On the Renormalization of the Two-Higgs-Doublet Model


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- Motivation
- Description of the Model
- Renormalization of the 2HDM
  - On-Shell Renormalized Fields and Masses
  - The Scalar Mixing Angles
  - The Parameter
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Motivation

- 2HDM: one of the simplest extensions of the SM
  - dark matter candidate (Inert Doublet Model)
  - source of CP-violation
  - extended scalar sector
  - renormalizable

- renormalization of the two **scalar mixing angles** in the 2HDM is non-trivial

- existing schemes are either numerically unstable, process-dependent or gauge-dependent

- search for a suitable renormalization scheme of the scalar mixing angles
  - full electroweak NLO corrections to all decays within the 2HDM
Description of the Model: Scalar Sector

- two complex $SU(2)_L$ Higgs doublets

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ v_1 + \rho_1 + i\eta_1 \end{pmatrix}^{\sqrt{2}}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ v_2 + \rho_2 + i\eta_2 \end{pmatrix}^{\sqrt{2}}$$

- non-vanishing vacuum expectation values (VEVs) $v_1, v_2$ with

$$v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$

- scalar Lagrangian with CP- and $\mathbb{Z}_2$-conserving 2HDM potential:

$$V_{2\text{HDM}}(\Phi_1, \Phi_2) = m_{11}^2 (\Phi_1^\dagger \Phi_1) + m_{22}^2 (\Phi_2^\dagger \Phi_2) - m_{12}^2 \left[ (\Phi_1^\dagger \Phi_2) + (\Phi_2^\dagger \Phi_1) \right]$$

$$+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1)$$

$$+ \frac{\lambda_5}{2} \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]$$
Description of the Model: Parameters

- **eight** real-valued potential parameters:
  - dimensionless \( \lambda_i \ (i = 1, \ldots, 5) \)
  - mass-squared parameters \( m_{11}^2, m_{22}^2 \) and \( m_{12}^2 \)

- transformation to the Higgs mass basis via **scalar mixing angles**
  - \( \alpha \) for the CP-even sector
  - \( \beta \) for the CP-odd and charged sectors
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- transformation to the Higgs mass basis via **scalar mixing angles**
  - $\alpha$ for the CP-even sector
  - $\beta$ for the CP-odd and charged sectors

- set of **free parameters** of the 2HDM (excluding CKM elements, …)
  \[ \left\{ m_{h^0}, m_{H^0}, m_{A^0}, m_{H^\pm}, \alpha, \beta, m^2_{12}, T_{h^0}, T_{H^0}, e, m_W, m_Z, m_\Psi \right\} \]
  - “physical” tadpole terms $T_{h^0}, T_{H^0}$
  - elementary charge $e$
  - fermion masses $m_\Psi$
  - gauge boson masses $m_W, m_Z$
  - soft-$\mathbb{Z}_2$-breaking scale $m^2_{12}$
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  \[
  \{ m_{h^0}, m_{H^0}, m_{A^0}, m_{H^\pm}, \alpha, \beta, m_{12}^2, T_{h^0}, T_{H^0}, e, m_W, m_Z, m_\Psi \} 
  \]

- “physical” tadpole terms $T_{H^0}, T_{h^0}$ => alternative tadpole scheme
- elementary charge $e$ => Thomson limit
- fermion masses $m_\Psi$ => on-shell
- gauge boson masses $m_W, m_Z$ => on-shell
- soft- $\mathbb{Z}_2$ -breaking scale $m_{12}^2$ => $\overline{\text{MS}}$
Renormalization: Alternative Tadpole Scheme

- **alternative scheme:** VEVs represent the same minimum as at **tree level**

- correct minimum conditions at NLO require a **shift in the VEVs**

- shifts are connected to **tadpole renormalization**
  ➔ tadpole diagrams appear
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- shifts are connected to *tadpole renormalization*
  
  - tadpole diagrams appear

- the shifts translate into **all CTs, wave function renormalization constants** and *Feynman rules for vertices*

- bare masses are expressed through gauge-independent **tree-level VEVs**
  
  - mass CTs **become gauge-independent**

- alternative tadpole scheme **worked out for the 2HDM**
  
  *(MK: Master's thesis, Karlsruhe Institute of Technology, 2016)*
Renormalization: Scalar Mixing Angles

- renormalization of mixing angles $\alpha$ and $\beta$ is **non-trivial** in the 2HDM

- simplest approach: $\overline{\text{MS}}$ conditions for $\alpha$ and $\beta$
  - **numerically unstable**  
  - **unsuitable scheme** in many cases
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  - numerically unstable  
  \textit{(R. Lorenz: Master’s thesis, Karlsruhe Institute of Technology, 2015)}
  - unsuitable scheme in many cases

- "**no-go theorem**" for the MSSM: a renormalization scheme for $\tan \beta$
  - cannot be simultaneously
    - gauge-independent
    - process-independent
    - numerically stable

- is there a renormalization scheme for the 2HDM **satisfying all three criteria** above?
Renormalization: Scalar Mixing Angles

- S. Kanemura et al. (KOSY scheme): connect definition of $\delta \alpha$ and $\delta \beta$ with inverse propagator of the scalar doublets
  - intricate gauge-dependence in $\delta \alpha$ and $\delta \beta$ (MK: Master’s thesis, KIT, 2016)
  - gauge-dependence cannot be removed unambiguously
  - for gauge-independent decay widths: gauge-dep. parameter treatment

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- process-dependent definition: $\Gamma^{\text{LO}} \overset{!}{=} \Gamma^{\text{NLO, weak}}$ for $\frac{H}{A^0} \rightarrow \tau^+ \tau^-$
  - gauge-independent S-matrix elements
  - $\delta \alpha$ and $\delta \beta$ are flavor- and process-dependent


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  - gauge-independent S-matrix elements
  - $\delta\alpha$ and $\delta\beta$ are flavor- and process-dependent

- gauge-independent scheme: use the pinch technique (PT)
  - consistency requires alternative tadpole scheme
  - PT was worked out for the 2HDM (MK: Master’s thesis, KIT, 2016)
  - $\delta\alpha$ and $\delta\beta$ are process-indep., gauge-indep. and unambiguously defined

(D. Binosi, J. Papavassiliou: j.physrep.2009.05.001, arXiv:0909.2536)

Numerical Results: \[ H^+ \rightarrow W^+ h \]

check for **numerical stability**: 

\[ \Delta \Gamma_{H+W+h} := \frac{\Gamma_{H+W+h}^{\text{NLO}} - \Gamma_{H+W+h}^{\text{LO}}}{\Gamma_{H+W+h}^{\text{LO}}} \]
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  \[ \Delta \Gamma_{H^+ W^+ h} := \frac{\Gamma_{H^+ W^+ h}^{\text{NLO}} - \Gamma_{H^+ W^+ h}^{\text{LO}}}{\Gamma_{H^+ W^+ h}^{\text{LO}}} \]

- points in parameter space still allowed by experiment and theory
Numerical Results: $H^+ \rightarrow W^+ h$

- check for numerical stability:

$$\Delta \Gamma_{H+W+h} := \frac{\Gamma_{H+W+h}^{NLO} - \Gamma_{H+W+h}^{LO}}{\Gamma_{H+W+h}^{LO}}$$

- points in parameter space still allowed by experiment and theory

- KOSY / pinched schemes: mostly within $\pm 30\%$

- process-dependent scheme: mostly up to $\pm 250\%$

- process-dependent scheme: numerically unstable for type II

- pinched schemes: numerically stable

(MK, Master's thesis)

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Conclusions

- the alternative tadpole scheme was worked out for the 2HDM
- the KOSY scheme can lead to gauge-dependent NLO decay widths
- the pinched scheme for $\delta\alpha$ and $\delta\beta$ is gauge-independent, process-independent and numerically stable
  ➔ no “no-go” theorem for the 2HDM


future work:
- provide 2HDMCalc, a new tool for calculating all 2HDM decay processes at NLO for various renormalization schemes (for comparison)
- analyze 2HDM types I, X and Y

scheduled: Fall 2017
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Description of the Model: Full Lagrangian

- consider the **electroweak** Lagrangian of the 2HDM:

\[
\mathcal{L}_{2\text{HDM}}^{\text{EW}} = \mathcal{L}_Y + \mathcal{L}_F + \mathcal{L}_{GF} + \mathcal{L}_{FP} + \mathcal{L}_S + \mathcal{L}_{Yuk}
\]

- \(\mathcal{L}_Y\) : Yang-Mills Lagrangian

- \(\mathcal{L}_F\) : Fermion Lagrangian (kinetic terms and interactions with gauge bosons)

- \(\mathcal{L}_{GF}\) : Gauge-fixing Lagrangian

- \(\mathcal{L}_{FP}\) : Faddeev-Popov Lagrangian

- \(\mathcal{L}_S\) : Scalar Lagrangian (kinetic terms and scalar potential)

- \(\mathcal{L}_{Yuk}\) : Yukawa Lagrangian (scalar-fermion interactions)

"SM-like"
Description of the Model: Scalar Sector

- expand the doublets around their VEVs:

\[ \Phi_1 = \left( \frac{\omega_1^+}{v_1 + \rho_1 + i\eta_1} \right), \quad \Phi_2 = \left( \frac{\omega_2^+}{v_2 + \rho_2 + i\eta_2} \right) \]

- eight real fields: two CP-even \( \rho_i \), two CP-odd \( \eta_i \), four charged \( \omega_i^{\pm} \)

- minimum of the potential \( \triangleq \) vanishing of the \textbf{tadpole parameters} \( T_1, T_2 \)
  (terms linear in \( \rho_i \)) at tree level:

\[
T_1 := m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2} \lambda_1 v_1^3 + \frac{1}{2} \lambda_{345} v_1 v_2^2 \overset{!}{=} 0 \\
T_2 := m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2} \lambda_2 v_2^3 + \frac{1}{2} \lambda_{345} v_1^2 v_2 \overset{!}{=} 0
\]
Description of the Model: Scalar Sector

- **two** complex $SU(2)_L$ Higgs **doublets** $\Phi_i \ (i = 1, 2)$

- scalar Lagrangian with 2HDM potential $V_{2\text{HDM}}$:

$$L_S = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V_{2\text{HDM}} (\Phi_1, \Phi_2)$$

- covariant derivative:

$$D_\mu = \partial_\mu + ig\frac{\sigma^a}{2} W^a_\mu (x) + ig' \frac{Y}{2} B_\mu (x) \quad \text{(sum over } a = 1, 2, 3)$$

- **non-vanishing** vacuum expectation values (VEVs) $v_1, v_2$

- **spontaneous symmetry breaking:** mass generation, e.g.

$$m_W^2 = g^2 \frac{v^2}{4}, \quad v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$
Description of the Model: Fermion Sector

- coupling between fermions and scalars: Yukawa Lagrangian

- in contrast to the SM: Yukawa matrices in mass terms and couplings are **not proportional** to each other
  - FCNC possible on tree level

- impose an **additional $\mathbb{Z}_2$ symmetry**: $\mathcal{L}_{2\text{HDM}}^{\text{EW}}$ invariant under $\Phi_1 \rightarrow -\Phi_1$
  - no FCNC on tree level

- exact realization of the symmetry determines the 2HDM type:
  - **type I**: RH quarks/leptons couple only to doublet $\Phi_2$
  - **type II**: RH up-type quarks couple to $\Phi_2$, RH down-type quarks/leptons to $\Phi_1$
  - ...

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Description of the Model: Scalar Sector

- terms **bilinear** in the field doublets:

\[ V_{\text{2HDM}}^{\text{bilin}} = \frac{1}{2} \begin{pmatrix} \rho_1 & \rho_2 \end{pmatrix} M_\rho^2 \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \eta_1 & \eta_2 \end{pmatrix} M_\eta^2 \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \omega_1^+ & \omega_2^+ \end{pmatrix} M_\omega^2 \begin{pmatrix} \omega_1^- \\ \omega_2^- \end{pmatrix} \]

- **diagonalization** with **two mixing angles**

\[ R_\alpha = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}, \quad R_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \quad (c_x := \cos(x), \ s_x := \sin(x)) \]

- \( \alpha \): CP-even sector; \( \beta \): CP-odd and charged sectors

- **2HDM potential is rotated to the mass basis:**
  - CP-even Higgses \( (H^0, h^0) \) with masses \( (m_{H^0}, m_{h^0}) \)
  - CP-odd Higgs and Goldstone \( (A^0, G^0) \) with masses \( (m_{A^0}, 0) \)
  - charged Higgses and Goldstones \( (H^\pm, G^\pm) \) with masses \( (m_{H^\pm}, 0) \)
  - soft-\( \mathbb{Z}_2 \)-breaking parameter \( \Lambda_5 \) (or equivalently, mass scale \( M \)):

\[ \Lambda_5 := \frac{2m_{12}^2}{v^2 s_\beta c_\beta} \equiv \frac{2M^2}{v^2} \]
Renormalization: 2HDM Parameters

- renormalization program for the 2HDM:
  - tadpole terms \(\rightarrow\) standard / alternative tadpole scheme
  - mass counterterms \(\rightarrow\) on-shell
  - field strength renormalization constants \(\rightarrow\) on-shell
  - elementary charge \(\rightarrow\) Thomson limit (without light fermion contributions)
  - parameter \(m_{12}\) \(\rightarrow\) \(\overline{\text{MS}}\)
  - scalar mixing angles \(\rightarrow\) ?
Renormalization: On-Shell Conditions

- **Consider scalar field doublet** \((\phi_1, \phi_2)\)

- **Field strength renormalization:**

  \[
  \begin{pmatrix}
  \phi_1 \\
  \phi_2
  \end{pmatrix}
  = \sqrt{Z}_\phi
  \begin{pmatrix}
  \phi_1 \\
  \phi_2
  \end{pmatrix}
  \approx
  \begin{pmatrix}
  12 \times 2 + \delta Z_\phi \\
  \delta Z_\phi/2
  \end{pmatrix}
  \begin{pmatrix}
  \phi_1 \\
  \phi_2
  \end{pmatrix}
  ,
  \delta Z_\phi/2
  =
  \begin{pmatrix}
  \delta Z_{\phi_1\phi_1} \\
  \delta Z_{\phi_2\phi_1} \\
  \delta Z_{\phi_2\phi_2}
  \end{pmatrix}
  \]

- **Two-point correlation function for the doublet with momentum** \(p^2\):

  \[
  \hat{\Gamma}_\phi(p^2) :=
  \begin{pmatrix}
  \hat{\Gamma}_{\phi_1\phi_1}(p^2) & \hat{\Gamma}_{\phi_1\phi_2}(p^2) \\
  \hat{\Gamma}_{\phi_2\phi_1}(p^2) & \hat{\Gamma}_{\phi_2\phi_2}(p^2)
  \end{pmatrix}
  \]

  \[
  = i \sqrt{Z}_\phi^t \left[ p^2 1_{2 \times 2} - D^2_\phi + \Sigma_\phi(p^2) - \delta D^2_\phi \right] \sqrt{Z}_\phi \approx i \left[ p^2 1_{2 \times 2} - D^2_\phi + \hat{\Sigma}_\phi(p^2) \right]
  \]

- **Mass matrices** ↔ **Mass CTs** ↔ **1PI self-energies** ↔ **Renormalized self-energies**

  \[
  i \Sigma(p^2) := \begin{array}{c}
  \text{1PI} \\
  \text{1PI}
  \end{array} = \begin{array}{c}
  \text{1PI} \\
  \text{1PI}
  \end{array} + \begin{array}{c}
  \text{1PI} \\
  \text{1PI}
  \end{array} + \cdots
  \]
Renormalization: On-Shell Conditions

**on-shell conditions:**

- mixing of fields vanishes for $p^2 = m^2_{\phi_i}$
- masses $m^2_{\phi_i}$ are the real parts of the pole of the propagator
- normalization: residue of the propagator at its pole equals $i$

**fixation of diagonal mass counterterms:**

$$\text{Re}\left[\delta D_{\phi_1\phi_1}^2\right] = \text{Re}\left[\Sigma_{\phi_1\phi_1}(m^2_{\phi_1})\right], \quad \text{Re}\left[\delta D_{\phi_2\phi_2}^2\right] = \text{Re}\left[\Sigma_{\phi_2\phi_2}(m^2_{\phi_2})\right]$$

**fixation of field strength renormalization constants:**

$$\delta Z_{\phi_1\phi_1} = -\text{Re}\left[\frac{\partial \Sigma_{\phi_1\phi_1}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_1}}, \quad \delta Z_{\phi_2\phi_2} = -\text{Re}\left[\frac{\partial \Sigma_{\phi_2\phi_2}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_2}}$$

$$\delta Z_{\phi_1\phi_2} = \frac{2}{m^2_{\phi_1} - m^2_{\phi_2}} \text{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta D_{\phi_1\phi_2}^2\right], \quad \delta Z_{\phi_2\phi_1} = \frac{2}{m^2_{\phi_2} - m^2_{\phi_1}} \text{Re}\left[\Sigma_{\phi_1\phi_2}(m^2_{\phi_1}) - \delta D_{\phi_1\phi_2}^2\right]$$

**the specific form of the $\delta D_{\phi_i\phi_j}^2$ depends on the tadpole scheme**
Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:

\[ i T_{H^0/h^0} - i \delta T_{H^0/h^0} = 0 \]

- **purpose**: restore the minimum conditions of the potential at NLO

- **practical effect**: no tadpole diagrams in NLO calculations
Renormalization: Standard Tadpole Scheme

- **standard scheme**: VEVs are derived from the loop-corrected potential
  (e.g. in A. Denner: arXiv:0709.1075)

- VEVs in the mass relations produce correct one-loop OS masses

- tadpole terms appear explicitly in the bare mass matrices
  \[ \Rightarrow \text{mass matrix counterterms contain the tadpole counterterms} \]

- one-loop corrected potential is gauge-dependent
  \[ \Rightarrow \text{VEVs are gauge-dependent} \]
  \[ \Rightarrow \text{mass counterterms become gauge-dependent} \]
Renormalization: Alternative Tadpole Scheme

**example: W boson mass**

\[ m_W^2 = g^2 v^2 \frac{v^2}{4} \rightarrow m_W^2 + g^2 v_1 \delta v_1 + \frac{v_2 \delta v_2}{2} = m_W^2 + i \left( \begin{array}{c} W^\pm \\ \bullet \end{array} H^0 \begin{array}{c} W^\pm \\ \bullet \end{array} \right) + i \left( \begin{array}{c} W^\pm \\ \bullet \end{array} h^0 \begin{array}{c} W^\pm \\ \bullet \end{array} \right) \]

**example: coupling between Higgs and Z bosons**

\[ i g_{H^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2} (c_\alpha v_1 + s_\alpha v_2), \quad i g_{H^0 H^0 Z^0 Z^0} = \frac{ig^2}{2c_W^2} \]

\[ i g_{H^0 Z^0 Z^0} \rightarrow i g_{H^0 Z^0 Z^0} + \frac{ig^2}{2c_W^2} (c_\alpha \delta v_1 + s_\alpha \delta v_2) = i g_{H^0 Z^0 Z^0} + \left( \begin{array}{c} H^0 \\ \bullet \end{array} \begin{array}{c} Z^0 \\ \bullet \end{array} \right) \text{trunc} \]

**effects** of the alternative tadpole scheme:

- **tadpole diagrams are added everywhere** where they exist in the 2HDM
- **mass counterterms become manifestly gauge-independent**
- **tadpole counterterms in the scalar sector are removed**
Renormalization: Scalar Mixing Angles

- approach by S. Kanemura et al.: connect the definition of $\alpha$ and $\beta$ with the inverse propagator matrix

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}_0 = R^T_{\delta \theta, 0} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}_0 \approx R^T_{\delta \theta} R^T_{\theta} \sqrt{Z_{\phi}} R_{\theta} R^T_{\theta} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}_0 \approx \begin{pmatrix}
1 + \frac{\delta Z_{\phi_1}}{2} \\
\delta C_{\phi_2} + \delta \theta
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}_0
\]

- mixing angle counterterms within the standard tadpole scheme:

\[
\delta \alpha = \frac{1}{2 (m_{H_0}^2 - m_{H^0}^2)} \Re \left[ \Sigma_{H^0 H^0} (m_{H_0}^2) + \Sigma_{H^0 H^0} (m_{H^0}^2) - 2 \delta T_{H^0 H^0} \right]
\]

\[
\delta \beta = -\frac{1}{2 m_{H^\pm}^2} \Re \left[ \Sigma_{G^\pm H^\pm} (m_{H^\pm}^2) + \Sigma_{G^\pm H^\pm} (0) - 2 \delta T_{G^\pm H^\pm} \right]
\]


- it was shown analytically that Kanemura’s scheme introduces an intricate gauge-dependence in $\delta \alpha$ and $\delta \beta$

(MK, Master’s thesis, Karlsruhe Institute of Technology, 2016)
Renormalization: Scalar Mixing Angles

- gauge-independent approach: use the pinch technique (PT)
  

- main ideas:
  - use a toy scattering process that includes to-be pinched self-energies
  - unambiguously isolate all gauge-dependences by triggering Ward identities
  - rearrange all gauge-dependences in the S-matrix elements
    - creation of manifestly gauge-independent self-energies

- consistent application of the PT requires the alternative tadpole scheme

- the PT was worked out...
  - for the singlet extension of the SM
  - for the scalar sector of the 2HDM  (MK, Master’s thesis, KIT, 2016)
Renormalization: Scalar Mixing Angles

- PT-based definition of the scalar mixing angle counterterms:

\[
\delta\alpha = \frac{\text{Re} \left[ \Sigma^{\text{tad}}_{H^0 H^0}(m^2_{H^0}) + \Sigma^{\text{tad}}_{H^0 H^0}(m^2_{H^0}) \right]_{\xi=1}}{2 \left( m^2_{H^0} - m^2_{h^0} \right)} \\
\delta\beta = -\frac{\text{Re} \left[ \Sigma^{\text{tad}}_{G^\pm H^\mp}(m^2_{H^\pm}) + \Sigma^{\text{tad}}_{G^\pm H^\mp}(0) \right]_{\xi=1}}{2m^2_{H^\pm}}
\]

- properties of the pinched scheme:
  - process-independent ✔
  - manifestly gauge-independent by construction ✔
  - unambiguous definition
  - numerically stable ?
  
\[
\delta\alpha = \frac{1}{m^2_{H^0} - m^2_{h^0}} \text{Re} \left[ \Sigma^{\text{tad}}_{H^0 H^0} \left( \frac{m^2_{H^0} + m^2_{h^0}}{2} \right) \right]_{\xi=1} \\
\delta\beta = -\frac{1}{m^2_{A^0}} \text{Re} \left[ \Sigma^{\text{tad}}_{G^0 A^0} \left( \frac{m^2_{A^0}}{2} \right) \right]_{\xi=1}
\]

"OS-pinched"  

"p*-pinched"
Renormalization: Scalar Mixing Angles

- another approach: process-dependent definition

- use the decays $H^0/A^0 \rightarrow \tau^+ \tau^-$ for defining $\delta \alpha$ and $\delta \beta$

- renormalization condition for the extraction of the angle counterterms:

  \[
  \Gamma^{\text{LO}} \equiv \Gamma^{\text{NLO,weak}}
  \]


- exclusion of QED-type diagrams due to IR divergences

- properties:
  - the process-dependent $\delta \alpha$ and $\delta \beta$ lead to gauge-independent S-matrix elements independently of the tadpole scheme
  - the angle counterterms are process- and flavor-dependent
Renormalization: The Parameter $m_{12}$

- no obvious “on-shell” condition applicable for potential parameter $m_{12}$

- implementation of two schemes:
  - $\overline{\text{MS}}$ condition for $\delta m_{12}^2$
  - process-dependent over $\Gamma^{LO} \neq \Gamma^{NLO}$ for the process $H^0 \rightarrow A^0 A^0$

- disadvantage of process-dependent scheme: kinematic constraint

- both schemes are gauge-independent
  - $\overline{\text{MS}}$ scheme is the preferable scheme if numerically stable
Numerical Results: Considered Processes

- choose 2HDM-specific Higgs decays for analysis on **numerical stability**

- considered processes:
  - $H^+ \rightarrow W^+ h/\bar{H}$
  - $H \rightarrow Z^0 Z^0$
  - $H \rightarrow h h$

- full **electroweak** one-loop corrections considered

\[ A_{HZ^0Z^0}^{CT} = \frac{g c_{\beta - \alpha} m_W}{c_W^2} (\varepsilon_2^* \cdot \varepsilon_3^*) \left[ \frac{\delta g}{g} + \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{2m_W^2} + \delta Z_{HH} + \delta ZZ + \frac{s_{\beta - \alpha}}{c_{\beta - \alpha}} \left( \frac{\delta Z_{hH}}{2} + \delta \alpha - \delta \beta \right) \right] \]
Numerical Results: Used Software

- development of the Python program 2HDMCalc
  - generates the (N)LO decay amplitudes in $R_\xi$ gauge with FeynArts 3.9
  - calculates the amplitudes with FeynCalc 8.2.0
  - generates the NLO counterterm of the decay with Mathematica
  - checks automatically for UV-finiteness and gauge-dependence
  - evaluates the (N)LO decay widths numerically with LoopTools 2.12

- SM parameters are fixed for all parameter points

- 2HDM parameter set generation with ScannerS:
  - variation of $m_{H^0}$, $m_{A^0}$, $m_{H^\pm}$, $\alpha$, $\beta$, $\Lambda_5$

- constraints on the parameter sets: theoretically & experimentally allowed
  (for details: P. M. Ferreira, R. Guedes, M.O.P. Sampaio, and R. Santos: arXiv:1409.6723)

- additional constraints: all decays happen on-shell
  (R. Coimbra, M.O.P. Sampaio, R. Santos: arXiv:1301.2599)
Numerical Results: $H^+ \rightarrow W^+ h^0$

- Numerical check of gauge-dependence for Kanemura’s scheme

$$\Delta \Gamma_{\xi_W} := \frac{\left[ \Gamma_{NLO}^{H+W+h^0} \right]_{\xi_W} - \left[ \Gamma_{NLO}^{H+W+h^0} \right]_{\xi_W=1}}{\left[ \Gamma_{NLO}^{H+W+h^0} \right]_{\xi_W=1}}$$

- Fix $\xi_Z = 1$, vary $\xi_W$

- Kinks due to kinematic thresholds

- Strongly gauge-dependent

- For large $\xi_W$: $\Delta \Gamma_{\xi_W}$ drops as

$$-(m_{H^0} - m_{h^0}) \ln(\xi_W)$$

- Kanemura’s scheme is not suitable for renormalizing the mixing angles

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$m_{H^0} = 742.84$ GeV, $m_{A^0} = 700.13$ GeV, $m_{H^\pm} = 780.00$ GeV, $\alpha = -0.57$, $\tan\beta = 1.46$, $\Lambda_5 = 14.18$; type II 2HDM
Numerical Results: \( H^0 \rightarrow Z^0 Z^0 \)

- check for numerical stability:

\[
\Delta \Gamma_{H^0 Z^0 Z^0} := \frac{\Gamma_{H^0 Z^0 Z^0}^{\text{NLO}} - \Gamma_{H^0 Z^0 Z^0}^{\text{LO}}}{\Gamma_{H^0 Z^0 Z^0}^{\text{LO}}}
\]

- all 70,000 points:

  - Kanemura/pinched schemes: mostly within \( \pm 30\% \)
  - process-dependent scheme: mostly up to \( \pm 250\% \)
  - process-dependent scheme: numerically unstable
  - pinched schemes: numerically stable

(MK, Master’s thesis)
Numerical Results: \[ H \rightarrow h \ h \]

- check for **numerical stability**:
  \[
  \Delta \Gamma_{Hhh} := \frac{\Gamma_{Hhh}^{NLO} - \Gamma_{Hhh}^{LO}}{\Gamma_{Hhh}^{LO}}
  \]

- points in parameter space still allowed by experiment and theory
  - up to few hundreds of percents for **all schemes**
  - process-dependent scheme: slightly **larger deviations**
  - numerical **instability** of all schemes?

(arXiv: 1609.04185 [hep-ph])
Numerical Results: \[ H \longrightarrow h h \]

- large electroweak corrections to **trilinear Higgs couplings** observed before

- scalar masses \( m_\phi^2 \) depend on 2HDM potential parameters \( \lambda_i \)

- decoupling limit:
  - \( m_\phi^2 \) can be large **without** \( \lambda_i \) becoming large
  - loop effects of heavy scalars are **suppressed**
    \( \Rightarrow \) small NLO corrections possible

- strong coupling limit:
  - large masses \( m_\phi^2 \) imply large \( \lambda_i \)
  - NLO corrections are a function of \( m_H^4 \)
    \( \Rightarrow \) NLO vertex corrections can become large

\[ \Rightarrow \text{“MSSM-like”} \]

\[ \Rightarrow \text{“2HDM-exclusive“} \]
Numerical Results: $H \rightarrow h\ h$

- analyze points in the **decoupling limit** only
- exclude points from the **wrong-sign regime** where $\sin(\alpha) < 0$

![Plot of $\Delta \Gamma_{Hhh}$ vs. $\Gamma_{H\rightarrow hh}^{\text{tree}}$ in GeV]

- pinched schemes: mostly within ±30%
- process-dependent scheme: mostly up to ±250%
- Higgs-to-Higgs decay features numerically stable results
- numerical instability: freedom of parameter choice in the 2HDM

(arXiv: 1609.04185 [hep-ph])
Numerical Results: \( H^0 \rightarrow h^0 h^0 \)

- alternative scheme for \( m^2_{12} \): process-dependent via \( H^0 \rightarrow A^0 A^0 \)

- check for numerical stability:

\[
\Delta \Gamma_{H^0 h^0 h^0} := \frac{\Gamma^{NLO}_{H^0 h^0 h^0} - \Gamma^{LO}_{H^0 h^0 h^0}}{\Gamma^{LO}_{H^0 h^0 h^0}}
\]

for most sets: negative

NLO partial decay width \( \Rightarrow \) unsuitable scheme?

- proc.-dep. scheme for mixing angles unsuitable in general

- more parameter sets needed for higher statistics

(\( m_{H^0} = 607.46 \text{ GeV}, m_{A^0} \) varied, \( m_{H^\pm} = 617.94 \text{ GeV}, \)
\( \alpha = -0.78, \tan \beta = 1.15, \Lambda_5 = 1.03; \) type II 2HDM)

29.03.2017 - DPG Münster

M. Krause: On the Renormalization of the Two-Higgs-Doublet Model
Numerical Results: \[ H \rightarrow h \ h \]

- consistency check: choose a parameter set in the decoupling limit

- Kanemura & pinched schemes numerically stable; proc.-dep.: unstable

- the Higgs-to-Higgs decay features numerically stable results as well

- numerical instability: consequence of freedom of parameter choice in the 2HDM

\[ m_{h^0} \ \text{in GeV} \]

\( m_{H^0} \) varied, \( m_{A^0} = 700.13 \) GeV, \( m_{H^\pm} = 700.35 \) GeV, 
\( \alpha = -0.57, \ \tan\beta = 1.46, \ \Lambda_5 = 14.18; \ \text{type II 2HDM} \)
Numerical Results: Used Software

- development of the Python program 2HDMCalc
  - generates the (N)LO decay amplitudes in $R_\xi$ gauge with FeynArts 3.9
  - calculates the amplitudes with FeynCalc 8.2.0
  - generates the NLO counterterm of the decay with Mathematica
  - checks automatically for UV-finiteness and gauge-dependence
  - evaluates the (N)LO decay widths numerically with LoopTools 2.12

- current status: works automatically (i.e. tested) for certain decay topologies

- possible future work:
  - fully automate 2HDMCalc
  - calculate all decays within the 2HDM @NLO
Numerical Results: Input Parameters

- **SM parameters are fixed** for all parameter points.

- 2HDM parameter set generation with ScannerS: variation of $m_{H^0}$, $m_{A^0}$, $m_{H^\pm}$, $\alpha$, $\beta$, $\Lambda_5$.

- Constraints on the parameter sets:
  - VEVs represent the global minimum of the potential.
  - Tree-level unitarity & boundedness from below.
  - Electroweak precision measurements (95% consistency with S, T, U).
  - LEP and recent LHC results (interfaced with SusHi and HDECAY).

- Additional **kinematic constraints**:
  - 70,000 points with $m_{H^\pm} \geq m_{h^0} + m_W$ $\land$ $m_{H^0} \geq 2m_{h^0}$
  - 20,000 points with $m_{H^0} \geq 2m_{h^0}$ $\land$ $m_{H^0} \geq 2m_{A^0}$

(R. Coimbra, M.O.P. Sampaio, R. Santos: arXiv:1301.2599)

(for details: P. M. Ferreira, R. Guedes, M.O.P. Sampaio, and R. Santos: arXiv:1409.6723)