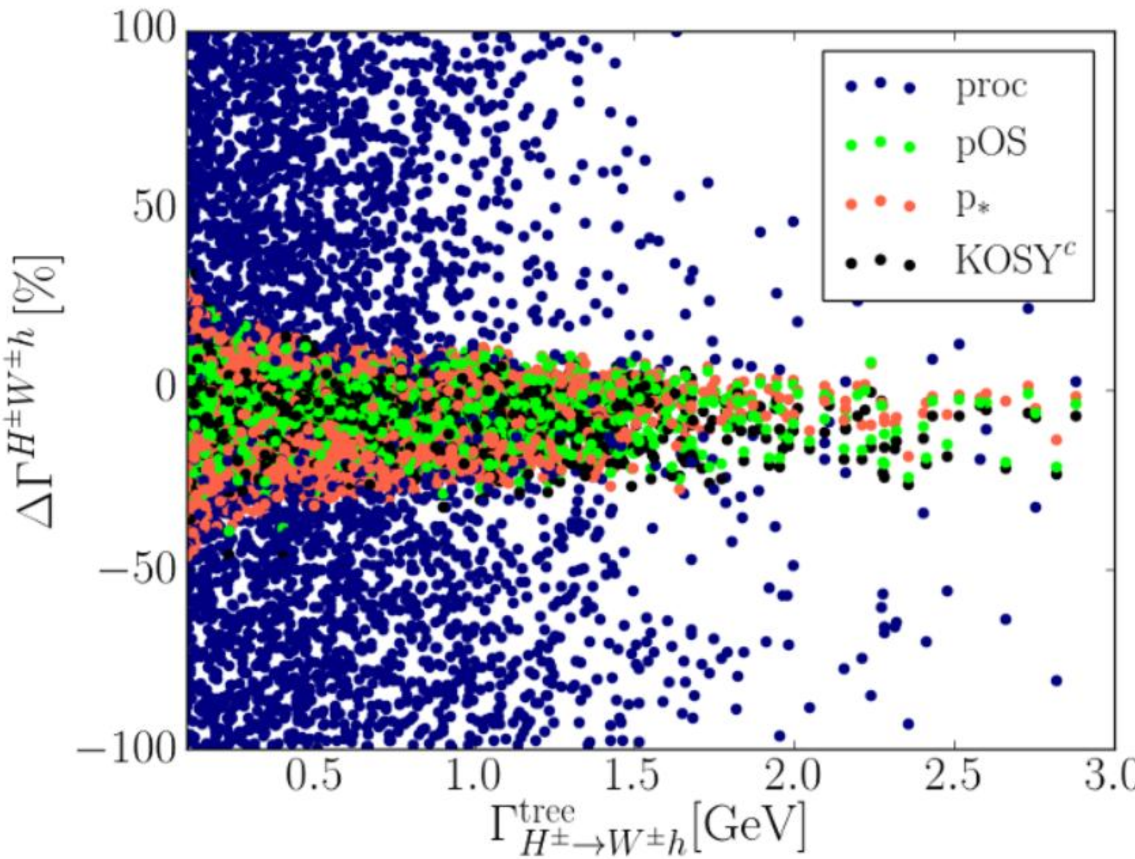

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- process-dependent scheme: **huge** NLO corrections (**unsuitable**)
- pinched schemes: well-behaving for **large parameter ranges**

Conclusions and Outlook

- gauge parameter independence: **key principle** for observables in QFTs
- certain renormalization schemes **spoil this independence** in the 2HDM
- a gauge-independent, process-independent and numerically stable scheme for $\delta\alpha$ and $\delta\beta$ was worked out **for the first time for the 2HDM**
- **full** electroweak one-loop corrections to 2HDM Higgs decays calculated
- combination with **state-of-the-art corrections from HDECAY**: development of new tool **2HDECAY**
- phenomenological studies (planned):
  - dependence of NLO corrections on **2HDM type**
  - analysis for certain **interesting limits** (decoupling, alignment, …)
  - effect of NLO electroweak corrections on **parameter space restriction**

Thanks!

Backup slides

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- the class of R_ξ gauges form an equivalence class of the gauge theory
  ➡ equations of motions, observables, ... **must not depend** on ξ
- higher-order calculations: cancellation of gauge dependences becomes very **intricate**
- in the 2HDM: unsuitable renormalization of mixing angles **spoils gauge parameter independence**

Gauge Invariance in QED

- consider Quantum Electrodynamics with spinors $\Psi(x)$, photon $A_\mu(x)$

- we demand **invariance** under **local U(1) gauge transformations**

$$\Psi(x) \ \rightarrow \ e^{i\alpha(x)}\Psi(x)$$

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- proper inclusion of the transformation: **covariant derivative**

$$D_\mu = \partial_\mu + ieA_\mu(x) \quad \Longrightarrow \quad D_\mu\Psi(x) \;\to\; e^{i\alpha(x)}D_\mu\Psi(x)$$

- renormalizability: **QED Lagrangian** up to dim-4 operators

$$\mathcal{L}_{\text{QED}} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

(m : fermion mass, $F_{\mu\nu}$: photon field strength tensor)

Gauge Dependences in QED

- **quantization** e.g. through the **Faddeev-Popov** method:

$$\mathcal{L}_{\mathrm{QED}} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{\xi}(\partial^\mu A_\mu)(\partial_\mu A^\mu)$$

- introduction of **gauge-fixing** and Lagrangian

  ➔ preservation of **unitarity**

  ➔ cancellation of **unphysical polarization** degrees of freedom

Gauge Dependences in QED

- **quantization** e.g. through the **Faddeev-Popov** method:

$$\mathcal{L}_{\text{QED}} = \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \underbrace{\frac{1}{\xi}(\partial^\mu A_\mu)(\partial_\mu A^\mu)}$$

- introduction of **gauge-fixing** and Lagrangian

  ➔ preservation of **unitarity**

  ➔ cancellation of **unphysical polarization** degrees of freedom

- Feynman rules depend on **gauge-fixing parameter** ξ_V :

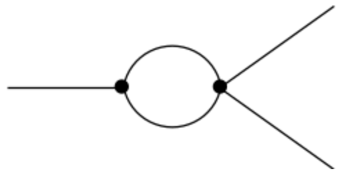
μ  ν

$$= \frac{-i}{k^2 - m_V^2}\left[g_{\mu\nu} - (1-\xi_V)\frac{k^\mu k^\nu}{k^2 - \xi_V m_V^2}\right]$$

➡ introduction of ξ_V dependence in (loop) calculations

Electroweak Corrections @1-Loop (III)

- many diagrams contain **UV divergences**, i.e. formally, we have



$$\xrightarrow{\quad p^2 \to \infty \quad} \infty$$

Electroweak Corrections @1-Loop (III)

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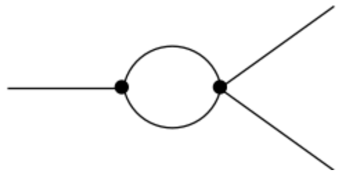
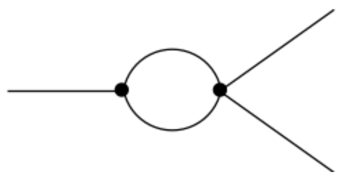


$$\xrightarrow{\quad p^2 \to \infty \quad} \infty$$

- use **dimensional regularization** ($d = 4 - 2\epsilon$), isolate the divergences:



$$\xrightarrow{\quad d = 4 - 2\epsilon \quad} \frac{1}{\epsilon} + \text{finite}$$

Electroweak Corrections @1-Loop (III)

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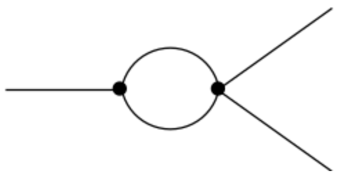

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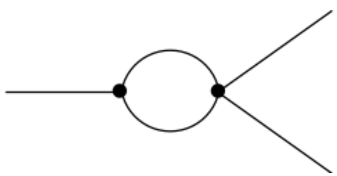


$$\xrightarrow{\;d = 4 - 2\epsilon\;} \frac{1}{\epsilon} + \text{finite}$$

- remove the divergences via **renormalization**

- idea: split ‘bare’ parameters into renormalized values and **counterterms**

$$m_i^2 \to m_i^2 + \delta m_i^2$$

- counterterms need to be fixed via **renormalization conditions**

Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet** $(\phi_1,\ \phi_2)$

- field strength renormalization:

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = \sqrt{Z_\phi}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \approx \left(1_{2\times 2} + \frac{\delta Z_\phi}{2}\right)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \quad , \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix}\frac{\delta Z_{\phi_1\phi_1}}{2} & \frac{\delta Z_{\phi_1\phi_2}}{2}\\ \frac{\delta Z_{\phi_2\phi_1}}{2} & \frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}$$

- two-point correlation function for the doublet with momentum p^2:

$$\widehat{\Gamma}_\phi(p^2) := \begin{pmatrix}\widehat{\Gamma}_{\phi_1\phi_1}(p^2) & \widehat{\Gamma}_{\phi_1\phi_2}(p^2)\\ \widehat{\Gamma}_{\phi_1\phi_2}(p^2) & \widehat{\Gamma}_{\phi_2\phi_2}(p^2)\end{pmatrix}$$

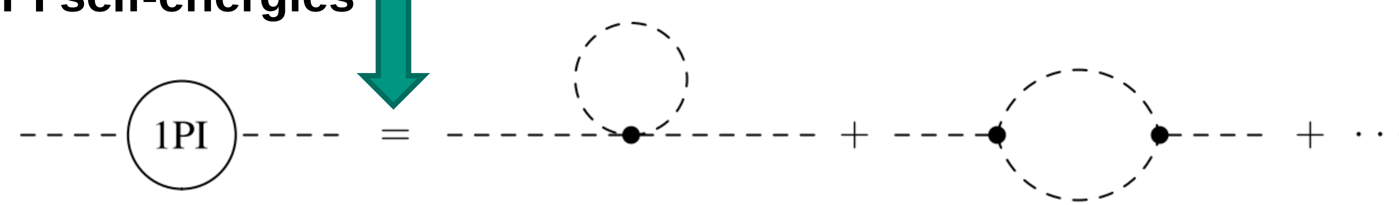
$$= i\sqrt{Z_\phi}^\dagger\left[p^2 1_{2\times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2\right]\sqrt{Z_\phi} \approx i\left[p^2 1_{2\times 2} - D_\phi^2 + \widehat{\Sigma}_\phi(p^2)\right]$$

mass matrices

mass CTs

renormalized self-energies

1PI self-energies

$i\Sigma(p^2) :=$ 

Renormalization: On-Shell Conditions (II)

- **on-shell conditions**:
  - mixing of fields vanishes for $p^2 = m^2_{\phi_i}$
  - masses $m^2_{\phi_i}$ are the real parts of the pole of the propagator
  - normalization: residue of the propagator at its pole equals i

- fixation of **diagonal** mass counterterms:

$$\mathrm{Re}\left[\delta D^2_{\phi_1\phi_1}\right] = \mathrm{Re}\left[\Sigma_{\phi_1\phi_1}(m^2_{\phi_1})\right] \quad , \quad \mathrm{Re}\left[\delta D^2_{\phi_2\phi_2}\right] = \mathrm{Re}\left[\Sigma_{\phi_2\phi_2}(m^2_{\phi_2})\right]$$

- fixation of field strength renormalization constants:

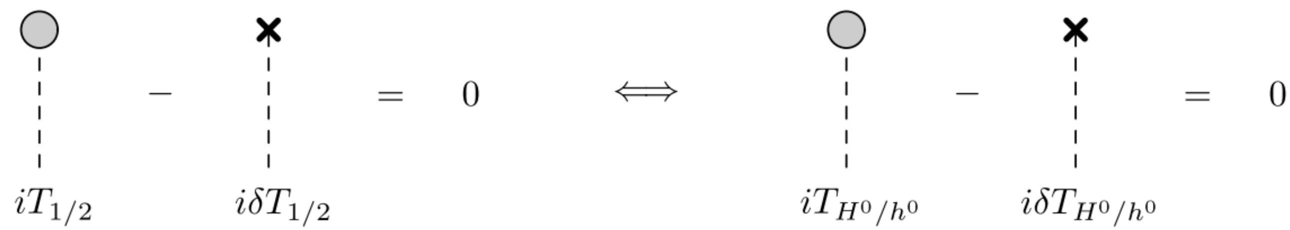
$$\delta Z_{\phi_1\phi_1} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_1\phi_1}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_1}} \quad , \quad \delta Z_{\phi_2\phi_2} = -\mathrm{Re}\left[\frac{\partial\Sigma_{\phi_2\phi_2}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_2}}$$

$$\delta Z_{\phi_1\phi_2} = \frac{2}{m^2_{\phi_1}-m^2_{\phi_2}}\mathrm{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta D^2_{\phi_1\phi_2}\right] \quad , \quad \delta Z_{\phi_2\phi_1} = \frac{2}{m^2_{\phi_2}-m^2_{\phi_1}}\mathrm{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_1}) - \delta D^2_{\phi_1\phi_2}\right]$$

- the **specific form** of the $\delta D^2_{\phi_i\phi_j}$ **depends on the tadpole scheme**

Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:



- conversion from gauge to **mass basis**:

$$\begin{pmatrix} \delta T_1 \\ \delta T_2 \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\alpha & -\mathrm{s}_\alpha \\ \mathrm{s}_\alpha & \mathrm{c}_\alpha \end{pmatrix} \begin{pmatrix} \delta T_{H^0} \\ \delta T_{h^0} \end{pmatrix} = \begin{pmatrix} \mathrm{c}_\alpha \delta T_{H^0} - \mathrm{s}_\alpha \delta T_{h^0} \\ \mathrm{s}_\alpha \delta T_{H^0} + \mathrm{c}_\alpha \delta T_{h^0} \end{pmatrix}$$

- **purpose**: restoring the minimum conditions of the potential at NLO

- **practical effect**: **no tadpole diagrams** in NLO calculations

Renormalization: Standard Tadpole Scheme

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- **standard scheme**: vevs are derived from the **loop-corrected potential**
  (e.g. in A. Denner: arXiv:0709.1075)

- vevs in the mass relations produce correct one-loop OS masses, e.g.

$$m_W^2 = g^2 \frac{v^2}{4} \quad , \qquad m_{A^0}^2 = v^2 \left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right)$$

- tadpole terms appear explicitly in the bare mass matrices
  ➔ mass matrix counterterms contain the **tadpole counterterms**:

$$\delta D_\phi^2 \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \begin{pmatrix} \delta T_{\phi_1 \phi_1} & \delta T_{\phi_1 \phi_2} \\ \delta T_{\phi_1 \phi_2} & \delta T_{\phi_2 \phi_2} \end{pmatrix}$$

- one-loop corrected potential is gauge-dependent
  ➔ **vevs** are gauge-dependent
  ➔ **mass counterterms** become **gauge-dependent**

Renormalization: Alternative Tadpole Scheme

- **alternative scheme**: vevs represent the same minimum as at **tree level**
  [based on: J. Fleischer, F. Jegerlehner, *Phys. Rev.* D **23** (1981) 2001-2026]

- bare masses are expressed through gauge-independent **tree-level vevs**
  ➔ **mass CTs become gauge-independent**

- correct minimum conditions @NLO require a **shift in the vevs**

$$v_1 \;\rightarrow\; v_1 + \delta v_1 \;, \qquad v_2 \;\rightarrow\; v_2 + \delta v_2$$

- fixation of the shifts by **applying the tadpole conditions**:

$$\begin{pmatrix} \delta v_1 \\ \delta v_2 \end{pmatrix} = \begin{pmatrix} \frac{\delta T_{H^0}}{m^2_{H^0}} c_\alpha - \frac{\delta T_{h^0}}{m^2_{h^0}} s_\alpha \\ \frac{\delta T_{H^0}}{m^2_{H^0}} s_\alpha + \frac{\delta T_{h^0}}{m^2_{h^0}} c_\alpha \end{pmatrix}$$

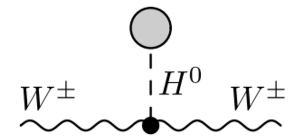
- the shifts translate into **every CT, wave function renormalization constants** and **Feynman rules**

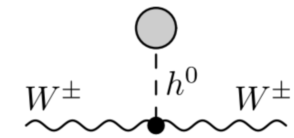
- alternative tadpole scheme **worked out for the 2HDM** ✔

Renormalization: Alternative Tadpole Scheme

- example: **W boson mass**

$$m_W^2 = g^2 \frac{v^2}{4} \quad \longrightarrow \quad m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} \quad = \quad m_W^2 + i \left(\quad \right) + i \left(\quad \right)$$

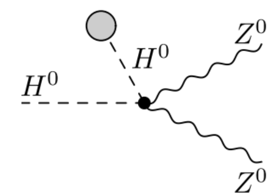




- example: **coupling between Higgs and Z bosons**

$$ig_{H^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2} \left(c_\alpha v_1 + s_\alpha v_2 \right) \quad , \qquad ig_{H^0 H^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2}$$

$$ig_{H^0 Z^0 Z^0} \ \rightarrow ig_{H^0 Z^0 Z^0} + \frac{ig^2}{2c_W^2} \left(c_\alpha \delta v_1 + s_\alpha \delta v_2 \right) = ig_{H^0 Z^0 Z^0} + \left(\quad \right)_{\text{trunc}}$$



- **effects** of the alternative tadpole scheme:
  - **tadpole diagrams are added everywhere** where they exist in the 2HDM
  - mass counterterms become **manifestly gauge-independent**
  - tadpole counterterms in the scalar sector are **removed**

Renormalization of the 2HDM (II)

- “**no-go theorem**” for the MSSM: a renormalization scheme for $\tan\beta$ **may not be simultaneously** [A. Freitas, D. Stöckinger, *Phys. Rev.* D**66** (2002) 095014]
  - gauge-independent
  - process-independent
  - numerically stable (i.e. leads to **moderate** NLO corrections)

Renormalization: Scalar Mixing Angles

- approach by S. Kanemura *et al.*: connect the definition of α and β with the **inverse propagator matrix** (S. Kanemura *et al.*: arXiv:hep-ph/0408364)

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = R^T_{\theta,0}\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix}_0 \approx R^T_{\delta\theta}R^T_{\theta}\sqrt{Z_{\widetilde{\phi}}}R_\theta R^T_\theta\begin{pmatrix}\widetilde{\phi}_1\\ \widetilde{\phi}_2\end{pmatrix} \approx \begin{pmatrix}1+\frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2}+\delta\theta\\ \delta C_{\phi_2}-\delta\theta & 1+\frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

- mixing angle counterterms **within the standard tadpole scheme**:

$$\delta\alpha = \frac{1}{2\left(m^2_{H^0}-m^2_{h^0}\right)}\mathrm{Re}\left[\Sigma_{H^0h^0}(m^2_{H^0})+\Sigma_{H^0h^0}(m^2_{h^0})-2\delta T_{H^0h^0}\right]$$

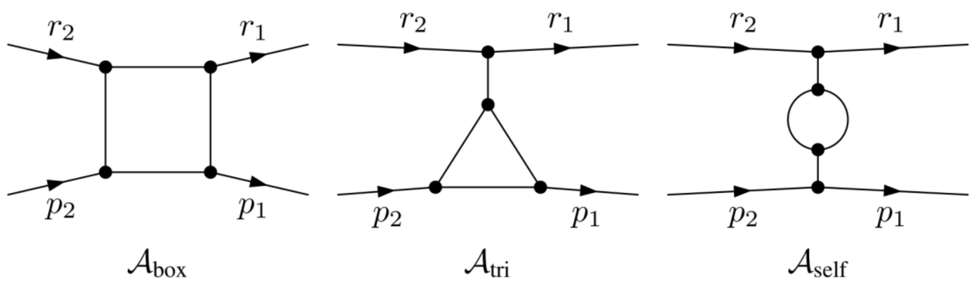
$$\delta\beta = -\frac{1}{2m^2_{H^\pm}}\mathrm{Re}\left[\Sigma_{G^\pm H^\pm}(m^2_{H^\pm})+\Sigma_{G^\pm H^\pm}(0)-2\delta T_{G^\pm H^\pm}\right]$$

(for details: R. Lorenz, Master's thesis, KIT, 2015)

- it was shown analytically that **Kanemura's scheme** introduces an **intricate gauge-dependence** in $\delta\alpha$ and $\delta\beta$

(M. Krause, Master's thesis, Karlsruhe Institute of Technology, 2016)

Pinch Technique: Introduction (I)



$$s = (r_1 + p_1)^2 = (r_2 + p_2)^2$$
$$t = (r_1 - r_2)^2 = (p_1 - p_2)^2$$

- we consider a **fermion scattering process** at one-loop QCD:

$$\mathcal{A}_{\text{full}}(s,t,m_1,m_2) = \mathcal{A}_{\text{box}}(s,t,m_1,m_2;\xi) + \mathcal{A}_{\text{tri}}(t,m_1,m_2;\xi) + \boxed{\mathcal{A}_{\text{self}}(t;\xi)}$$

- the gauge dependences **have to cancel** within the individual topologies
  ➔ rearrangement of the contributions is **always possible**
  ➔ rearrangement shows that **all** gauge dependences have **self-energy-like** or triangle-like form

$$\mathcal{A}_{\text{full}}(s,t,m_1,m_2) = \tilde{\mathcal{A}}_{\text{box}}(s,t,m_1,m_2) + \tilde{\mathcal{A}}_{\text{tri}}(t,m_1,m_2) + \tilde{\mathcal{A}}_{\text{self}}(t) \quad ,$$

$$\mathcal{A}_{\text{tri}}(t,m_1,m_2;\xi) \;\to\; \tilde{\mathcal{A}}_{\text{tri}}(t,m_1,m_2) + f_{\text{self}}(t;\xi) \quad , \quad \text{etc.}$$

Pinch Technique: Introduction (II)

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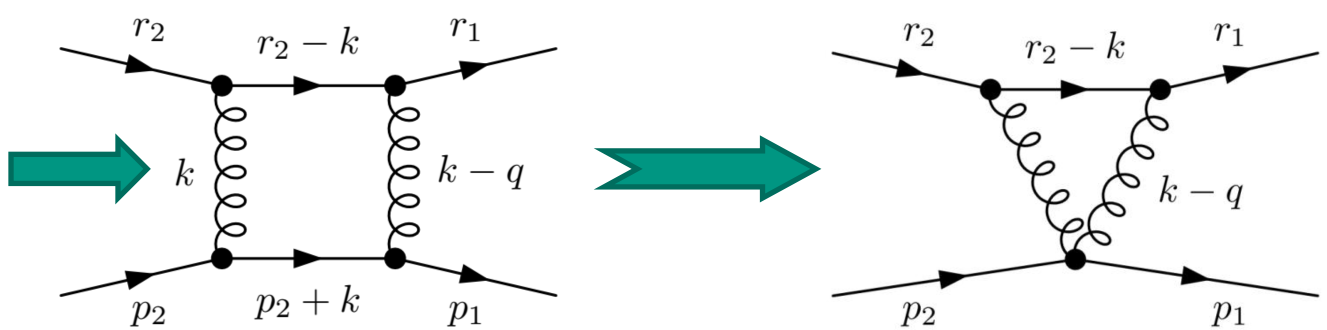
- determination of the gauge-dependent contributions: "**pinching**"
- main idea: trigger the **elementary Ward identity** for the loop momentum

$$\not{k} = (\not{k} + \not{p} - m) - (\not{p} - m) = \underbrace{S^{-1}(k+p) - S^{-1}(p)}$$

inverse fermion propagators

- right expression: vanishes OS between spinors
- left expression: **cancels** ("pinches out") an **internal fermion** propagator

$$\not{k} \equiv S^{-1}(p_2 + k) - S^{-1}(p_2)$$

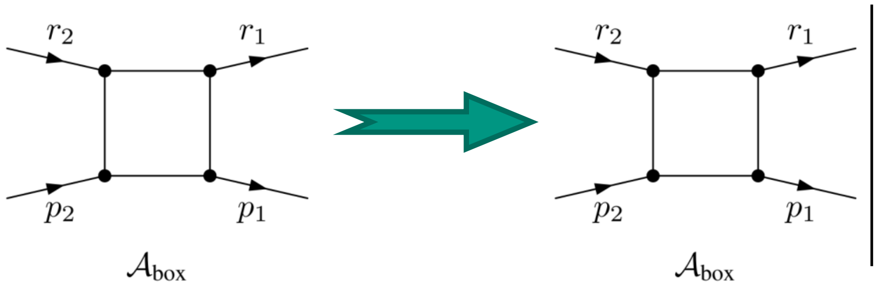


$S(p_2 + k)$

"pinched out"

Pinch Technique: Results (I)

- (almost) all pinch contributions are **proportional** to $(1-\xi)$
- the non-pinched contributions are **equivalent** to diagrams calculated in **Feynman-‘t Hooft gauge**, i.e. for $\xi \equiv 1$



$\xi \equiv 1$

$$+ \underbrace{(1-\xi) \cdot f(t)}_{\textbf{pinch contributions}}$$

gauge-independent part

- the **pinch contributions** are **self-energy like**, i.e. functions of only t
  ➔ **reallocation** of pinch contributions to the **gluon self-energy** possible

Pinch Technique: Results (II)

- sum of all pinch contributions ➔ **cancellation of gauge dependences**

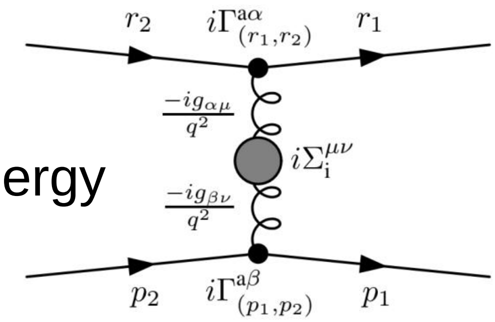
	$g_s^2 t(1-\xi)^2 \int_k \frac{k^\mu k^\nu}{k^4(k+q)^4}$	$g_s^2 t(1-\xi) \int_k \frac{k^\mu k^\nu}{k^4(k+q)^2}$	$g_s^2 t(1-\xi) \int_k \frac{g^{\mu\nu}}{k^2(k+q)^4}$	$g_s^2 t(1-\xi) \int_k \frac{g^{\mu\nu}}{k^4}$
$i\Sigma^{\mu\nu}_{\text{box}}$	$t\frac{C_A}{2}$	0	$-tC_A$	0
$i\Sigma^{\mu\nu}_{\text{tri1}}$	0	0	0	$C_A - 2C_f$
$i\Sigma^{\mu\nu}_{\text{tri2}}$	$-tC_A$	$2C_A$	$2tC_A$	$-2C_A$
$i\Sigma^{\mu\nu}_{\text{self,q}}$	0	0	0	$2C_f$
$i\Sigma^{\mu\nu}_{\text{self,g}}$	$t\frac{C_A}{2}$	$-2C_A$	$-tC_A$	C_A
Sum	0	0	0	0

$(q^2 \equiv t)$

C_A, C_f : Casimir operators

- **main results** from the application of the pinch technique:
  - demonstration of **intricate cancellation** of gauge dependences
  - cancellation is **not accidental**, but follows from **Ward identities**

Gauge-Independent Self-Energies via PT



- all pinch contributions are self-energy-like
  ➔ **reallocate** pinch contributions to the gluon self-energy

- the pinched self-energy is equivalent to the one evaluated for $\xi = 1$ after the cancellation of all gauge dependences
  ➔ Feynman-‘t Hooft-gauge is a **special gauge choice**

- **interesting properties** of the pinched gluon self-energy:
  - analogy to the gluon self-energy given by the **Background Field Method**
  - **uniquely defined** by the pinch technique framework
  - manifestly **gauge-independent** ➔ allows for gauge-independent **counterterms**
  - obeys **QED-like Ward identities** instead of complicated Slavnov-Taylor identities

[for more details cf. e.g. D. Binosi, J. Papavassiliou, Phys. Rep. **479** (2009) 1]

Applications of the Pinch Technique

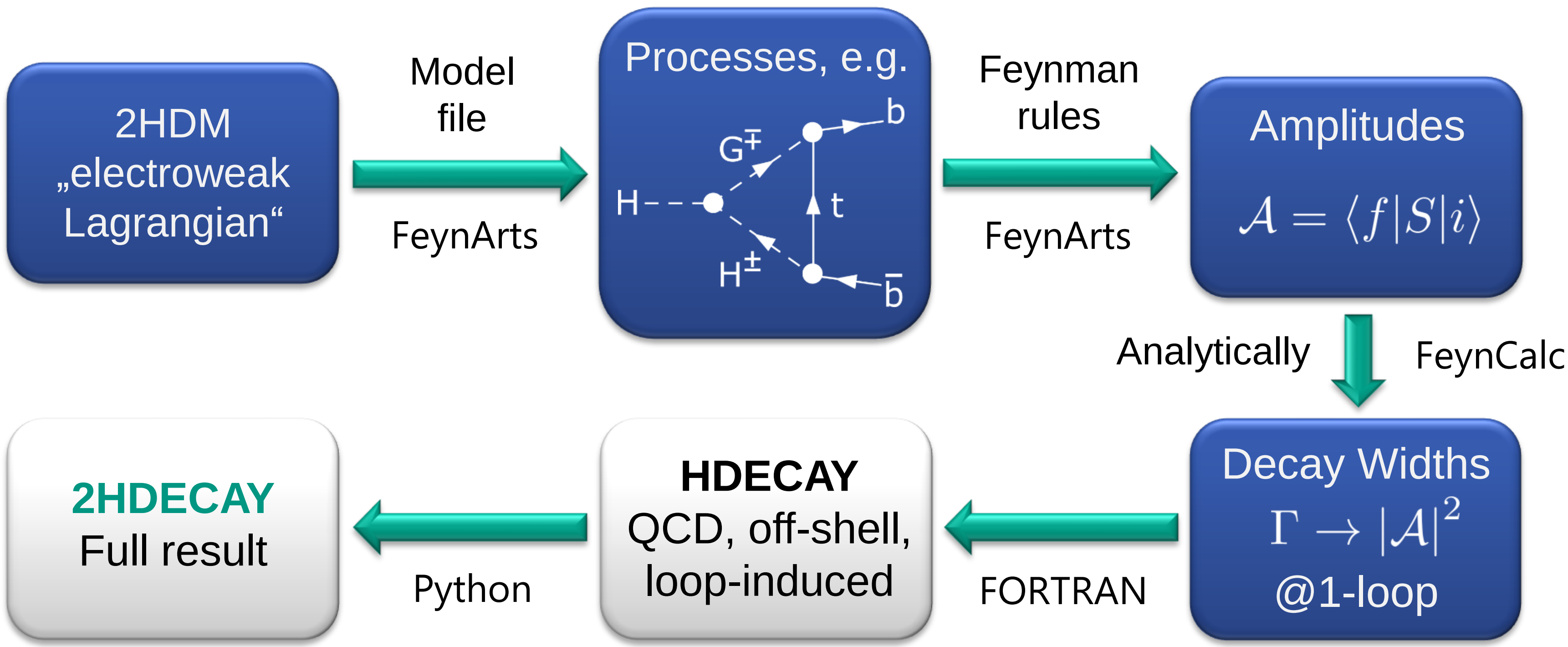
- the pinch technique can be applied to e.g. the SM, MSSM, **(N)2HDM**, …

- for consistency: **tadpole diagrams** have to be taken into account
  ➔ "alternative tadpole scheme" is **needed** (cf. part II of the talk)

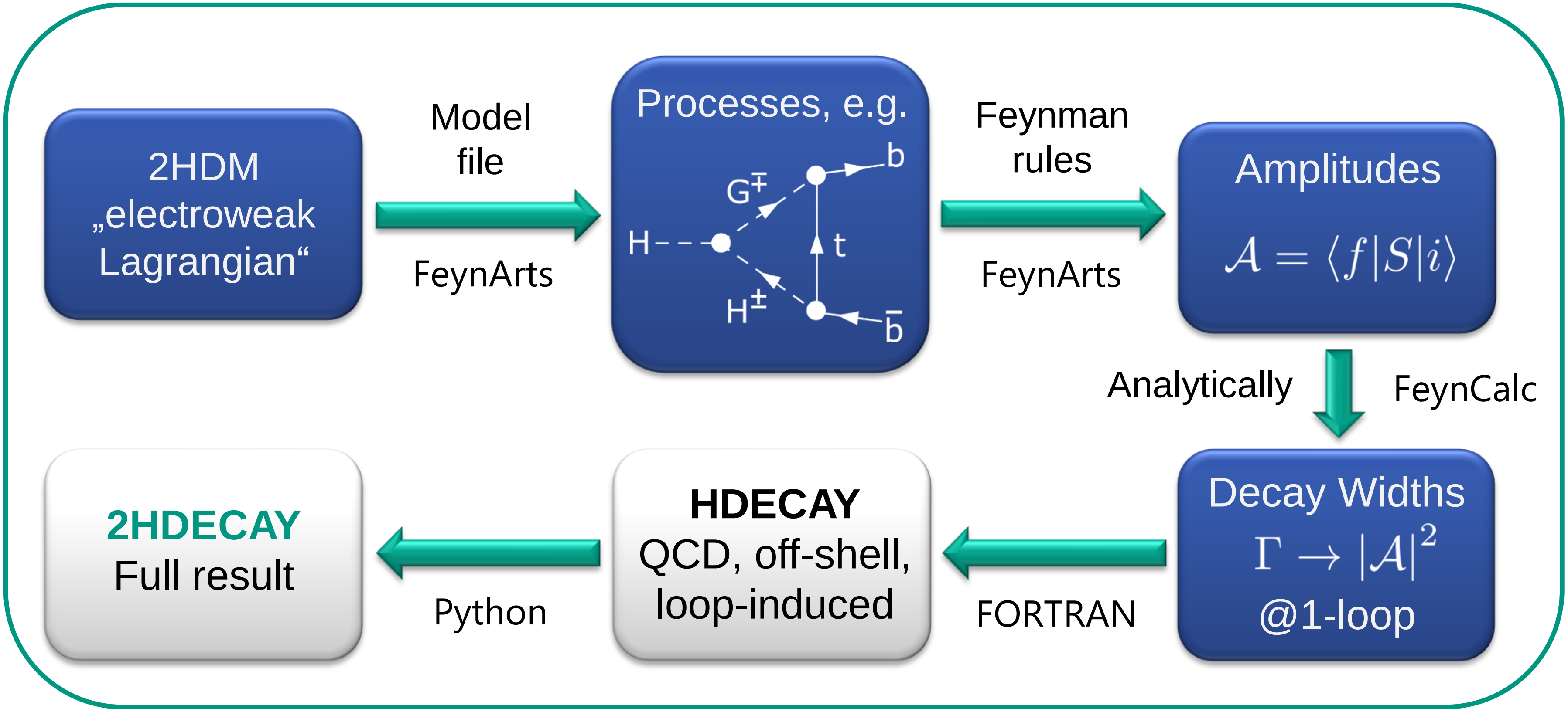
- applications of the pinched self-energies:
  - definition of **gauge-independent counterterms** (cf. part III of the talk)
  - general analysis of gauge dependence cancellations [D. Binosi, J. Papavassiliou, Phys. Rev. **D65** (2002) 085003]
  - generalization to **all orders** [D. Binosi, J. Phys. **G30** (2004) 1021]
  - construction of **QED-like Ward identities** for e.g. QCD
  - gauge-independent definition of **electroweak parameters**
  - consistent resummation for resonant transition amplitudes
  - extraction of gauge-independent part of **BFM** self-energies

  → [D. Binosi, J. Papavassiliou, Phys. Rep. **479** (2009) 1; J. Papavassiliou, Phys. Rev. **D50** (1994) 5958]

Implementation: 2HDECAY (I)



Implementation: 2HDECAY (I)

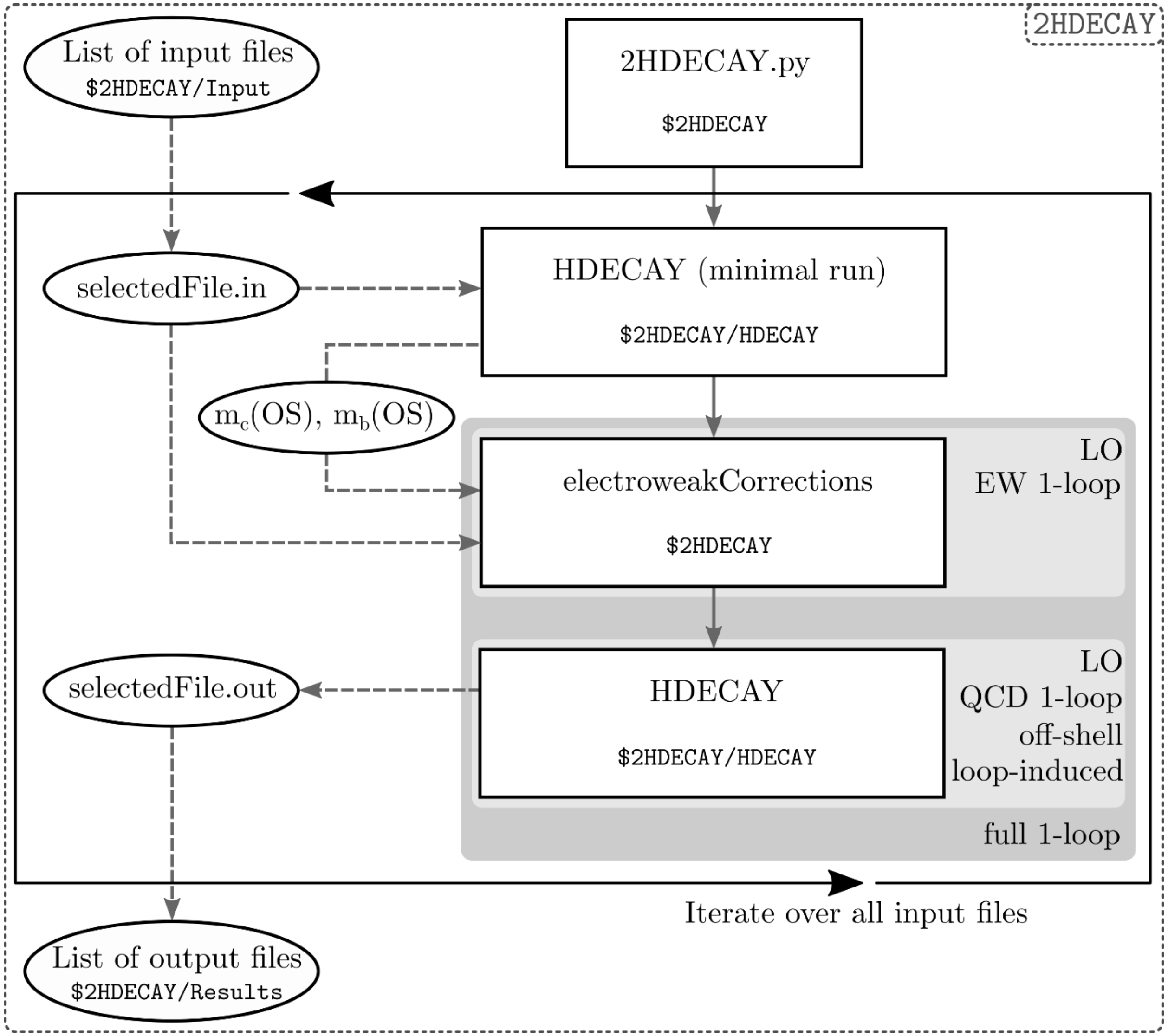


2HDECAY: “2HDM HDECAY“

A program for the calculation One-Loop Electroweak Corrections to Higgs Decays in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

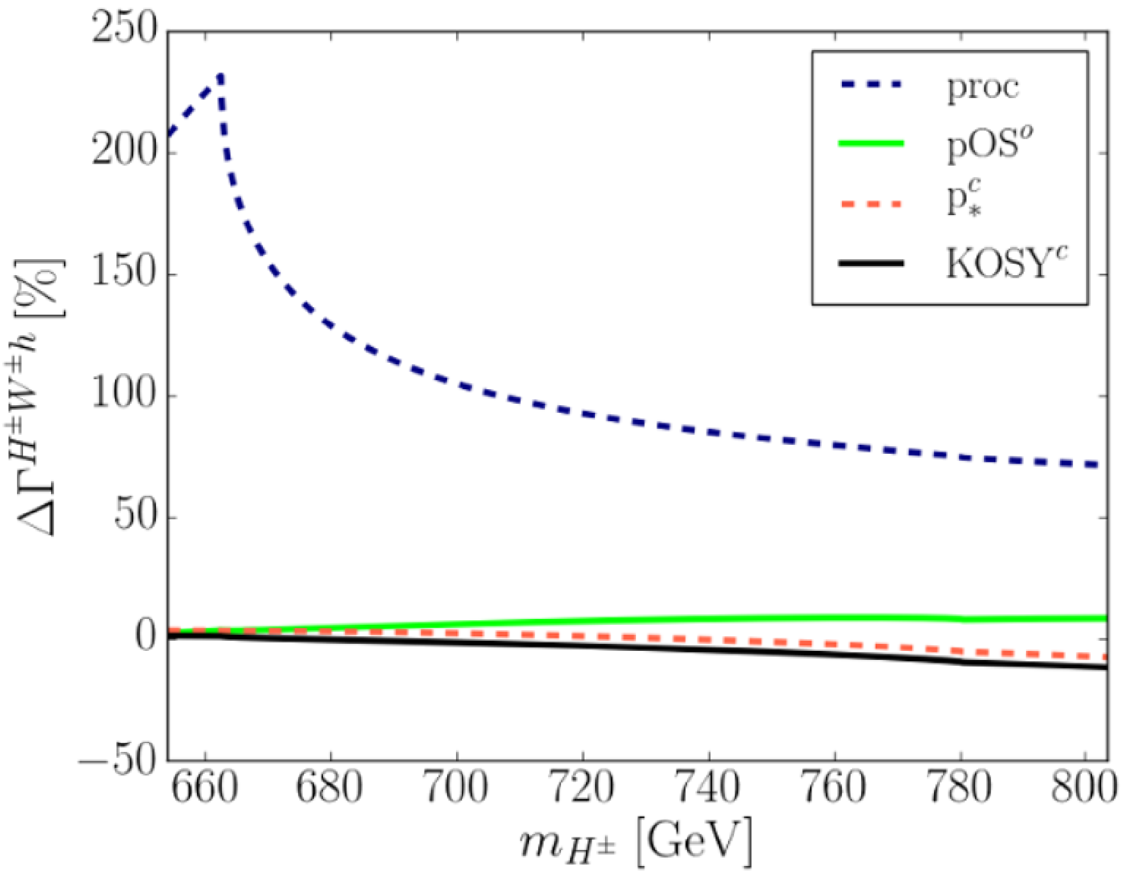
[MK, M. M. Mühlleitner, M. Spira, *in preparation,* arxiv:18MM.XXXXX]

Implementation: 2HDECAY (II)



[MK, M. M. Mühlleitner, M. Spira, *in preparation,* arxiv:18MM.XXXXX]

Numerical Analysis (II)



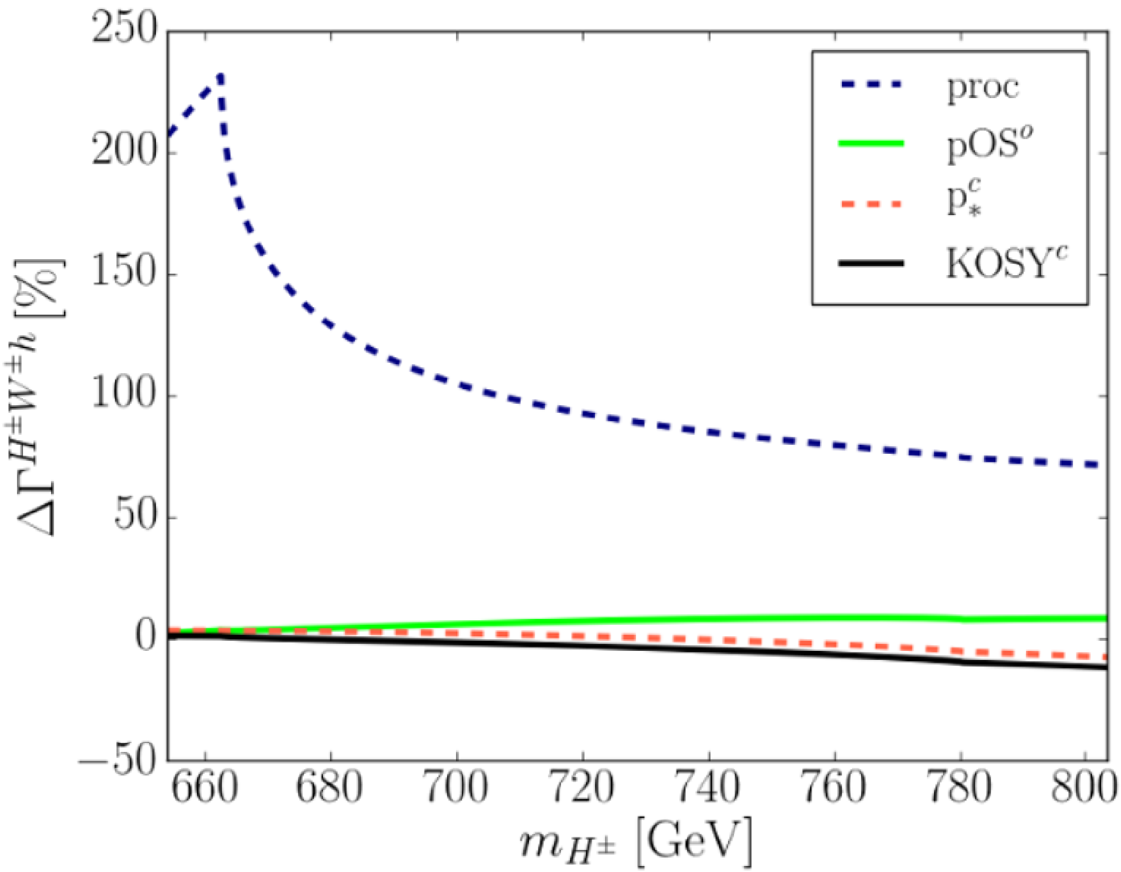
$$m_h = 125.09\,\mathrm{GeV},\; m_H = 742.84\,\mathrm{GeV}$$
$$m_A = 700.13\,\mathrm{GeV},\; m_{12} = 440.57\,\mathrm{GeV}$$
$$\tan\beta = 1.46,\; \alpha = -0.57$$

- proc: process-dependent
- pOS: “on-shell pinched”
- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

$$\Delta\Gamma = \frac{\Gamma_{\mathrm{NLO}} - \Gamma_{\mathrm{LO}}}{\Gamma_{\mathrm{LO}}}$$

Numerical Analysis (II)



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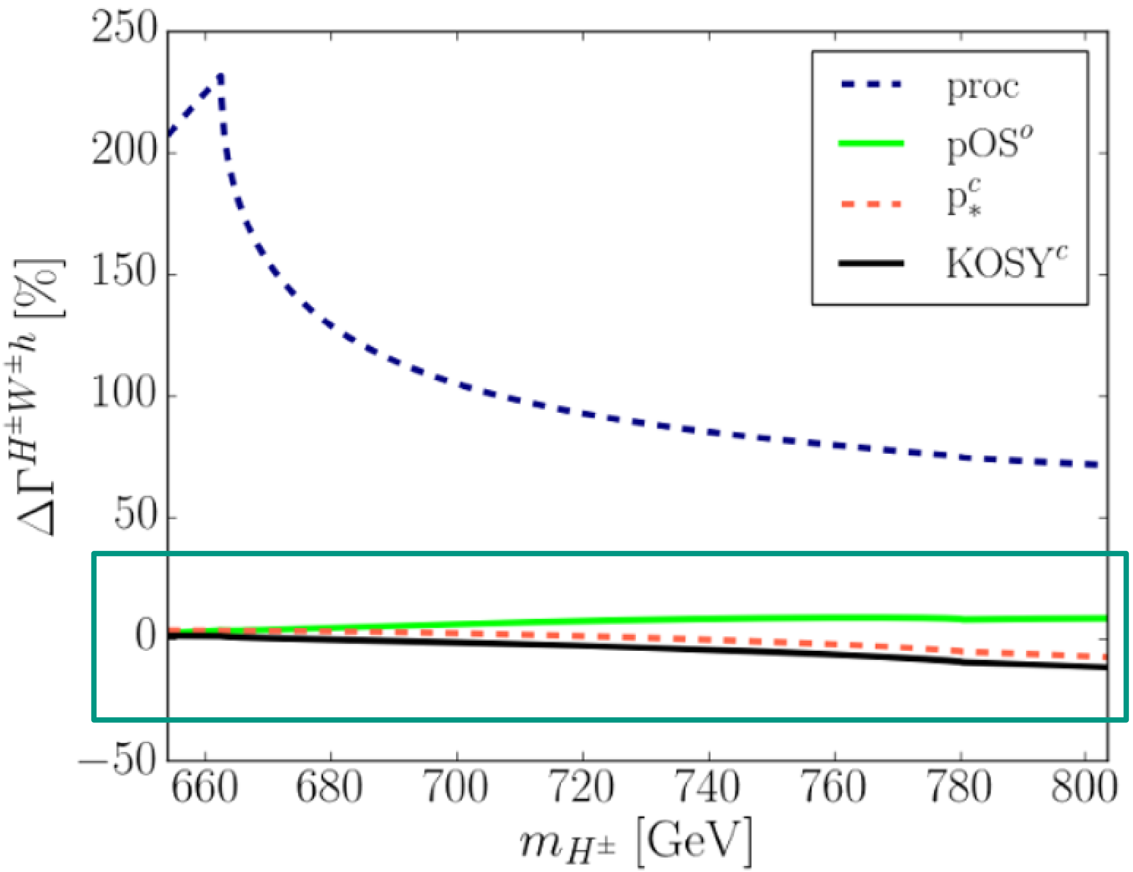
- kinks: **thresholds** for certain mass configurations
- process-dependent scheme is often **unsuitable** (large NLO corrections)

Numerical Analysis (II)



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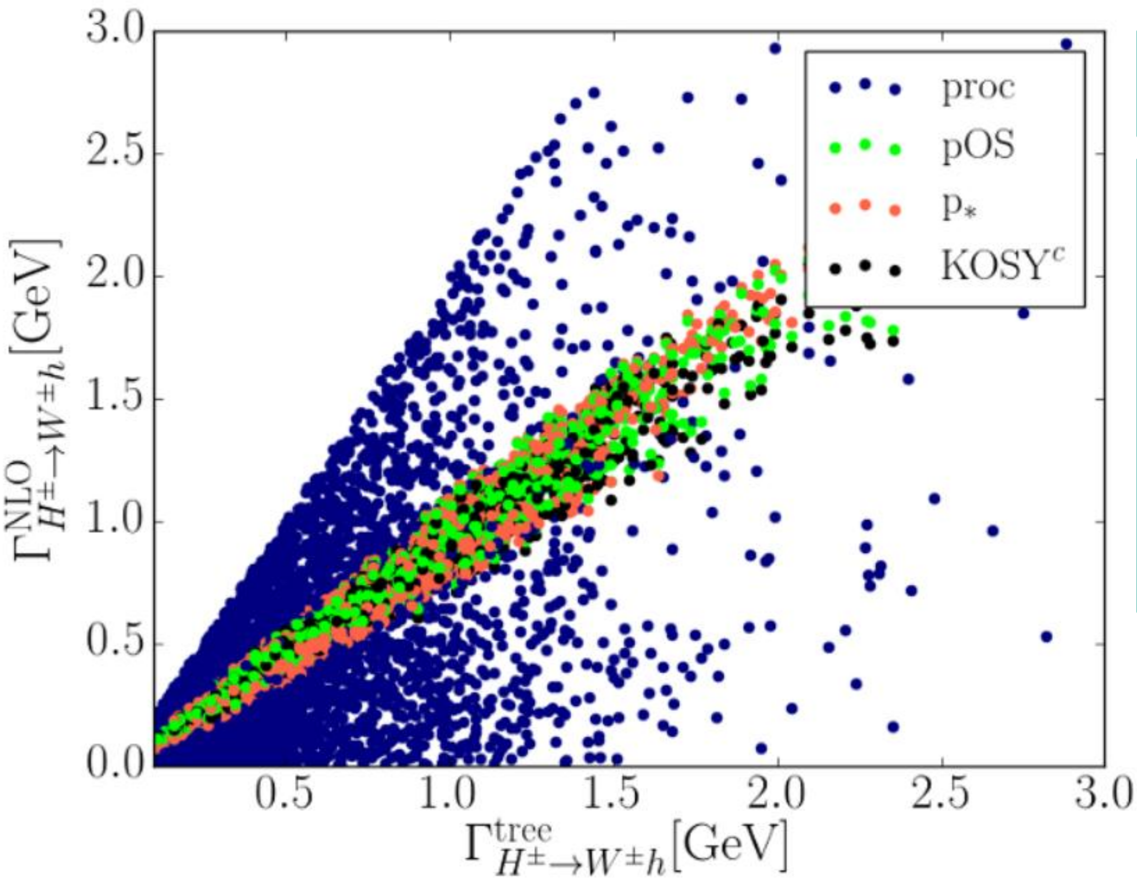
Numerical Analysis (IV)



scan over large parameter ranges

- proc: process-dependent
- pOS: “on-shell pinched”
- p*: “p*-pinched”
- KOSY: gauge-dependent scheme

superscript “c”: definition over charged sector

$$\Delta\Gamma = \frac{\Gamma_{\mathrm{NLO}} - \Gamma_{\mathrm{LO}}}{\Gamma_{\mathrm{LO}}}$$

- for LO approaching zero, $\Delta\Gamma$ may become large (**numerical instability**)
- numerical instability is “artificial” (**no** problem of renormalization scheme)