

# Corrections to 2HDM Higgs Decays with 2HDECAY

Marcel Krause

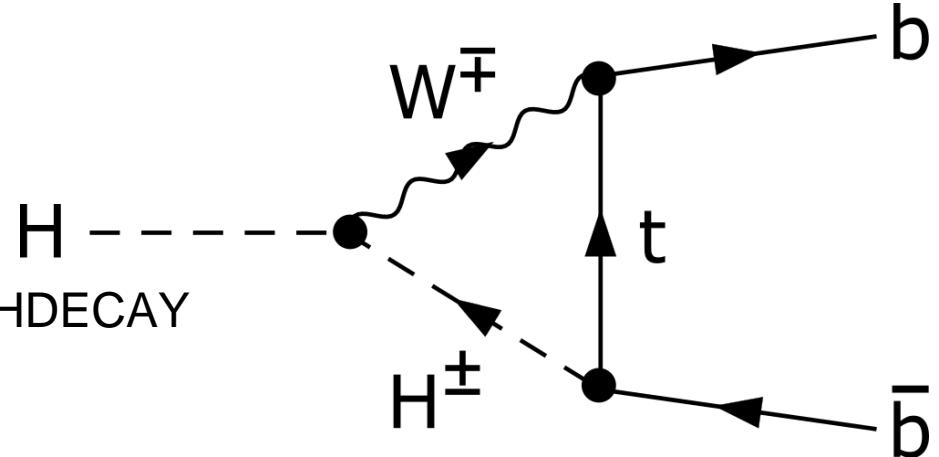
Institute for Theoretical Physics (ITP), Karlsruhe Institute of Technology (KIT)

[ MK, M. M. Mühlleitner, M. Spira, *arXiv:1810.00768*; <https://github.com/marcel-krause/2HDECAY> ]

Seminar Elementarteilchentheorie

May 09, 2019

- Motivation
- Principle of Gauge Invariance
- Introduction to the 2HDM
- Automated 1-Loop Calculations with 2HDECAY
- Renormalization of the 2HDM
- Numerical Results



# Motivation (I): Two-Higgs-Doublet Model

- 2HDM: one of the simplest extensions of the SM
  - dark matter candidate (*Inert Doublet Model*)
  - source of CP violation
  - **extended scalar sector**
  - renormalizable

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  - source of CP violation
  - **extended scalar sector**
  - renormalizable
- renormalization of the two **scalar mixing angles** in the 2HDM is non-trivial
- previously existing schemes are either numerically unstable, process-dependent or **gauge-dependent**
- search for a suitable renormalization scheme of the scalar mixing angles
  - ➡ full electroweak NLO corrections to all decays within the 2HDM

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  - off-shell decay modes for final-state massive vector bosons / heavy quarks
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- previous analysis has shown: they can be of **relevant size**
- **interesting theoretical studies** with one-loop electroweak corrections:
  - differences w.r.t. MSSM one-loop corrections (integrate out SUSY masses)  
→ 2HDM as effective theory for the MSSM with heavy sparticles
  - studies on renormalization scheme dependence (estimate of theoretical errors due to missing higher orders)
  - phenomenologically interesting limits (decoupling, alignment, wrong-sign, ...)

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- the class of  $R_\xi$  gauges form an equivalence class of the gauge theory
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  - equations of motions, observables, ... **must not depend** on  $\xi$
- higher-order calculations: cancellation of gauge dependences becomes very **intricate**
- in the 2HDM: unsuitable renormalization of mixing angles **spoils gauge parameter independence**

# Gauge Invariance in QED

- consider Quantum Electrodynamics with spinors  $\Psi(x)$ , photon  $A_\mu(x)$
- we demand **invariance** under **local U(1) gauge transformations**

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- renormalizability: **QED Lagrangian** up to dim-4 operators

$$\mathcal{L}_{\text{QED}} = \bar{\Psi} (iD^\mu - m) \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

( $m$  : fermion mass,  $F_{\mu\nu}$  : photon field strength tensor  
 $D$  : covariant derivative )

# Gauge Dependences in QED / QFTs

- **quantization** e.g. through the **Faddeev-Popov** method:

$$\mathcal{L}_{\text{QED}} = \bar{\Psi} (i \not{D} - m) \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial^\mu A_\mu)(\partial_\mu A^\mu)$$


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  - preservation of **unitarity**
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- introduction of **gauge-fixing** and Lagrangian
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  - cancellation of **unphysical polarization** degrees of freedom
- Feynman rules depend on **gauge-fixing parameter**  $\xi_V$ :

$$\mu \sim \sim \sim \sim \sim \sim \nu = \frac{-i}{k^2 - m_V^2} \left[ g_{\mu\nu} - (1 - \xi_V) \frac{k^\mu k^\nu}{k^2 - \xi_V m_V^2} \right]$$

- introduction of  $\xi_V$  dependence in (loop) calculations
- **individual Feynman diagrams dependent** on  $\xi_V$

# Cancellation of Gauge Dependences

- $\xi_V$  encodes **redundant** (unphysical) degrees of freedom
  - observables, decay amplitudes, etc. **must not depend** on  $\xi_V$
  - cancellation is ensured by BRST symmetry

[ C. Becchi, A. Rouet, R. Stora, *Ann. Phys.* **98** (1976) 287; M. Z. Iofa, I. V. Tyutin, *Theor. Math. Phys.* **27** (1976) 316 ]

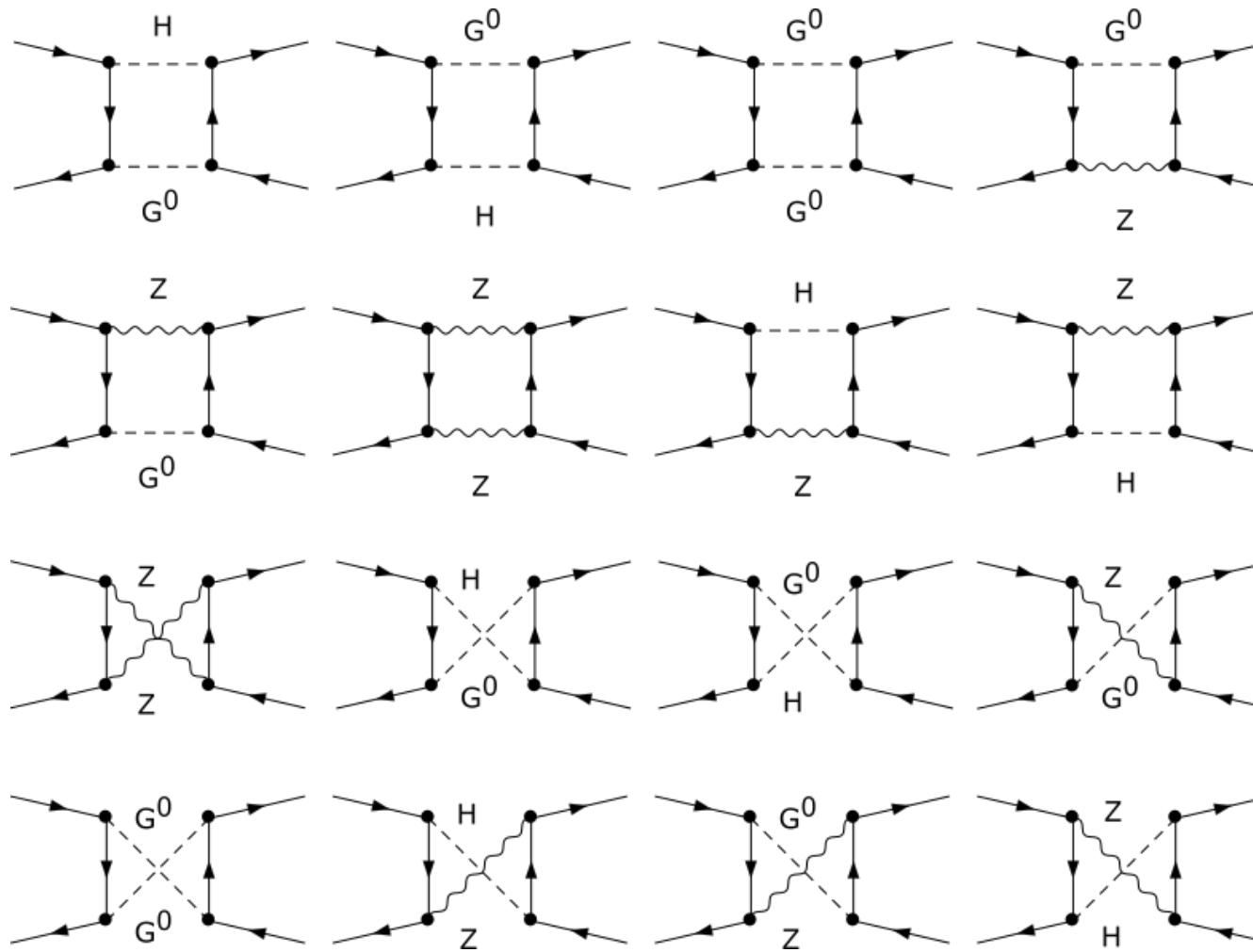
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- at higher orders, the cancellation becomes **very intricate**
- possible **violation** of the cancellation: renormalization conditions for mixing angles
  - SM: CKM matrix → solved
    - [ P. Gambino, P.A. Grassi, F. Madrigal, *Phys.Lett.* **B454** (1999) 98-104;  
A. Barroso, L. Brucher, R. Santos, *Phys.Rev.* **D62** (2000) 096003;  
B.A. Kniehl, F. Madrigal, M. Steinhauser, *Phys.Rev.* **D62** (2000) 073010;  
Y. Yamada, *Phys.Rev.* **D64** (2001) 036008;  
A. Denner, E. Kraus, M. Roth, *Phys.Rev.* **D70** (2004) 033002 ]
  - 2HDM: **scalar mixing angles** → ?

# Introduction to the 2HDM (I): Potential

- two complex  $SU(2)_L$  Higgs **doublets**

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- non-vanishing **vacuum expectation values (VEVs)**  $v_1, v_2$  with

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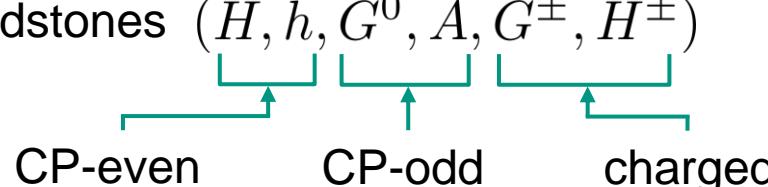
- scalar Lagrangian with **CP- and  $\mathbb{Z}_2$ -conserving** 2HDM potential:

$$\begin{aligned} V_{\text{2HDM}}(\Phi_1, \Phi_2) = & m_{11}^2 (\Phi_1^\dagger \Phi_1) + m_{22}^2 (\Phi_2^\dagger \Phi_2) - m_{12}^2 [(\Phi_1^\dagger \Phi_2) + (\Phi_2^\dagger \Phi_1)] \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2} \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right] \end{aligned}$$

# Introduction to the 2HDM (II): Parameters

- **eight** real-valued potential parameters:
  - dimensionless  $\lambda_i$  ( $i = 1, \dots, 5$ )
  - mass-squared parameters  $m_{11}^2$ ,  $m_{22}^2$  and  $m_{12}^2$
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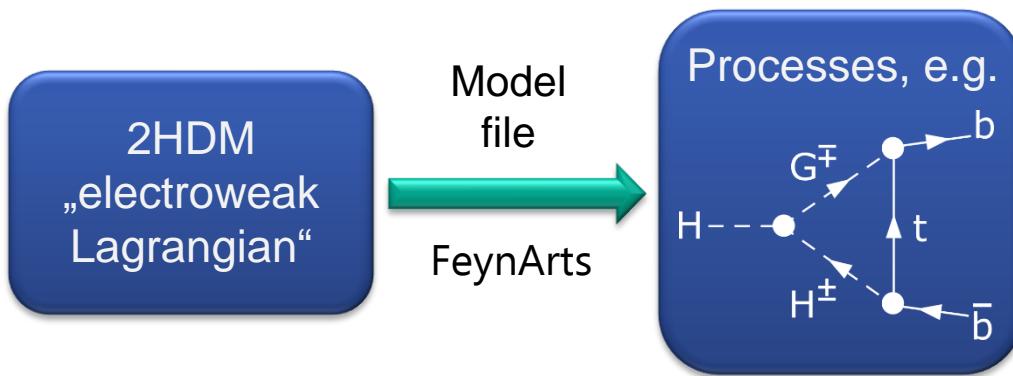
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  - difference w.r.t. MSSM: constants are **not fixed through SUSY relations**
  - transformation to the Higgs mass basis via **scalar mixing angles**
    - $\alpha$  for the CP-even sector
    - $\beta$  for the CP-odd and charged sectors
- physical Higgs bosons and Goldstones  $(H, h, G^0, A, G^\pm, H^\pm)$
- 
- The diagram illustrates the decomposition of scalar fields into their respective sectors. At the top, a horizontal line represents the full set of scalar fields:  $(H, h, G^0, A, G^\pm, H^\pm)$ . Below this line, three brackets group the fields into sectors: a green bracket groups  $H$  and  $h$  under the label "CP-even"; a blue bracket groups  $G^0$  and  $A$  under the label "CP-odd"; and a red bracket groups  $G^\pm$  and  $H^\pm$  under the label "charged".

# Electroweak Corrections @1-Loop (I)

- aim: calculate all 2HDM Higgs boson decays **@1-loop (electroweak)**
- use **perturbation theory** to solve the scattering matrix S at 1-loop level

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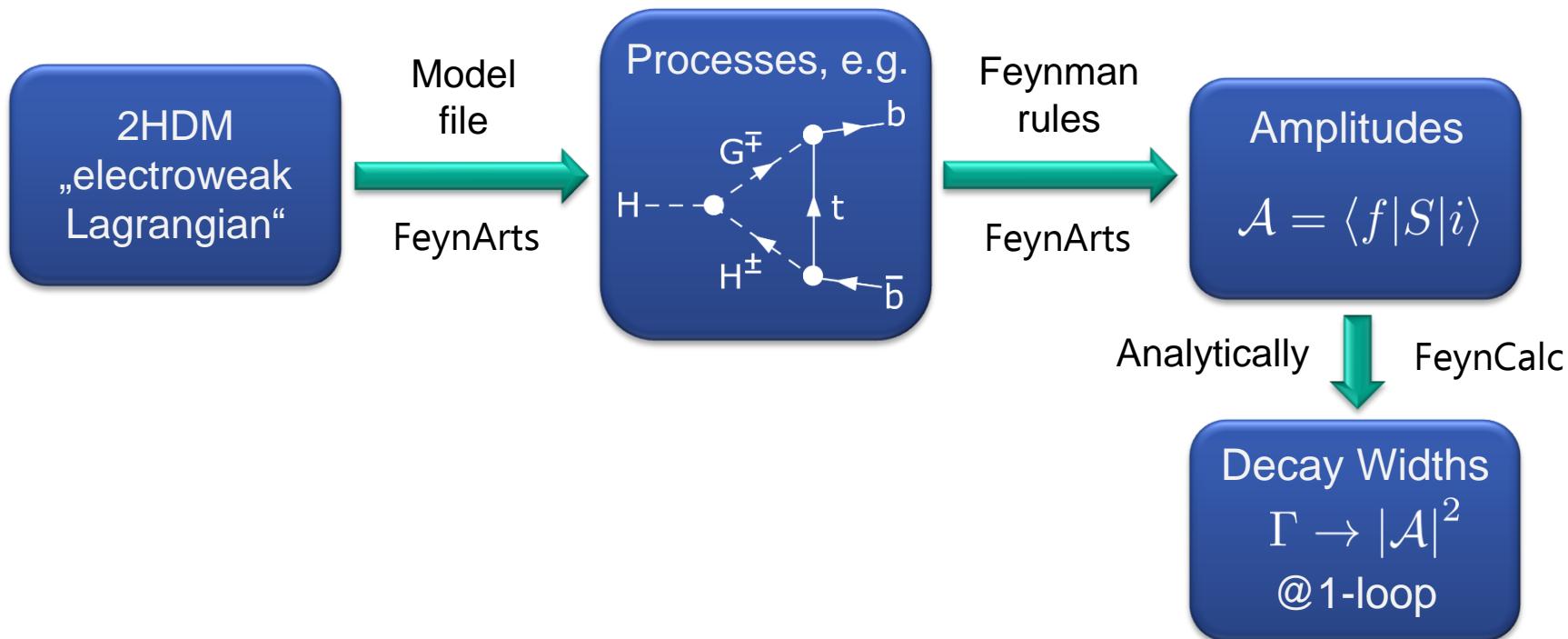
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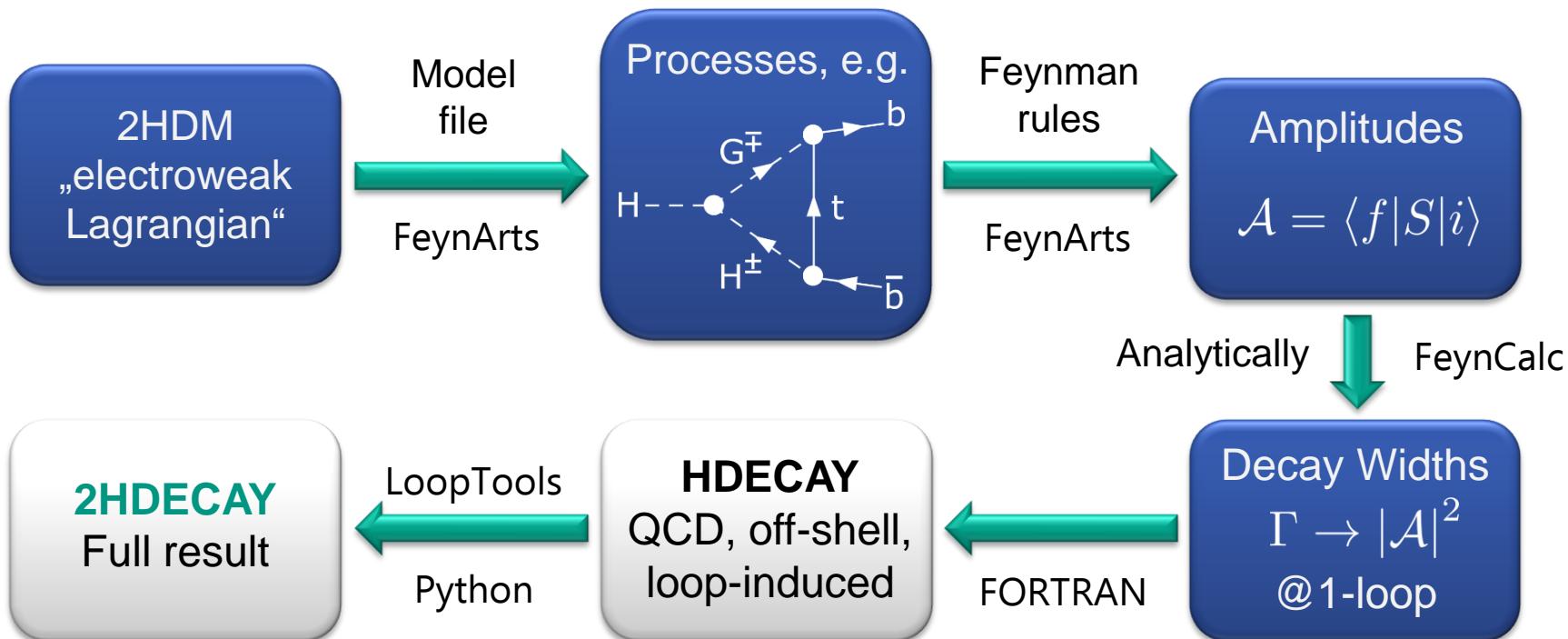
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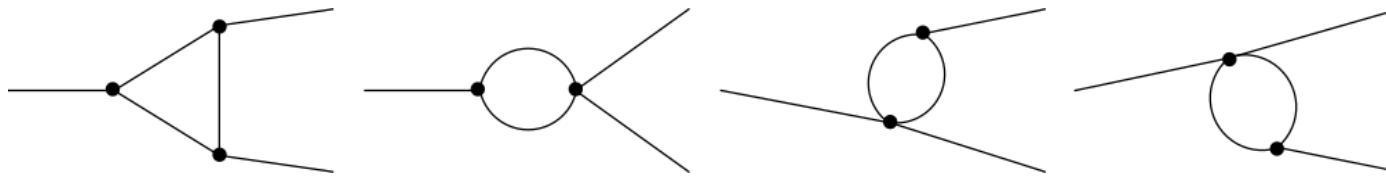
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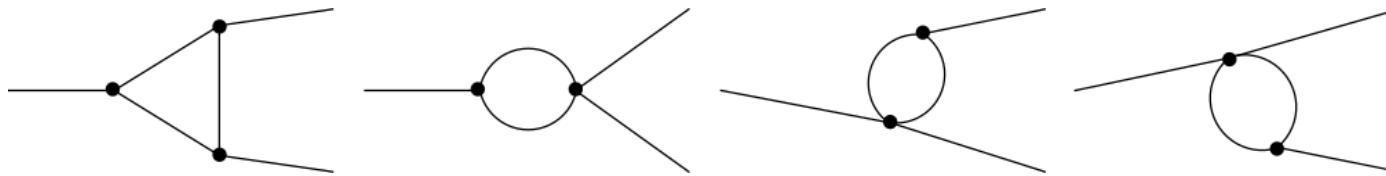
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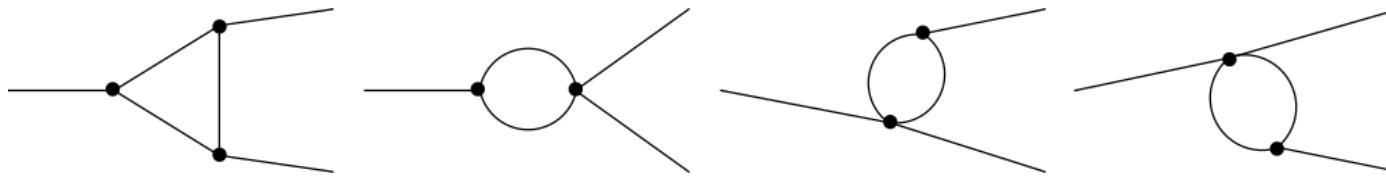


- decay channels that are considered:

- $h/H/A \rightarrow f\bar{f}$  ( $f = c, s, t, b, \mu, \tau$ )
- $h/H \rightarrow VV$  ( $V = W^\pm, Z$ )
- $h/H \rightarrow VS$  ( $V = Z, W^\pm, S = A, H^\pm$ )
- $H^\pm \rightarrow f\bar{f}$  ( $f = c, t, \nu_\mu, \nu_\tau$ ,  $\bar{f} = \bar{s}, \bar{b}, \mu^+, \tau^+$ )
- $h/H \rightarrow SS$  ( $S = A, H^\pm$ )
- $H^\pm \rightarrow VS$  ( $V = W^\pm, S = h, H, A$ )
- $A \rightarrow VS$  ( $V = Z, W^\pm, S = h, H, H^\pm$ )
- $H \rightarrow hh$

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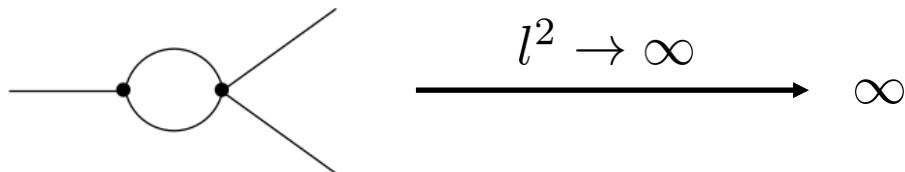
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- (semi-)**automated** calculation of the decays

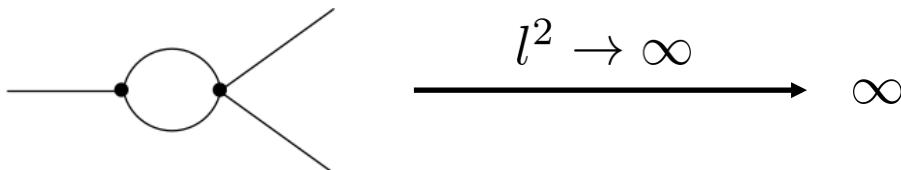
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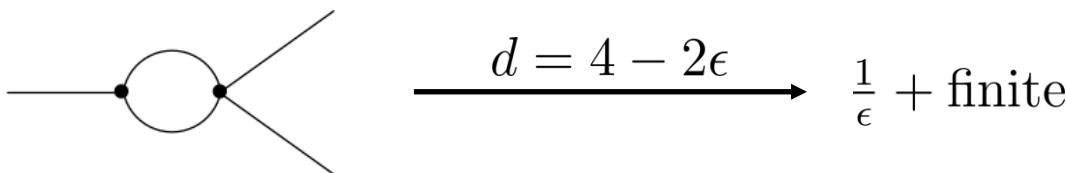


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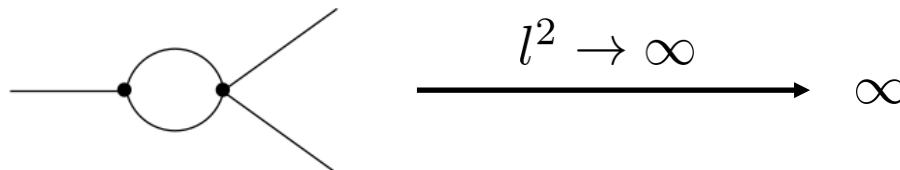


- use **dimensional regularization** ( $d = 4 - 2\epsilon$ ), isolate the divergences:

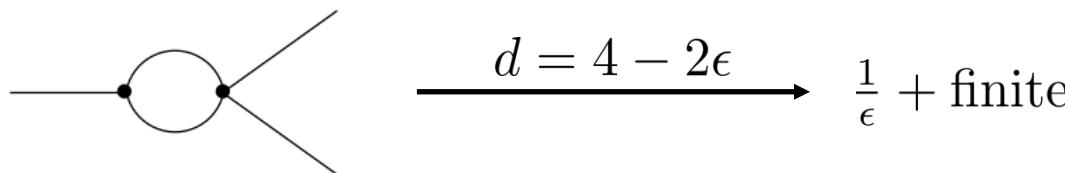


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- use **dimensional regularization** ( $d = 4 - 2\epsilon$ ), isolate the divergences:



- consistently remove the divergences via **renormalization**
- idea: split ‘bare’ parameters into **renormalized** values and **counterterms**

$$m_i^2 \rightarrow m_i^2 + \delta m_i^2$$

- counterterms need to be fixed via **renormalization conditions**

# Renormalization of the 2HDM

- set of free parameters of the 2HDM (excluding CKM elements, ...)

$$\left\{ T_{h/H}, \alpha_{\text{em}}, m_W, m_Z, m_f, m_h, m_H, m_A, m_{H^\pm}, \alpha, \beta, m_{12}^2, \dots \right\}$$

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- renormalization program for the 2HDM:

- tadpole terms  $\rightarrow$  standard / **alternative** tadpole scheme
- mass counterterms  $\rightarrow$  on-shell
- fine-structure constant  $\rightarrow$  at Z mass
- soft- $\mathbb{Z}_2$ -breaking scale  $m_{12}^2 \rightarrow \overline{\text{MS}}$
- **scalar mixing angles**  $\rightarrow$  ?

[ MK, *Master's thesis* (2016), KIT;  
MK, R. Lorenz, M. M. Mühlleitner, R. Santos, H. Ziesche, *J. High Energ. Phys.* **2016** (2016) 143;  
MK, M. M. Mühlleitner, R. Santos, H. Ziesche, *Phys. Rev. D* **95** (2017) 075019 ]

# Renormalization: On-Shell Conditions (I)

- consider **scalar field doublet**  $(\phi_1, \phi_2)$

- field strength renormalization:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_0 = \sqrt{Z_\phi} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \approx \left( 1_{2 \times 2} + \frac{\delta Z_\phi}{2} \right) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} , \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix} \frac{\delta Z_{\phi_1 \phi_1}}{2} & \frac{\delta Z_{\phi_1 \phi_2}}{2} \\ \frac{\delta Z_{\phi_2 \phi_1}}{2} & \frac{\delta Z_{\phi_2 \phi_2}}{2} \end{pmatrix}$$

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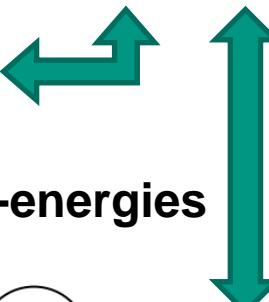
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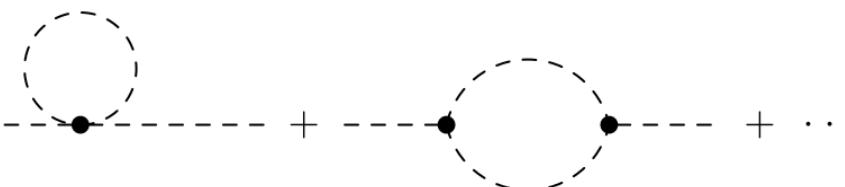
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- two-point correlation function for the doublet with momentum  $p^2$ :

$$\begin{aligned} \hat{\Gamma}_\phi(p^2) &:= \begin{pmatrix} \hat{\Gamma}_{\phi_1 \phi_1}(p^2) & \hat{\Gamma}_{\phi_1 \phi_2}(p^2) \\ \hat{\Gamma}_{\phi_2 \phi_1}(p^2) & \hat{\Gamma}_{\phi_2 \phi_2}(p^2) \end{pmatrix} \\ &= i \sqrt{Z_\phi}^\dagger \left[ p^2 1_{2 \times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2 \right] \sqrt{Z_\phi} \approx i \left[ p^2 1_{2 \times 2} - D_\phi^2 + \hat{\Sigma}_\phi(p^2) \right] \end{aligned}$$

mass matrices  **1PI self-energies**  
 **mass CTs**  
 **renormalized self-energies**

$$i\Sigma(p^2) := \text{---} \circled{1\text{PI}} \text{---} = \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \dots$$


# Renormalization: On-Shell Conditions (II)

## ■ on-shell conditions:

- mixing of fields vanishes for  $p^2 = m_{\phi_i}^2$
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## ■ fixation of **diagonal** mass counterterms:

$$\text{Re}\left[\delta D_{\phi_1 \phi_1}^2\right] = \text{Re}\left[\Sigma_{\phi_1 \phi_1}(m_{\phi_1}^2)\right], \quad \text{Re}\left[\delta D_{\phi_2 \phi_2}^2\right] = \text{Re}\left[\Sigma_{\phi_2 \phi_2}(m_{\phi_2}^2)\right]$$

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## ■ fixation of field strength renormalization constants:

$$\delta Z_{\phi_1 \phi_1} = -\text{Re}\left[\frac{\partial \Sigma_{\phi_1 \phi_1}(p^2)}{\partial p^2}\right]_{p^2=m_{\phi_1}^2}, \quad \delta Z_{\phi_2 \phi_2} = -\text{Re}\left[\frac{\partial \Sigma_{\phi_2 \phi_2}(p^2)}{\partial p^2}\right]_{p^2=m_{\phi_2}^2}$$

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## ■ the **specific form** of the $\delta D_{\phi_i \phi_j}^2$ **depends on the tadpole scheme**

# Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:

$$\begin{array}{c} \text{---} \\ | \\ \text{---} \\ iT_{1/2} \end{array} - \begin{array}{c} \text{---} \\ | \\ \text{---} \\ i\delta T_{1/2} \end{array} = 0 \iff \begin{array}{c} \text{---} \\ | \\ \text{---} \\ iT_H/h \end{array} - \begin{array}{c} \text{---} \\ | \\ \text{---} \\ i\delta T_H/h \end{array} = 0$$

- conversion from gauge to **mass basis**:

$$\begin{pmatrix} \delta T_1 \\ \delta T_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \delta T_H \\ \delta T_h \end{pmatrix} = \begin{pmatrix} c_\alpha \delta T_H - s_\alpha \delta T_h \\ s_\alpha \delta T_H + c_\alpha \delta T_h \end{pmatrix}$$

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- purpose:** restoring the minimum conditions of the potential at NLO
- practical effect:** no tadpole diagrams in NLO calculations

# Renormalization: Standard Tadpole Scheme

- **standard scheme**: vevs are derived from the **loop-corrected potential**
- vevs in the mass relations produce correct one-loop OS masses, e.g.

$$m_W^2 = g^2 \frac{v^2}{4} , \quad m_A^2 = v^2 \left( \frac{m_{12}^2}{v_1 v_2} - \lambda_5 \right)$$

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→ mass matrix counterterms contain the **tadpole counterterms**:

$$\delta D_\phi^2 \approx \begin{pmatrix} \delta m_{\phi_1}^2 & 0 \\ 0 & \delta m_{\phi_2}^2 \end{pmatrix} + \begin{pmatrix} \delta T_{\phi_1 \phi_1} & \delta T_{\phi_1 \phi_2} \\ \delta T_{\phi_1 \phi_2} & \delta T_{\phi_2 \phi_2} \end{pmatrix}$$

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- one-loop corrected potential is gauge-dependent  
→ **vevs** are gauge-dependent  
→ **mass counterterms** become **gauge-dependent**

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- **alternative scheme:** vevs represent the same minimum as at **tree level**  
[ based on: J. Fleischer, F. Jegerlehner, *Phys. Rev. D* **23** (1981) 2001-2026 ]
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- the shifts translate into **every CT, wave function renormalization constants** and **Feynman rules**
- alternative tadpole scheme **worked out for the 2HDM at one-loop**

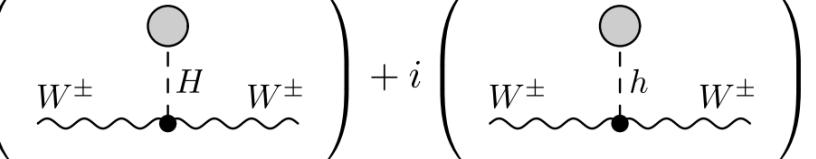
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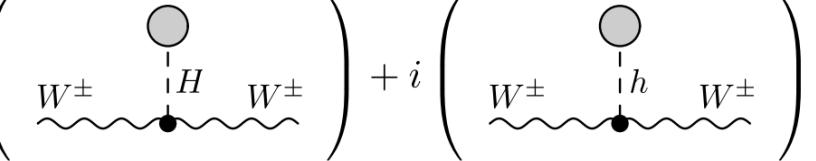
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$$m_W^2 = g^2 \frac{v^2}{4} \longrightarrow m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} = m_W^2 + i \left( \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right) + i \left( \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right)$$


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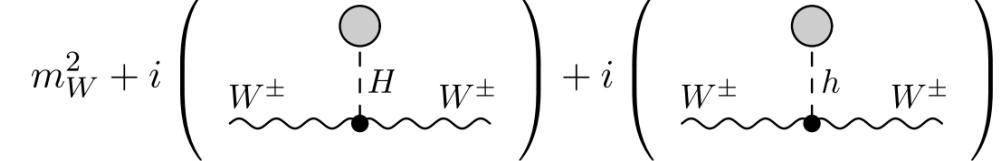
The diagram shows two contributions to the W boson mass. The first contribution is a loop with a Higgs boson (H) vertex and two W± bosons. The second contribution is a loop with a higgs boson (h) vertex and two W± bosons.

## ■ example: coupling between Higgs and Z bosons

$$ig_{HZZ} = \frac{ig^2}{2c_W^2} (c_\alpha v_1 + s_\alpha v_2) , \quad ig_{HHZZ} = \frac{ig^2}{2c_W^2}$$

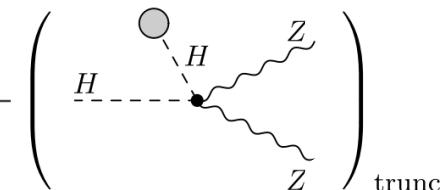
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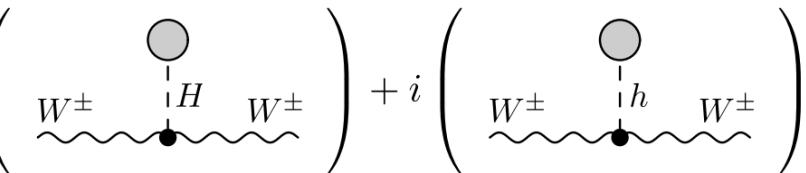
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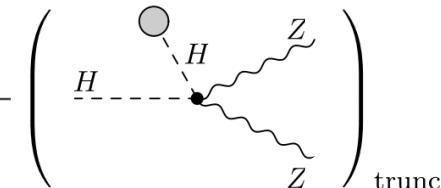
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## ■ effects of the alternative tadpole scheme:

- tadpole diagrams are added everywhere where they exist in the 2HDM
- mass counterterms become manifestly gauge-independent
- tadpole counterterms in the scalar sector are removed

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- at NLO, the other tree-level parameters  $m_W$  and  $g$  still **have to be renormalized**:

$$\frac{2m_W}{g} \Big|^{\text{tree}} \rightarrow \frac{2m_W}{g} \Big|^{\text{ren}}_{\text{alt}} + \underbrace{\frac{2m_W}{g} \left( \frac{\delta m_W^2}{2m_W^2} - \frac{\delta g}{g} \right)}_{\equiv \Delta v} \Big|_{\text{FJ}}$$

- the quantity  $\Delta v$  combines the effect of the CTs of  $m_W$  and  $g$

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  - [ MK, D. Lopez-Val, M. M. Mühlleitner, R. Santos, *J. High Energ. Phys.* **2017** (2017) 77 ]
  - renormalization through  $\overline{\text{MS}}$ , process-dependent scheme, ...

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    - can be **numerically unstable** in one-to-two-body decays
    - **divergences for degenerate masses / “dead corners”** of parameter space
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[ A. Freitas, D. Stöckinger, *Phys. Rev. D* **66** (2002) 095014 ]
- is there a renormalization scheme **for the 2HDM** satisfying these **“three desirable criteria”?**

# Renormalization: Scalar Mixing Angles (II)

- approach by S. Kanemura *et al.*: connect the definition of  $\alpha$  and  $\beta$  with the **inverse propagator matrix** (“KOSY scheme”)

[ S. Kanemura *et al.*, Phys. Rev. D70 (2004) 115002 ]

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_0 = R_{\theta,0}^T \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix}_0 \approx R_{\delta\theta}^T R_\theta^T \sqrt{Z_{\tilde{\phi}}} R_\theta R_\theta^T \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix} \approx \begin{pmatrix} 1 + \frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2} + \delta\theta \\ \delta C_{\phi_2} - \delta\theta & 1 + \frac{\delta Z_{\phi_2\phi_2}}{2} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

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- mixing angle counterterms **within the standard tadpole scheme**:

$$\begin{aligned} \delta\alpha &= \frac{1}{2(m_H^2 - m_h^2)} \text{Re} \left[ \Sigma_{Hh}(m_H^2) + \Sigma_{Hh}(m_h^2) - 2\delta T_{Hh} \right] \\ \delta\beta &= -\frac{1}{2m_{H^\pm}^2} \text{Re} \left[ \Sigma_{G^\pm H^\pm}(m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm}(0) - 2\delta T_{G^\pm H^\pm} \right] \end{aligned}$$

- it was shown analytically that the **KOSY scheme** introduces an **intricate gauge dependence** in  $\delta\alpha$  and  $\delta\beta$

[ MK, *Master's thesis* (2016), KIT ]

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- the PT was worked out
  - to all orders in the SM [ [D. Binosi, \*J. Phys. G\* \*\*30\*\* \(2004\) 1021](#) ]
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use the pinched scalar self-energies instead of the usual ones
- properties of the pinched scheme:
  - **process-independent**, symmetric in the fields
  - manifestly **gauge-independent** per construction
    - ➡ gauge-independent NLO **amplitudes**
  - **numerically stable** (depending on the point in parameter space)
    - ➡ proposed solution for renormalizing  $\delta\alpha$  and  $\delta\beta$  in the 2HDM
- possible downside: contains off-diagonal two-point functions (truly “OS”?)

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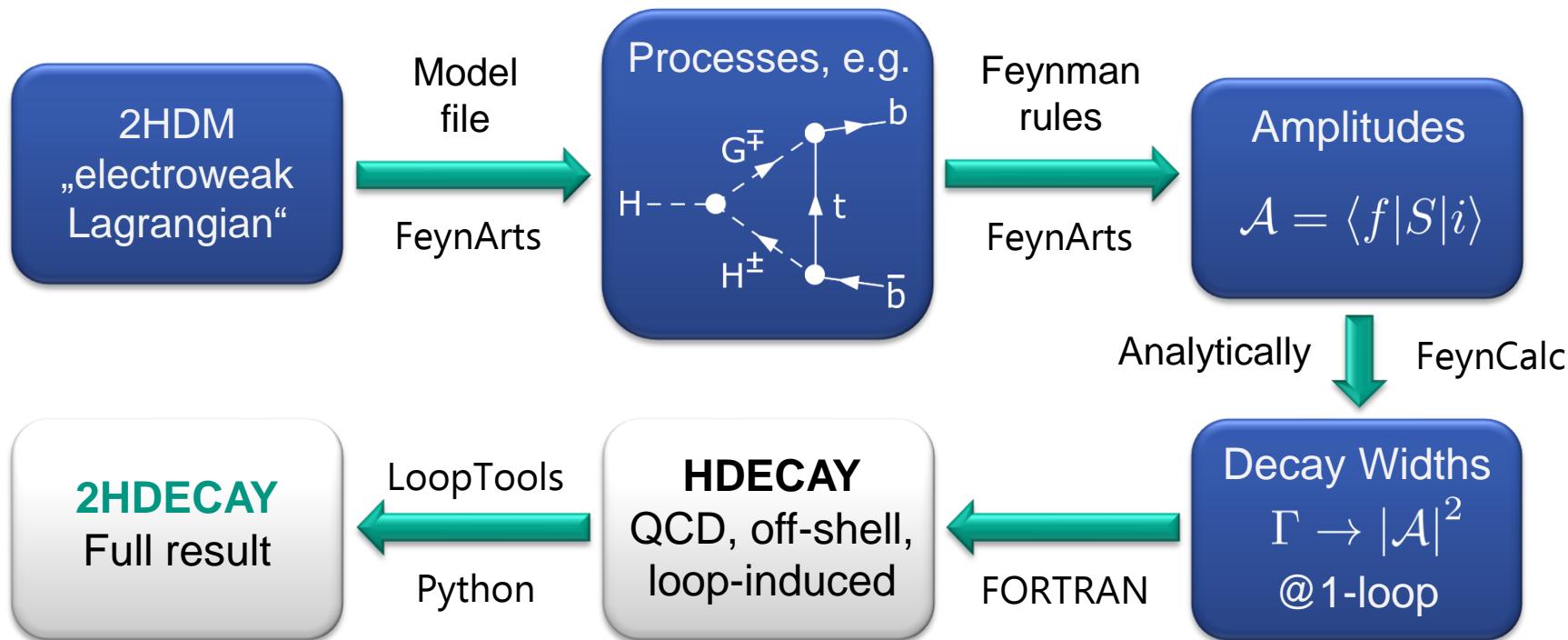
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$$\frac{\mathcal{A}_1^{H\nu_i\nu_i}}{\mathcal{A}_1^{h\nu_i\nu_i}} \equiv \frac{\mathcal{A}_0^{H\nu_i\nu_i}}{\mathcal{A}_0^{h\nu_i\nu_i}} \quad (i = 1, 2)$$

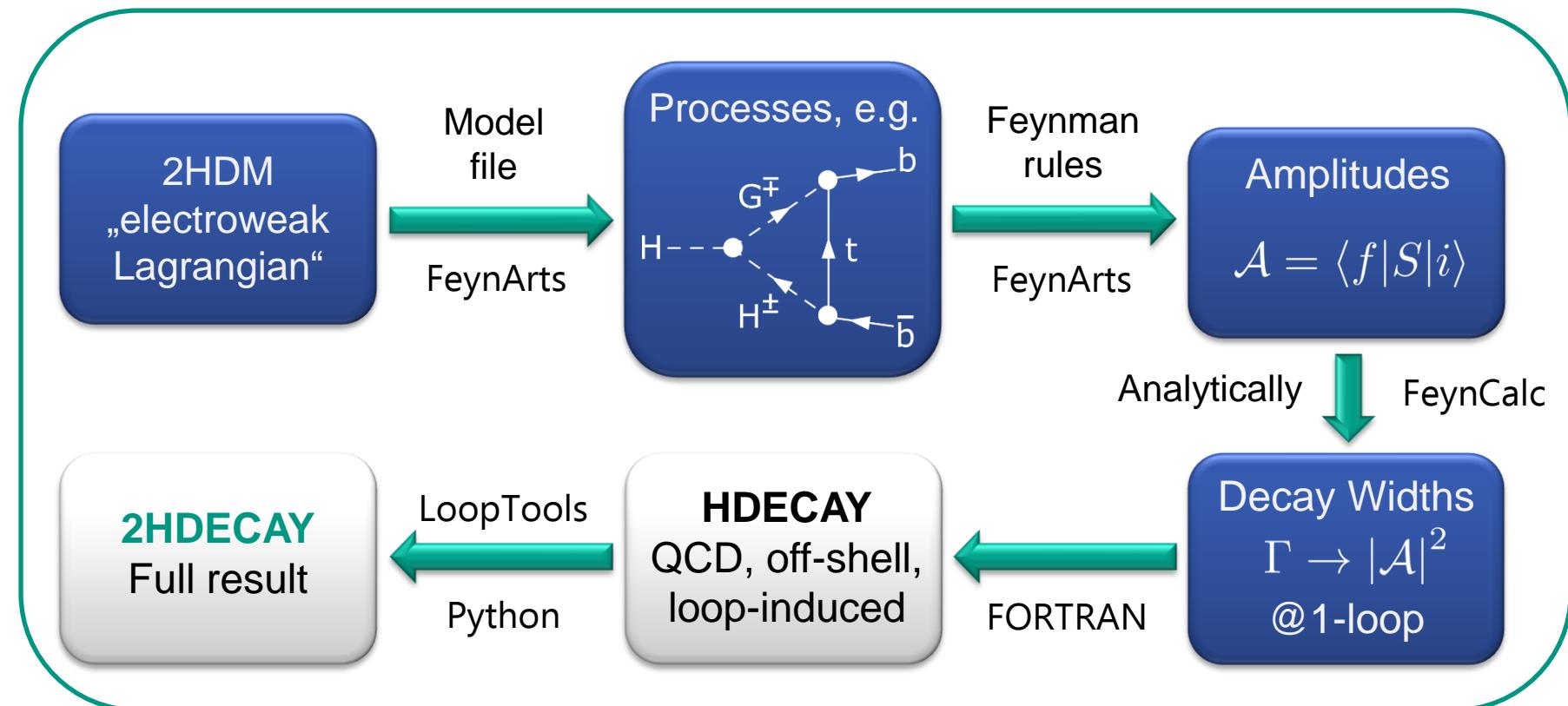
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- gauge-independent “physical OS approach”: use S matrix elements through a process [A. Denner, S. Dittmaier, J.-N. Lang, arXiv:1808.03466]
- idea: introduce **two right-handed fermion singlets**  $\nu_{iR}$  with an additional  $\mathbb{Z}_2$  symmetries to prevent generation mixing
  - massive neutrinos with Yukawa couplings  $y_{\nu_i}$
- renormalization of  $\delta\alpha$  and  $\delta\beta$  through requirement that ratios of tree-level and one-loop S matrix elements are equivalent, e.g.:
$$\frac{\mathcal{A}_1^{H\nu_i\nu_i}}{\mathcal{A}_1^{h\nu_i\nu_i}} \equiv \frac{\mathcal{A}_0^{H\nu_i\nu_i}}{\mathcal{A}_0^{h\nu_i\nu_i}} \quad (i = 1, 2)$$
- properties of the “physical OS approach”:
  - CTs are defined purely through gauge-independent S matrix elements
    - manifestly **gauge-independent** per construction
  - **numerically stable** (depending on the point in parameter space)
- possible downside: contains process-specific contributions (universality?)

# Implementation: 2HDECAY (I)



# Implementation: 2HDECAY (I)



## 2HDECAY: “2HDM HDECAY”

A program for the calculation One-Loop Electroweak Corrections to Higgs Decays  
in the Two-Higgs-Doublet Model Including State-of-the-Art QCD Corrections

[ MK, M. M. Mühlleitner, M. Spira, arXiv:1810.00768; <https://github.com/marcel-krause/2HDECAY> ]

# Implementation: 2HDECAY (II)

- renormalization schemes that are (to be) implemented in **2HDECAY**:

included in  
the original version  
**2HDECAY 1.0.0**

added in  
the current version  
**2HDECAY 1.1.0**

Input ID	Tadpole scheme	$\delta\alpha$	$\delta\beta$	Gauge-indep. $\Gamma$
1	standard	KOSY	KOSY (odd)	$\times$
2	standard	KOSY	KOSY (charged)	$\times$
3	alternative (FJ)	KOSY	KOSY (odd)	$\times$
4	alternative (FJ)	KOSY	KOSY (charged)	$\times$
5	alternative (FJ)	$p_*$ -pinched	$p_*$ -pinched (odd)	✓
6	alternative (FJ)	$p_*$ -pinched	$p_*$ -pinched (charged)	✓
7	alternative (FJ)	OS-pinched	OS-pinched (odd)	✓
8	alternative (FJ)	OS-pinched	OS-pinched (charged)	✓
9	standard	proc.-dep. 1	proc.-dep. 1	✓
10	alternative (FJ)	proc.-dep. 1	proc.-dep. 1	✓
11	standard	proc.-dep. 2	proc.-dep. 2	✓
12	alternative (FJ)	proc.-dep. 2	proc.-dep. 2	✓
13	standard	proc.-dep. 3	proc.-dep. 3	✓
14	alternative (FJ)	proc.-dep. 3	proc.-dep. 3	✓
15	standard	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\times$
16	alternative (FJ)	$\overline{\text{MS}}$	$\overline{\text{MS}}$	✓
17	alternative (FJ)	physical OS1	physical OS1	✓
18	alternative (FJ)	physical OS2	physical OS2	✓
19	alternative (FJ)	physical OS12	physical OS12	✓

[ MK, M. M. Mühlleitner, M. Spira, arXiv:1810.00768; <https://github.com/marcel-krause/2HDECAY> ]

# Conclusions and Outlook

- gauge parameter independence: **key principle** for observables in QFTs
- certain renormalization schemes **spoil this independence** in the 2HDM
- a gauge-independent, process-independent and numerically stable scheme for  $\delta\alpha$  and  $\delta\beta$  was worked out **for the first time for the 2HDM**
- full electroweak one-loop corrections to 2HDM Higgs decays calculated
- combination with **state-of-the-art corrections from HDECAY**: development of new tool **2HDECAY**
- several different renormalization schemes included
- phenomenological studies (planned):
  - dependence of NLO corrections on **2HDM type**
  - analysis for certain **interesting limits** (decoupling, alignment, ...)
  - effect of NLO electroweak corrections on **parameter space restriction**

# Thanks!



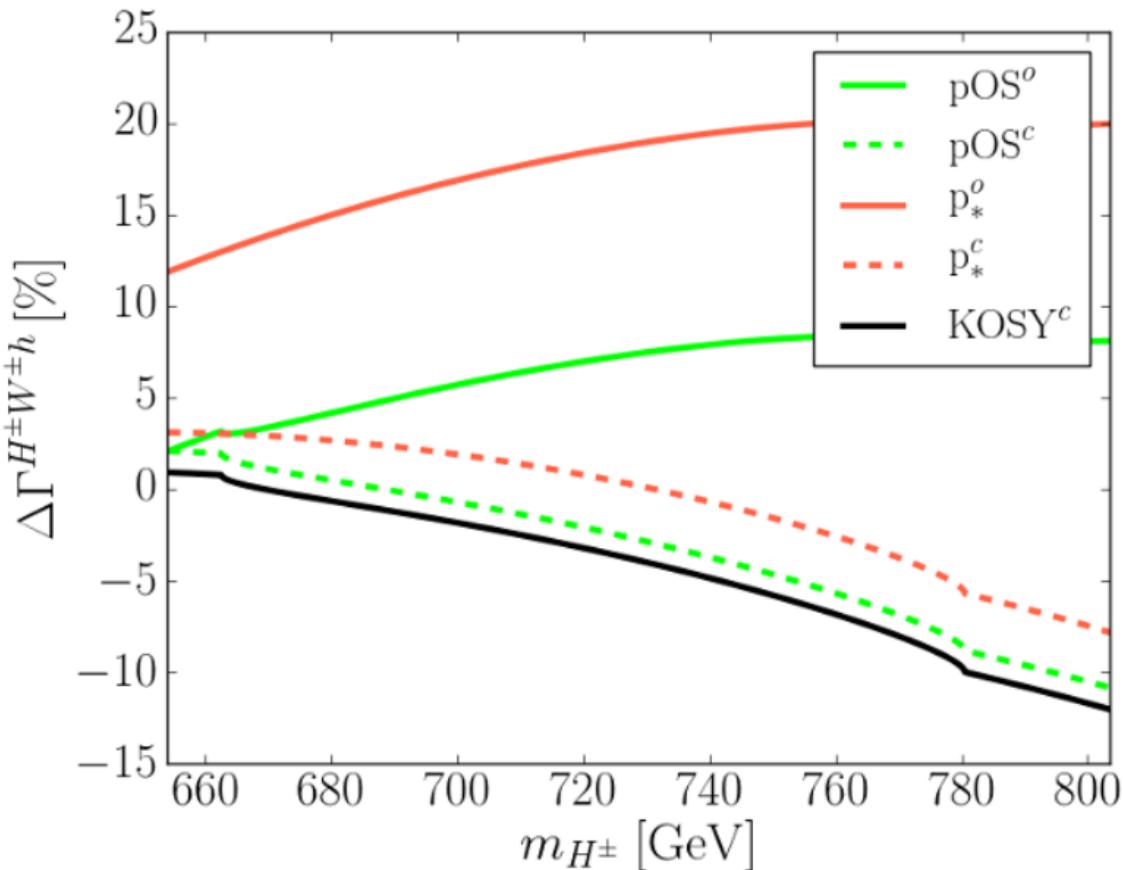
# Backup slides



# Numerical Analysis (I)

- we consider the **exemplary process**  $H^\pm \rightarrow W^\pm h$
- exemplary parameter points (all other parameters: SM-like):  
 $m_h = 125.09 \text{ GeV}$ ,  $m_H = 742.84 \text{ GeV}$ ,  $m_A = 700.13 \text{ GeV}$ ,  $m_{12} = 440.57 \text{ GeV}$   
 $\tan \beta = 1.46$ ,  $\alpha = -0.57$ ,  $m_{H^\pm} = (654 \cdots 804) \text{ GeV}$
- keep in mind: the 2HDM contains **a lot of free parameters**
  - ➡ scanning through the parameter space is possible
- chosen parameter points respect **several constraints**:
  - theoretical (boundedness from below, tree-level unitarity, global minimum)
  - experimental (S/T/U parameters, lower bound on  $m_{H^\pm}$ , ...)

# Numerical Analysis (II)



$m_h = 125.09 \text{ GeV}, m_H = 742.84 \text{ GeV}$   
 $m_A = 700.13 \text{ GeV}, m_{12} = 440.57 \text{ GeV}$   
 $\tan \beta = 1.46, \alpha = -0.57$

- pOS: “on-shell pinched”
- $p^*$ : “ $p^*$ -pinched”
- KOSY: gauge-dependent scheme

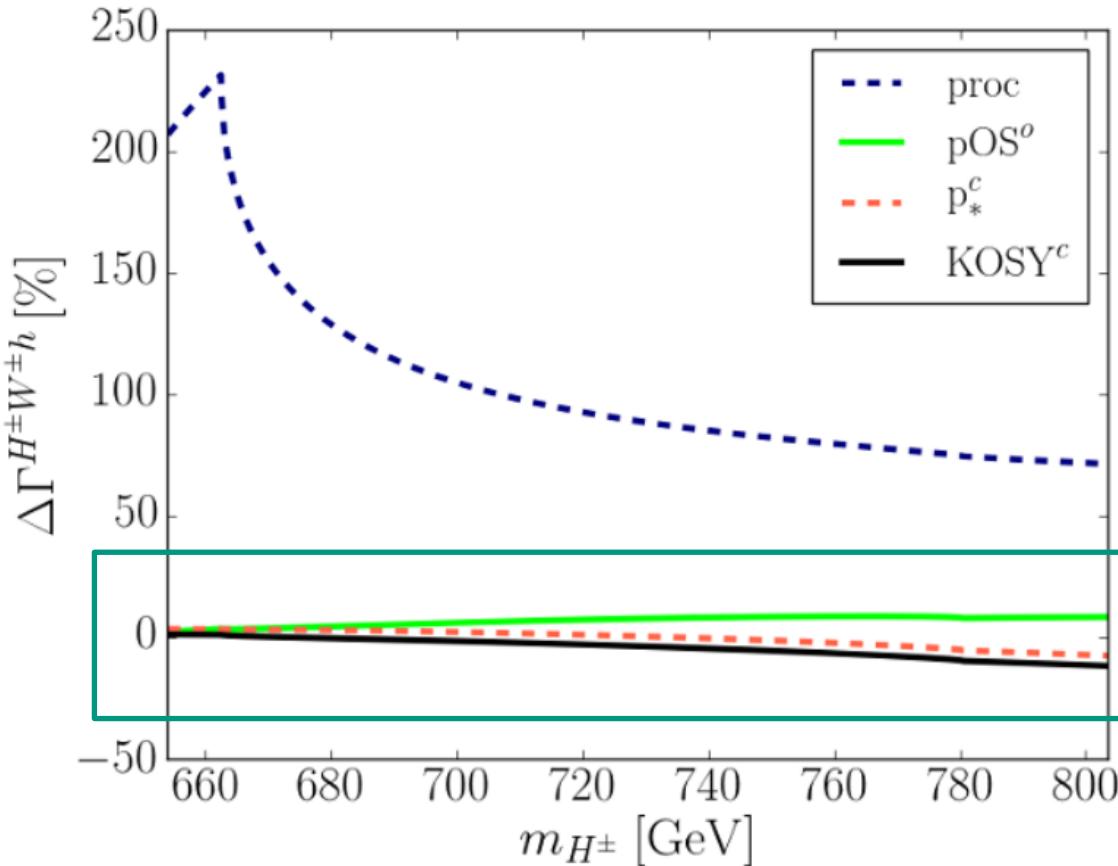
superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

**relative size of NLO corrections:**

$$\Delta\Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- for pinched schemes: NLO corrections are **moderate** (up to 20%)
- relatively large difference in finite parts → missing higher orders  
(full analysis: **rescale the parameters** → **t.b.a.** in **2HDECAY 1.1.0**)

# Numerical Analysis (II)



$$m_h = 125.09 \text{ GeV}, \quad m_H = 742.84 \text{ GeV}$$

$$m_A = 700.13 \text{ GeV}, \quad m_{12} = 440.57 \text{ GeV}$$

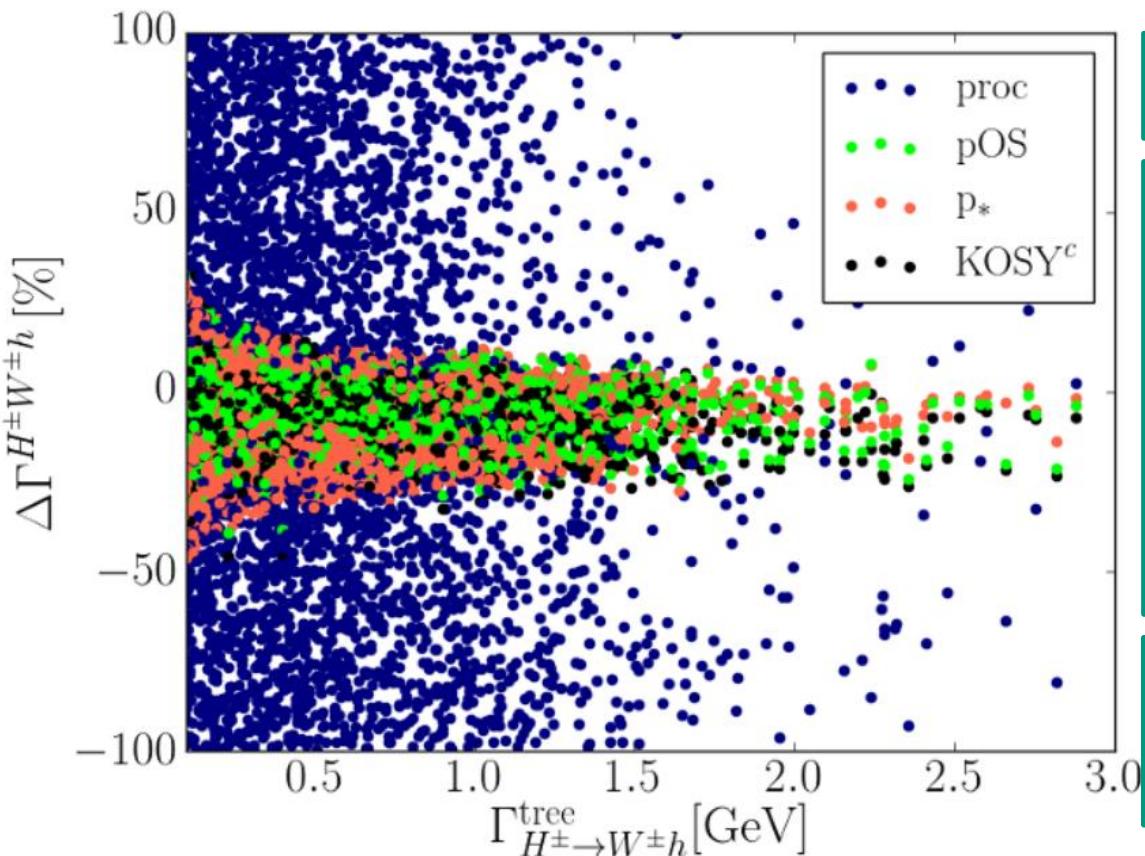
$$\tan \beta = 1.46, \quad \alpha = -0.57$$

- proc: process-dependent
  - pOS: “on-shell pinched”
  - p<sup>\*</sup>: “p<sup>\*</sup>-pinched”
  - KOSY: gauge-dependent scheme
- superscripts “o”, “c”: definition over CP-odd / charged sectors, resp.

$$\Delta\Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- kinks: **thresholds** for certain mass configurations
- process-dependent scheme is often **unsuitable** (large NLO corrections)

# Numerical Analysis (III)



scan over large parameter ranges

- proc: process-dependent
- pOS: "on-shell pinched"
- p\*: "p\*-pinched"
- KOSY: gauge-dependent scheme

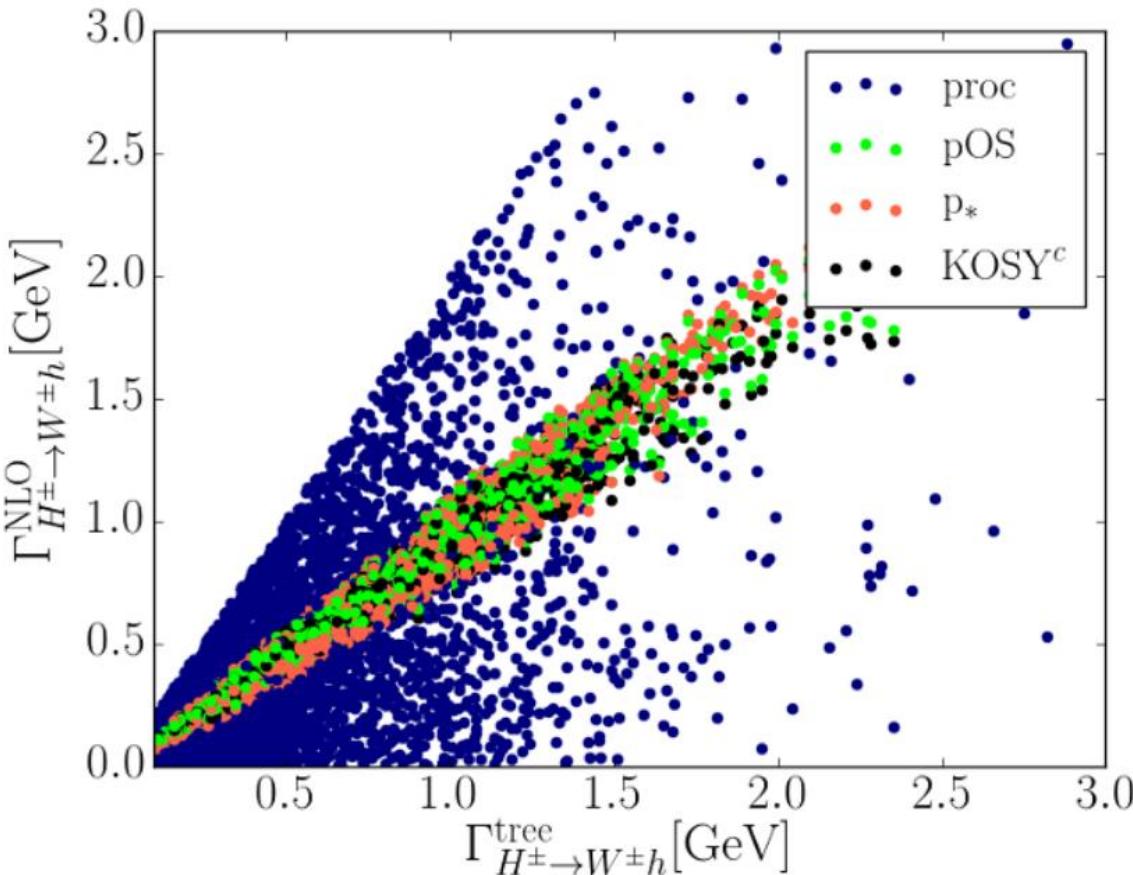
superscript "c": definition over charged sector

relative size of NLO corrections:

$$\Delta\Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- process-dependent scheme: **huge NLO corrections (*unsuitable*)**
- pinched schemes: well-behaving for **large parameter ranges**

# Numerical Analysis (IV)



scan over large parameter ranges

proc: process-dependent  
pOS: "on-shell pinched"  
 $p^*$ : " $p^*$ -pinched"  
KOSY: gauge-dependent scheme

superscript "c": definition over charged sector

$$\Delta\Gamma = \frac{\Gamma_{\text{NLO}} - \Gamma_{\text{LO}}}{\Gamma_{\text{LO}}}$$

- for LO approaching zero,  $\Delta\Gamma$  may become large (**numerical instability**)
- numerical instability is "artificial" (**no** problem of renormalization scheme)

# Gauge Invariance in Electrodynamics

- consider classical electrodynamics (“Theo C”):  $\vec{E}$  and  $\vec{B}$  fields

$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}$$

$\Phi$  : scalar potential

$\vec{A}$  : vector potential

- fields are **invariant** under simultaneous **gauge transformations**

$$\Phi \longrightarrow \Phi - \frac{\partial\Lambda}{\partial t}, \quad \vec{A} \longrightarrow \vec{A} + \nabla\Lambda$$

$\Lambda$  : arbitrary field

→ **Maxwell's equations** are invariant as well

- a **gauge fixing** sets conditions on  $\Lambda$  (and hence, on the potentials)

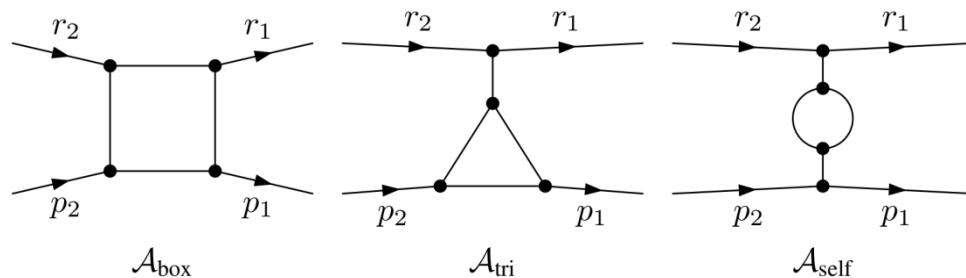
■ Coulomb gauge:  $\nabla \cdot \vec{A} = 0$

■ Lorenz gauge:  $\nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial\Phi}{\partial t} = 0$



can be used to simplify  
Maxwell's equations

# Pinch Technique: Introduction (I)



$$s = (r_1 + p_1)^2 = (r_2 + p_2)^2$$

$$t = (r_1 - r_2)^2 = (p_1 - p_2)^2$$

- we consider a **fermion scattering process** at one-loop QCD:

$$\mathcal{A}_{\text{full}}(s, t, m_1, m_2) = \mathcal{A}_{\text{box}}(s, t, m_1, m_2; \xi) + \mathcal{A}_{\text{tri}}(t, m_1, m_2; \xi) + \boxed{\mathcal{A}_{\text{self}}(t; \xi)}$$

- the gauge dependences **have to cancel** within the individual topologies  
 → rearrangement of the contributions is **always possible**  
 → rearrangement shows that **all** gauge dependences have **self-energy-like** or triangle-like form

$$\mathcal{A}_{\text{full}}(s, t, m_1, m_2) = \tilde{\mathcal{A}}_{\text{box}}(s, t, m_1, m_2) + \tilde{\mathcal{A}}_{\text{tri}}(t, m_1, m_2) + \tilde{\mathcal{A}}_{\text{self}}(t) ,$$

$$\mathcal{A}_{\text{tri}}(t, m_1, m_2; \xi) \rightarrow \tilde{\mathcal{A}}_{\text{tri}}(t, m_1, m_2) + f_{\text{self}}(t; \xi) , \quad \text{etc.}$$

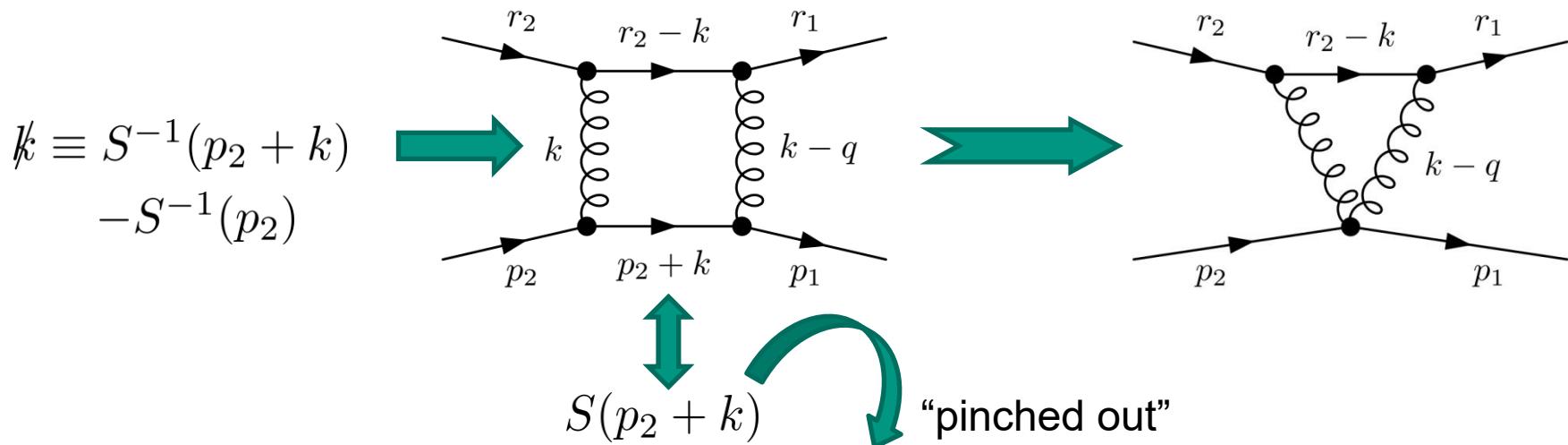
# Pinch Technique: Introduction (II)

- determination of the gauge-dependent contributions: “pinching”
- main idea: trigger the **elementary Ward identity** for the loop momentum

$$\not{k} = (\not{k} + \not{p} - m) - (\not{p} - m) = S^{-1}(k + p) - S^{-1}(p)$$

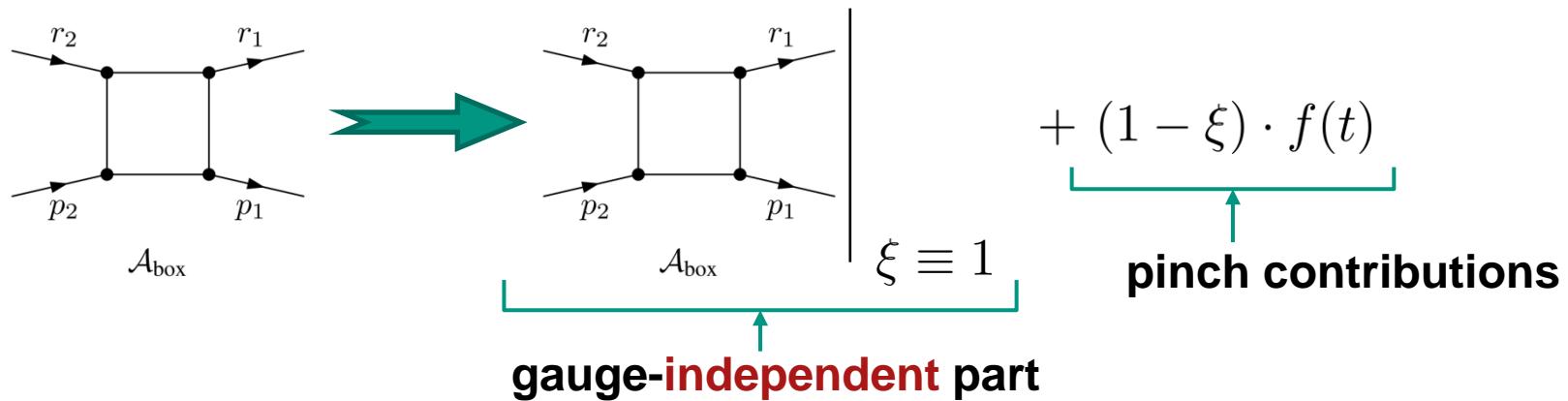
inverse fermion  
propagators

- right expression: vanishes OS between spinors
- left expression: **cancels** (“pinches out”) an **internal fermion** propagator



# Pinch Technique: Results (I)

- (almost) all pinch contributions are **proportional** to  $(1 - \xi)$
- the non-pinched contributions are **equivalent** to diagrams calculated in **Feynman-'t Hooft gauge**, i.e. for  $\xi \equiv 1$



- the **pinch contributions** are **self-energy like**, i.e. functions of only  $t$   
→ **reallocation** of pinch contributions to the **gluon self-energy** possible

# Pinch Technique: Results (II)

- sum of all pinch contributions → cancellation of gauge dependences

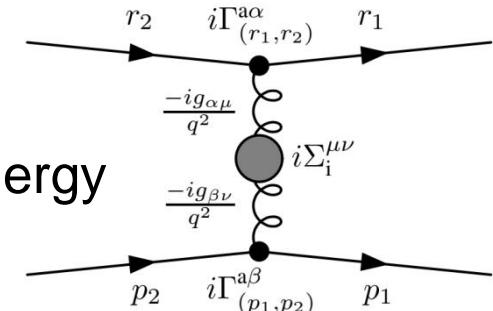
	$g_s^2 t(1 - \xi)^2 \int_k \frac{k^\mu k^\nu}{k^4(k+q)^4}$	$g_s^2 t(1 - \xi) \int_k \frac{k^\mu k^\nu}{k^4(k+q)^2}$	$g_s^2 t(1 - \xi) \int_k \frac{g^{\mu\nu}}{k^2(k+q)^4}$	$g_s^2 t(1 - \xi) \int_k \frac{g^{\mu\nu}}{k^4}$	$(q^2 \equiv t)$
$i\Sigma_{\text{box}}^{\mu\nu}$	$t \frac{C_A}{2}$	0	$-tC_A$	0	
$i\Sigma_{\text{tril1}}^{\mu\nu}$	0	0	0	$C_A - 2C_f$	
$i\Sigma_{\text{tril2}}^{\mu\nu}$	$-tC_A$	$2C_A$	$2tC_A$	$-2C_A$	
$i\Sigma_{\text{self,q}}^{\mu\nu}$	0	0	0	$2C_f$	
$i\Sigma_{\text{self,g}}^{\mu\nu}$	$t \frac{C_A}{2}$	$-2C_A$	$-tC_A$	$C_A$	
Sum	0	0	0	0	

$C_A, C_f$  : Casimir operators

- main results from the application of the pinch technique:
  - demonstration of intricate cancellation of gauge dependences
  - cancellation is not accidental, but follows from Ward identities

# Gauge-Independent Self-Energies via PT

- all pinch contributions are self-energy-like  
→ **reallocates** pinch contributions to the gluon self-energy



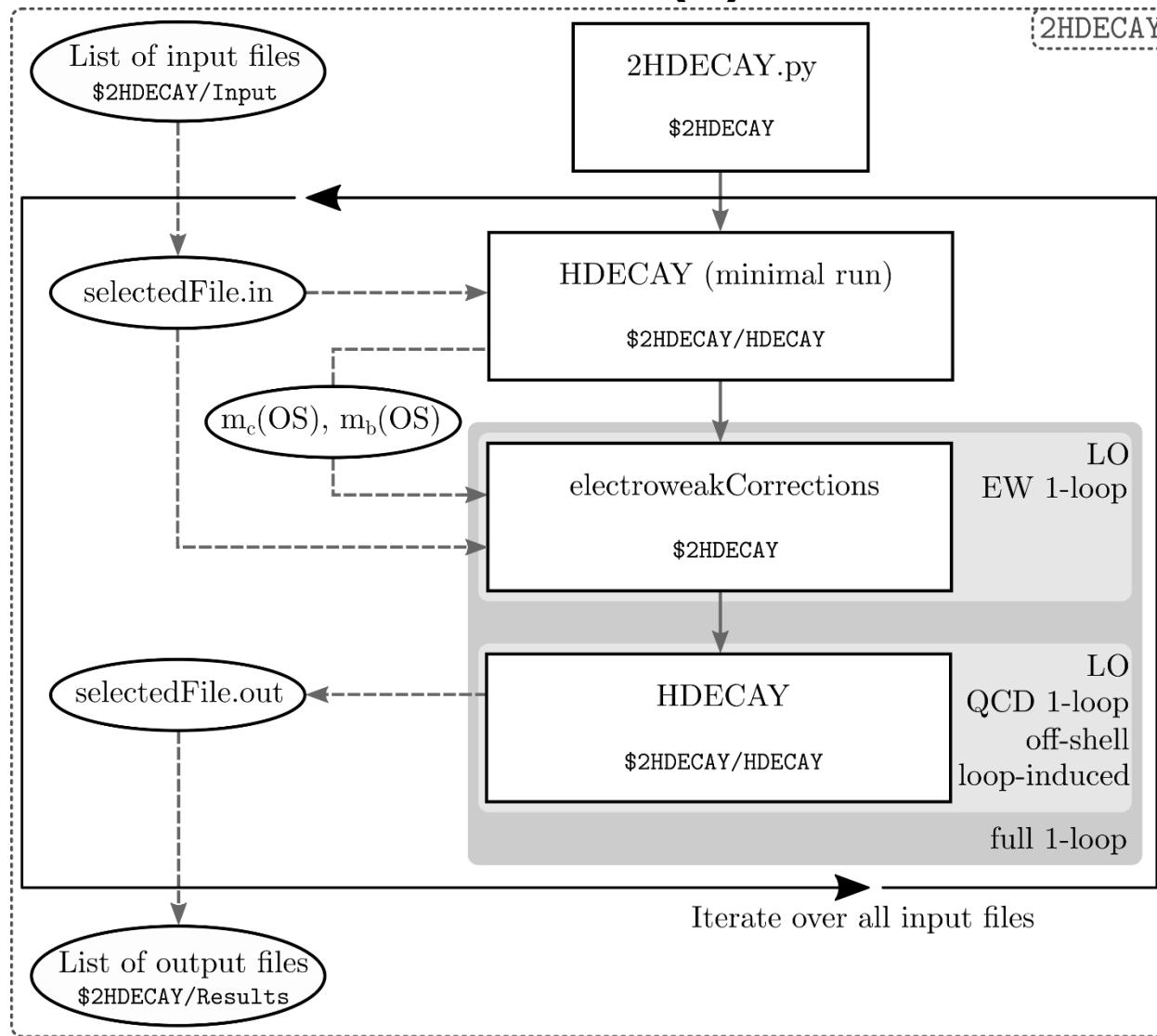
- the pinched self-energy is equivalent to the one evaluated for  $\xi \equiv 1$  after the cancellation of all gauge dependences  
→ Feynman-'t Hooft-gauge is a **special gauge choice**
- **interesting properties** of the pinched gluon self-energy:
  - analogy to the gluon self-energy given by the **Background Field Method**
  - **uniquely defined** by the pinch technique framework
  - manifestly **gauge-independent** → allows for gauge-independent **counterterms**
  - obeys **QED-like Ward identities** instead of complicated Slavnov-Taylor identities

[for more details cf. e.g. D. Binosi, J. Papavassiliou, Phys. Rep. **479** (2009) 1]

# Applications of the Pinch Technique

- the pinch technique can be applied to e.g. the SM, MSSM, **(N)2HDM**, ...
- for consistency: **tadpole diagrams** have to be taken into account  
→ “**alternative tadpole scheme**” is **needed** (cf. part II of the talk)
- applications of the pinched self-energies:
  - definition of **gauge-independent counterterms** (cf. part III of the talk)
  - general analysis of gauge dependence cancellations [D. Binosi, J. Papavassiliou, Phys. Rev. **D65** (2002) 085003]
  - generalization to **all orders** [D. Binosi, J. Phys. **G30** (2004) 1021]
  - construction of **QED-like Ward identities** for e.g. QCD
  - gauge-independent definition of **electroweak parameters**
  - consistent resummation for resonant transition amplitudes
  - extraction of gauge-independent part of **BFM** self-energies

# Implementation: 2HDECAY (II)



[ MK, M. M. Mühlleitner, M. Spira, arXiv:1810.00768; <https://github.com/marcel-krause/2HDECAY> ]