On the Renormalization of the Two-Higgs-Doublet Model

(MK, R. Lorenz, M. Muehlleitner, R. Santos, H. Ziesche: JHEP09(2016)143, arXiv:1605.04853 & MK, M. Muehlleitner, R. Santos, H. Ziesche: to appear in Phys. Rev. D, arXiv:1609.04185)

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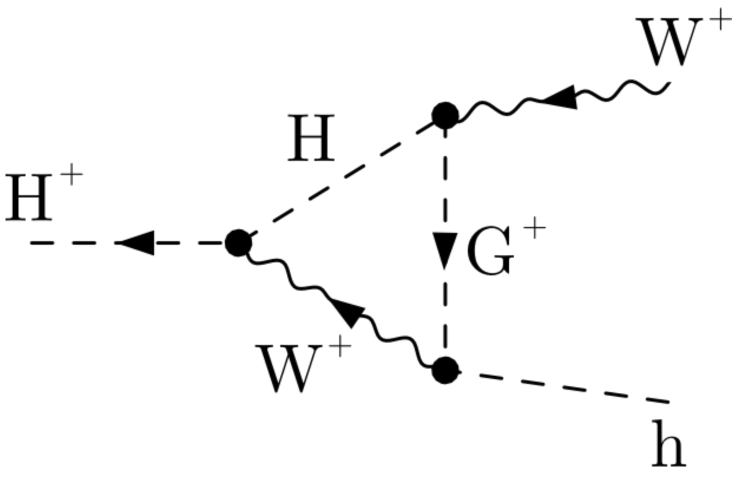
- Motivation
- Description of the Model
- Renormalization of the 2HDM
  - On-Shell Renormalized Fields and Masses
  - The Scalar Mixing Angles
  - The Parameter
- Numerical Results



Motivation

- 2HDM: one of the simplest extensions of the SM
  - dark matter candidate (Inert Doublet Model)
  - source of CP-violation
  - extended scalar sector
  - renormalizable
- renormalization of the two **scalar mixing angles** in the 2HDM is non-trivial
- existing schemes are either numerically unstable, process-dependent or gauge-dependent
- search for a suitable renormalization scheme of the scalar mixing angles
  ➔ full electroweak NLO corrections to all decays within the 2HDM

Description of the Model: Scalar Sector

- **two** complex $SU(2)_L$ Higgs **doublets**

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix} \,, \qquad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- non-vanishing **vacuum expectation values** (VEVs) v_1, v_2 with

$$v^2 := v_1^2 + v_2^2 \approx (246 \text{ GeV})^2$$

- scalar Lagrangian with **CP- and $\mathbb{Z}_2$-conserving** 2HDM potential:

$$\begin{aligned} V_{\text{2HDM}}(\Phi_1, \Phi_2) = {} & m_{11}^2 \left(\Phi_1^\dagger \Phi_1\right) + m_{22}^2 \left(\Phi_2^\dagger \Phi_2\right) - m_{12}^2 \left[\left(\Phi_1^\dagger \Phi_2\right) + \left(\Phi_2^\dagger \Phi_1\right)\right] \\ & + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_2^\dagger \Phi_2\right) \\ & + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right] \end{aligned}$$

Description of the Model: Parameters

- **eight** real-valued potential parameters:
  - dimensionless λ_i $(i = 1, ..., 5)$
  - mass-squared parameters m_{11}^2, m_{22}^2 and m_{12}^2
- transformation to the Higgs mass basis via **scalar mixing angles**
  - α for the CP-even sector
  - β for the CP-odd and charged sectors

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- set of **free parameters** of the 2HDM (excluding CKM elements, ...)

$$\left\{ m_{h^0},\; m_{H^0},\; m_{A^0},\; m_{H^\pm},\; \alpha,\; \beta,\; m_{12}^2,\; T_{h^0},\; T_{H^0},\; e,\; m_W,\; m_Z,\; m_\Psi \right\}$$

  - "physical" tadpole terms T_{H^0}, T_{h^0}
  - elementary charge e
  - fermion masses m_Ψ
  - gauge boson masses m_W, m_Z
  - soft- $\mathbb{Z}_2$ -breaking scale m_{12}^2

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  - "physical" tadpole terms T_{H^0}, T_{h^0} ➡ **alternative tadpole scheme**
  - elementary charge e ➡ **Thomson limit**
  - fermion masses m_Ψ ➡ **on-shell**
  - gauge boson masses m_W, m_Z ➡ **on-shell**
  - soft-$\mathbb{Z}_2$-breaking scale m_{12}^2 ➡ $\overline{\mathrm{MS}}$

Renormalization: Alternative Tadpole Scheme

- alternative scheme: VEVs represent the same minimum as at **tree level**
  (based on J. Fleischer, F. Jegerlehner: Phys. Rev. D 23 (1981) 2001-2026)

- correct minimum conditions at NLO require a **shift in the VEVs**

- shifts are connected **to tadpole renormalization**
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- shifts are connected **to tadpole renormalization**
  ➔ tadpole diagrams appear
- the shifts translate into **all CTs**, **wave function renormalization constants** and **Feynman rules for vertices**
- bare masses are expressed through gauge-independent **tree-level VEVs**
  ➔ mass CTs **become gauge-independent**
- alternative tadpole scheme **worked out for the 2HDM** ✔
  (MK: Master's thesis, Karlsruhe Institute of Technology, 2016)

Renormalization: Scalar Mixing Angles

- renormalization of mixing angles α and β is **non-trivial** in the 2HDM
- simplest approach: $\overline{\mathrm{MS}}$ conditions for α and β
  ➔ **numerically unstable** (R. Lorenz: Master's thesis, Karlsruhe Institute of Technology, 2015)
  ➔ **unsuitable scheme** in many cases

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- "**no-go theorem**" for the MSSM: a renormalization scheme for $\tan\beta$ **cannot be simultaneously** (A. Freitas, D. Stoeckinger: PhysRevD.66.095014, arXiv:hep-ph/0205281)
  - gauge-independent
  - process-independent
  - numerically stable

- is there a renormalization scheme for the 2HDM **satisfying all three criteria** above?

Renormalization: Scalar Mixing Angles

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- S. Kanemura *et al.* (KOSY scheme): connect definition of $\delta\alpha$ and $\delta\beta$ with **inverse propagator** of the scalar doublets (S. Kanemura *et al.*: PhysRevD.70.115002, arXiv:hep-ph/0408364)
  - intricate **gauge-dependence** in $\delta\alpha$ and $\delta\beta$ (MK: Master's thesis, KIT, 2016)
  - gauge-dependence **cannot** be removed **unambiguously**
  - for gauge-independent decay widths: **gauge-dep. parameter** treatment ⚡

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  - for gauge-independent decay widths: **gauge-dep. parameter** treatment
- process-dependent definition: $\Gamma^{\mathrm{LO}} \overset{!}{=} \Gamma^{\mathrm{NLO,weak}}$ for $H/A^0 \longrightarrow \tau^+ \tau^-$
  - gauge-independent S-matrix elements
  - $\delta\alpha$ and $\delta\beta$ are **flavor-** and **process-dependent**

(inspired by A. Freitas, D. Stoeckinger: PhysRevD.66.095014, arXiv:hep-ph/0205281)

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  - gauge-independent S-matrix elements
  - $\delta\alpha$ and $\delta\beta$ are **flavor-** and **process-dependent**
- gauge-independent scheme: use the **pinch technique** (PT) (D. Binosi, J. Papavassiliou: j.physrep.2009.05.001, arXiv:0909.2536)
  - consistency **requires** alternative tadpole scheme
  - PT was worked out for the 2HDM (MK: Master's thesis, KIT, 2016)
  - $\delta\alpha$ and $\delta\beta$ are **process-indep.**, **gauge-indep.** and **unambiguously** defined

(MK, R. Lorenz, M. Muehlleitner, R. Santos, H. Ziesche: "Gauge-independent Renormalization of the 2-Higgs-Doublet Model", JHEP09(2016)143, arXiv:1605.04853)

Numerical Results: $H^+ \longrightarrow W^+ h$

- check for **numerical stability**:

$$\Delta\Gamma_{H^+W^+h} := \frac{\Gamma^{\text{NLO}}_{H^+W^+h} - \Gamma^{\text{LO}}_{H^+W^+h}}{\Gamma^{\text{LO}}_{H^+W^+h}}$$

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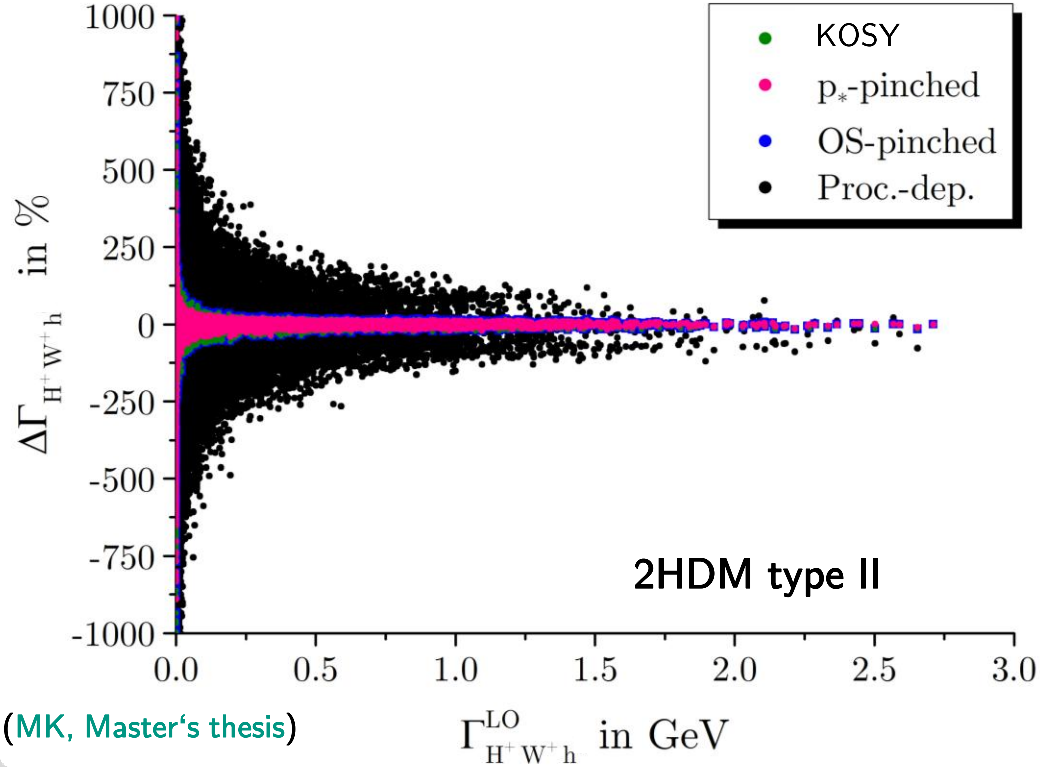
- points in parameter space still allowed by experiment and theory



(MK, Master's thesis)

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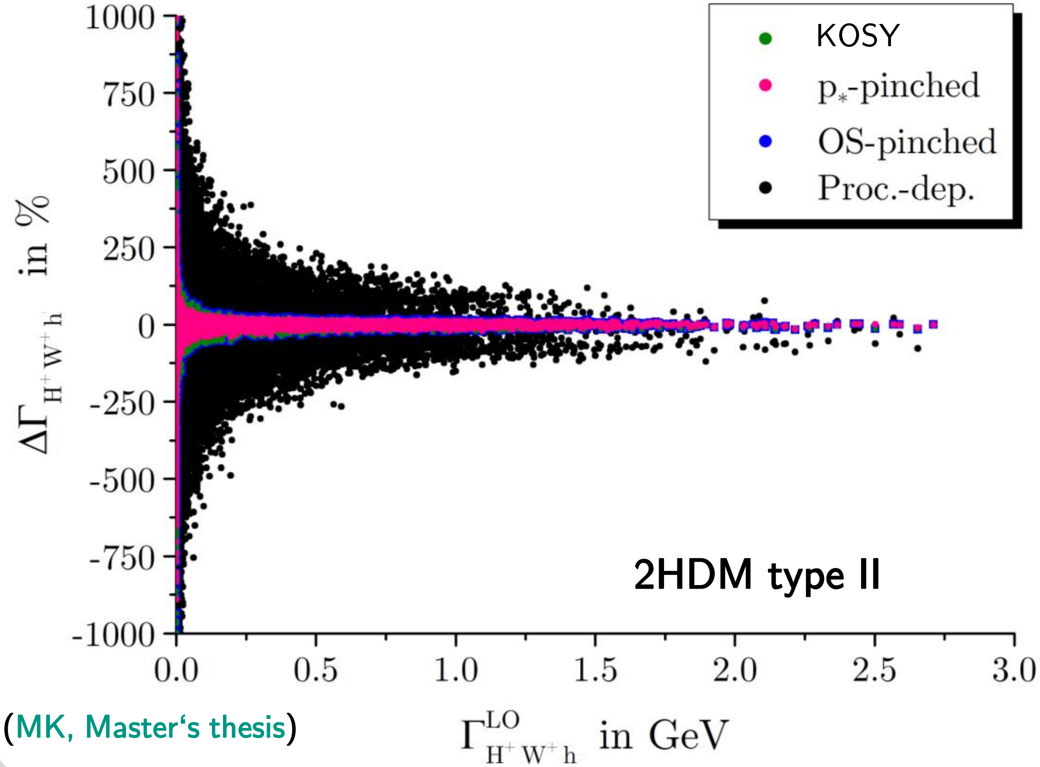

(MK, Master‘s thesis)

- KOSY / pinched schemes: mostly within $\pm$ **30%**
- process-dependent scheme: mostly up to $\pm$ **250%**

- **process-dependent** scheme: numerically **unstable** for type II
- **pinched** schemes: numerically **stable**

Conclusions

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- the **alternative tadpole** scheme was worked out for the 2HDM
- the KOSY scheme can lead to **gauge-dependent** NLO decay widths
- the pinched scheme for $\delta\alpha$ and $\delta\beta$ is gauge-independent, process-independent and numerically stable

  ➔ **no "no-go" theorem for the 2HDM**

- for details: (MK, R. Lorenz, M. Muehlleitner, R. Santos, H. Ziesche: "Gauge-independent Renormalization of the 2-Higgs-Doublet Model", JHEP09(2016)143, arXiv:1605.04853)

  (MK, M. Muehlleitner, R. Santos, H. Ziesche: "2HDM Higgs-to-Higgs Decays at Next-to-Leading Order", arXiv: 1609.04185)

- future work:
  - provide 2HDMCalc, a new tool for calculating all 2HDM decay processes at NLO for various renormalization schemes (for comparison)
  - analyze 2HDM types I, X and Y

scheduled: Fall 2017

Backup slides

Description of the Model: Full Lagrangian

- consider the **electroweak** Lagrangian of the 2HDM:

$$\mathcal{L}^{\text{EW}}_{\text{2HDM}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{S}} + \mathcal{L}_{\text{Yuk}}$$

  - $\mathcal{L}_{\text{YM}}$: Yang-Mills Lagrangian
  - $\mathcal{L}_{\text{F}}$: Fermion Lagrangian (kinetic terms and interactions with gauge bosons)
  - $\mathcal{L}_{\text{GF}}$: Gauge-fixing Lagrangian
  - $\mathcal{L}_{\text{FP}}$: Faddeev-Popov Lagrangian

  "SM-like"

  - $\mathcal{L}_{\text{S}}$: Scalar Lagrangian (kinetic terms and scalar potential)
  - $\mathcal{L}_{\text{Yuk}}$: Yukawa Lagrangian (scalar-fermion interactions)

Description of the Model: Scalar Sector

- expand the doublets around their VEVs:

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{v_1+\rho_1+i\eta_1}{\sqrt{2}} \end{pmatrix} \,, \qquad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{v_2+\rho_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$$

- eight real fields: two CP-even ρ_i, two CP-odd η_i, four charged $\omega_i^\pm$
- minimum of the potential $\triangleq$ vanishing of the **tadpole parameters** $T_1, \ T_2$ (terms linear in ρ_i) at tree level:

$$T_1 := m_{11}^2 v_1 - m_{12}^2 v_2 + \frac{1}{2}\lambda_1 v_1^3 + \frac{1}{2}\lambda_{345} v_1 v_2^2 \stackrel{!}{=} 0$$
$$T_2 := m_{22}^2 v_2 - m_{12}^2 v_1 + \frac{1}{2}\lambda_2 v_2^3 + \frac{1}{2}\lambda_{345} v_1^2 v_2 \stackrel{!}{=} 0$$

Description of the Model: Scalar Sector

- **two** complex $\mathrm{SU}(2)_\mathrm{L}$ Higgs **doublets** $\Phi_i \quad (i=1,2)$
- scalar Lagrangian with 2HDM potential V_{2HDM} :

$$\mathcal{L}_\mathrm{S} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V_{\mathrm{2HDM}}(\Phi_1, \Phi_2)$$

- covariant derivative:

$$D_\mu = \partial_\mu + \underbrace{ig\frac{\sigma^a}{2}W_\mu^a(x)}_{SU(2)_L} + \underbrace{ig'\frac{Y}{2}B_\mu(x)}_{U(1)_Y} \qquad (\text{sum over } a=1,2,3)$$

SU(2)$_L$ x *U(1)$_Y$* coupling constants, generators, gauge fields

- non-vanishing **vacuum expectation values** (VEVs) $v_1,\ v_2$
- **spontaneous symmetry breaking**: mass generation, e.g.

$$m_W^2 = g^2\frac{v^2}{4} \quad , \qquad v^2 := v_1^2 + v_2^2 \approx (246\ \mathrm{GeV})^2$$

Description of the Model: Fermion Sector

- coupling between fermions and scalars: Yukawa Lagrangian
- in contrast to the SM: Yukawa matrices in mass terms and couplings are **not proportional** to each other

  ➔ FCNC **possible** on tree level
- impose an **additional** $\mathbb{Z}_2$ **symmetry**: $\mathcal{L}^{\text{EW}}_{\text{2HDM}}$ invariant under $\Phi_1 \to -\Phi_1$

  ➔ **no FCNC on tree level**
- exact realization of the symmetry determines the 2HDM type:
  - **type I**: RH quarks/leptons couple only to doublet Φ_2
  - **type II**: RH up-type quarks couple to Φ_2, RH down-type quarks/leptons to Φ_1
  - ...

Description of the Model: Scalar Sector

- terms **bilinear** in the field doublets:

$$V_{\text{2HDM}}\Big|_{\text{bilin}} = \frac{1}{2}\begin{pmatrix}\rho_1 & \rho_2\end{pmatrix} M_\rho^2 \begin{pmatrix}\rho_1 \\ \rho_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\eta_1 & \eta_2\end{pmatrix} M_\eta^2 \begin{pmatrix}\eta_1 \\ \eta_2\end{pmatrix} + \frac{1}{2}\begin{pmatrix}\omega_1^+ & \omega_2^+\end{pmatrix} M_\omega^2 \begin{pmatrix}\omega_1^- \\ \omega_2^-\end{pmatrix}$$

- **diagonalization** with **two mixing angles**

$$R_\alpha = \begin{pmatrix} \mathrm{c}_\alpha & -\mathrm{s}_\alpha \\ \mathrm{s}_\alpha & \mathrm{c}_\alpha \end{pmatrix} , \qquad R_\beta = \begin{pmatrix} \mathrm{c}_\beta & -\mathrm{s}_\beta \\ \mathrm{s}_\beta & \mathrm{c}_\beta \end{pmatrix} \qquad (\mathrm{c}_x := \cos(x),\ \mathrm{s}_x := \sin(x))$$

- α : CP-even sector; β : CP-odd **and** charged sectors

- 2HDM potential is rotated to the **mass basis**:
  - CP-even Higgses (H^0, h^0) with masses (m_{H^0}, m_{h^0})
  - CP-odd Higgs and Goldstone (A^0, G^0) with masses $(m_{A^0}, 0)$
  - charged Higgses and Goldstones $(H^\pm, G^\pm)$ with masses $(m_{H^\pm}, 0)$
  - soft-$\mathbb{Z}_2$-breaking parameter Λ_5 (or equivalently, mass scale M):

$$\Lambda_5 := \frac{2m_{12}^2}{v^2 \mathrm{s}_\beta \mathrm{c}_\beta} \equiv \frac{2M^2}{v^2}$$

Renormalization: 2HDM Parameters

- renormalization program for the 2HDM:
  - tadpole terms ➔ **standard** / alternative **tadpole scheme**
  - mass counterterms ➔ **on-shell**
  - field strength renormalization constants ➔ **on-shell**
  - elementary charge ➔ **Thomson limit** (without light fermion contributions)
  - **parameter** m_{12} ➔ $\overline{\mathrm{MS}}$
  - **scalar mixing angles** ➔ **?**

Renormalization: On-Shell Conditions

- consider **scalar field doublet** $(\phi_1,\ \phi_2)$
- field strength renormalization:

$$\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}_0 = \sqrt{Z_\phi}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \approx \left(1_{2\times 2} + \frac{\delta Z_\phi}{2}\right)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \quad , \quad \frac{\delta Z_\phi}{2} = \begin{pmatrix}\frac{\delta Z_{\phi_1\phi_1}}{2} & \frac{\delta Z_{\phi_1\phi_2}}{2}\\ \frac{\delta Z_{\phi_2\phi_1}}{2} & \frac{\delta Z_{\phi_2\phi_2}}{2}\end{pmatrix}$$

- two-point correlation function for the doublet with momentum p^2:

$$\widehat{\Gamma}_\phi(p^2) := \begin{pmatrix}\widehat{\Gamma}_{\phi_1\phi_1}(p^2) & \widehat{\Gamma}_{\phi_1\phi_2}(p^2)\\ \widehat{\Gamma}_{\phi_1\phi_2}(p^2) & \widehat{\Gamma}_{\phi_2\phi_2}(p^2)\end{pmatrix}$$

$$= i\sqrt{Z_\phi}^{\dagger}\left[p^2 1_{2\times 2} - D_\phi^2 + \Sigma_\phi(p^2) - \delta D_\phi^2\right]\sqrt{Z_\phi} \approx i\left[p^2 1_{2\times 2} - D_\phi^2 + \widehat{\Sigma}_\phi(p^2)\right]$$

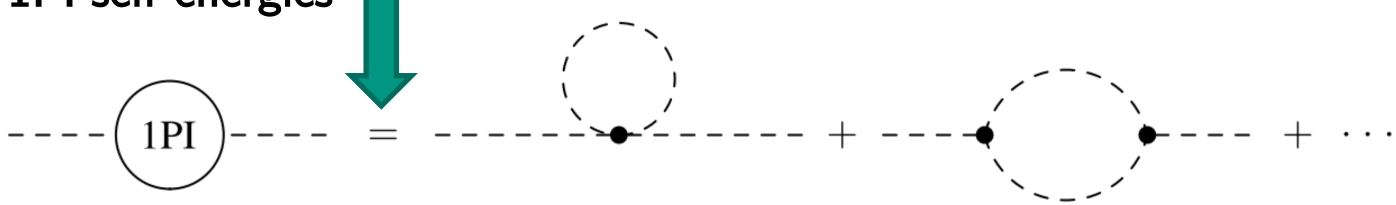
mass matrices

mass CTs

renormalized self-energies

1PI self-energies

$i\Sigma(p^2) :=$



Renormalization: On-Shell Conditions

- **on-shell conditions**:
  - mixing of fields vanishes for $p^2 = m^2_{\phi_i}$
  - masses $m^2_{\phi_i}$ are the real parts of the pole of the propagator
  - normalization: residue of the propagator at its pole equals i
- fixation of **diagonal** mass counterterms:

$$\text{Re}\Big[\delta D^2_{\phi_1\phi_1}\Big] = \text{Re}\Big[\Sigma_{\phi_1\phi_1}(m^2_{\phi_1})\Big] \quad , \quad \text{Re}\Big[\delta D^2_{\phi_2\phi_2}\Big] = \text{Re}\Big[\Sigma_{\phi_2\phi_2}(m^2_{\phi_2})\Big]$$

- fixation of field strength renormalization constants:

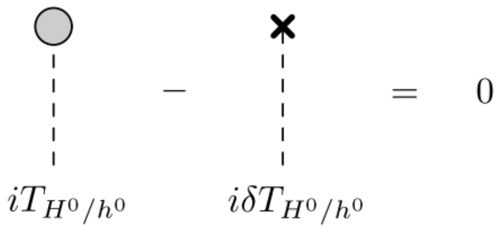
$$\delta Z_{\phi_1\phi_1} = -\text{Re}\left[\frac{\partial\Sigma_{\phi_1\phi_1}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_1}} \quad , \quad \delta Z_{\phi_2\phi_2} = -\text{Re}\left[\frac{\partial\Sigma_{\phi_2\phi_2}(p^2)}{\partial p^2}\right]_{p^2=m^2_{\phi_2}}$$

$$\delta Z_{\phi_1\phi_2} = \frac{2}{m^2_{\phi_1}-m^2_{\phi_2}}\text{Re}\Big[\Sigma_{\phi_1\phi_2}(m^2_{\phi_2}) - \delta D^2_{\phi_1\phi_2}\Big] \quad , \quad \delta Z_{\phi_2\phi_1} = \frac{2}{m^2_{\phi_2}-m^2_{\phi_1}}\text{Re}\Big[\Sigma_{\phi_1\phi_2}(m^2_{\phi_1}) - \delta D^2_{\phi_1\phi_2}\Big]$$

- the **specific form** of the $\delta D^2_{\phi_i\phi_j}$ **depends on the tadpole scheme**

Renormalization: General Tadpole Conditions

- renormalization conditions for the tadpole terms:



- **purpose**: restore the minimum conditions of the potential at NLO
- **practical effect**: **no tadpole diagrams** in NLO calculations

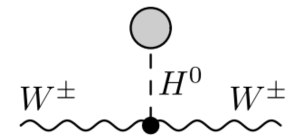
Renormalization: Standard Tadpole Scheme

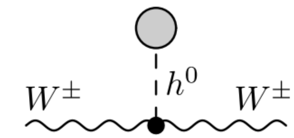
- standard scheme: VEVs are derived from the **loop-corrected potential** (e.g. in A. Denner: arXiv:0709.1075)
- VEVs in the mass relations produce correct one-loop OS masses
- tadpole terms appear explicitly in the bare mass matrices
  ➔ mass matrix counterterms contain the **tadpole counterterms**
- one-loop corrected potential is gauge-dependent
  ➔ **VEVs** are gauge-dependent
  ➔ **mass counterterms** become gauge-dependent

Renormalization: Alternative Tadpole Scheme

- example: **W boson mass**

$$m_W^2 = g^2 \frac{v^2}{4} \quad \longrightarrow \quad m_W^2 + g^2 \frac{v_1 \delta v_1 + v_2 \delta v_2}{2} \quad = \quad m_W^2 + i \left(\vphantom{\int} \cdots \right) + i \left(\vphantom{\int} \cdots \right)$$

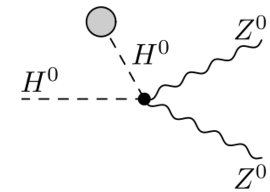




- example: **coupling between Higgs and Z bosons**

$$i g_{H^0 Z^0 Z^0} = \frac{i g^2}{2 c_W^2} \left(c_\alpha v_1 + s_\alpha v_2 \right) \quad , \qquad i g_{H^0 H^0 Z^0 Z^0} = \frac{i g^2}{2 c_W^2}$$

$$i g_{H^0 Z^0 Z^0} \;\to i g_{H^0 Z^0 Z^0} + \frac{i g^2}{2 c_W^2} \left(c_\alpha \delta v_1 + s_\alpha \delta v_2 \right) = i g_{H^0 Z^0 Z^0} + \left(\vphantom{\int} \cdots \right)_{\text{trunc}}$$



- **effects** of the alternative tadpole scheme:
  - **tadpole diagrams are added everywhere** where they exist in the 2HDM
  - mass counterterms become **manifestly gauge-independent**
  - tadpole counterterms in the scalar sector are **removed**

Renormalization: Scalar Mixing Angles

- approach by S. Kanemura *et al.*: connect the definition of α and β with the **inverse propagator matrix** (S. Kanemura *et al.*: arXiv:hep-ph/0408364)

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_0 = R^T_{\theta,0} \begin{pmatrix} \widetilde{\phi}_1 \\ \widetilde{\phi}_2 \end{pmatrix}_0 \approx R^T_{\delta\theta} R^T_\theta \sqrt{Z_{\widetilde{\phi}}} R_\theta R^T_\theta \begin{pmatrix} \widetilde{\phi}_1 \\ \widetilde{\phi}_2 \end{pmatrix} \approx \begin{pmatrix} 1 + \frac{\delta Z_{\phi_1\phi_1}}{2} & \delta C_{\phi_2} + \delta\theta \\ \delta C_{\phi_2} - \delta\theta & 1 + \frac{\delta Z_{\phi_2\phi_2}}{2} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

- mixing angle counterterms **within the standard tadpole scheme**:

$$\delta\alpha = \frac{1}{2\left(m^2_{H^0} - m^2_{h^0}\right)} \mathrm{Re}\left[\Sigma_{H^0 h^0}(m^2_{H^0}) + \Sigma_{H^0 h^0}(m^2_{h^0}) - 2\delta T_{H^0 h^0}\right]$$

$$\delta\beta = -\frac{1}{2m^2_{H^\pm}} \mathrm{Re}\left[\Sigma_{G^\pm H^\pm}(m^2_{H^\pm}) + \Sigma_{G^\pm H^\pm}(0) - 2\delta T_{G^\pm H^\pm}\right]$$

(**for details:** R. Lorenz: Master's thesis, Karlsruhe Institute of Technology, 2015)

- it was shown analytically that **Kanemura's scheme** introduces an **intricate gauge-dependence** in $\delta\alpha$ and $\delta\beta$

(MK, Master's thesis, Karlsruhe Institute of Technology, 2016)

Renormalization: Scalar Mixing Angles

- gauge-independent approach: use the **pinch technique (PT)**

(D. Binosi, J. Papavassiliou: arXiv:0909.2536 [hep-ph])

- main ideas:
  - use a toy scattering process that includes **to-be pinched self-energies**
  - **unambiguously** isolate all gauge-dependences by triggering Ward identities
  - rearrange all gauge-dependences in the S-matrix elements

  ➔ creation of **manifestly gauge-independent self-energies**

- **consistent** application of the PT **requires** the alternative tadpole scheme

- the PT was worked out…
  - to all orders in the SM (D. Binosi: arXiv:hep-ph/0401182)
  - for the MSSM (J. R. Espinosa, Y. Yamada: arXiv:hep-ph/0207351)
  - for the singlet extension of the SM
  - for the scalar sector of the 2HDM (MK, Master's thesis, KIT, 2016)

Renormalization: Scalar Mixing Angles

Karlsruhe Institute of Technology

- PT-based definition of the scalar mixing angle counterterms:

$$\delta\alpha = \frac{\mathrm{Re}\Big[\left[\Sigma^{\mathrm{tad}}_{H^0h^0}(m^2_{H^0}) + \Sigma^{\mathrm{tad}}_{H^0h^0}(m^2_{h^0})\right]_{\xi=1} + \Sigma^{\mathrm{add}}_{H^0h^0}(m^2_{H^0}) + \Sigma^{\mathrm{add}}_{H^0h^0}(m^2_{h^0})\Big]}{2\left(m^2_{H^0} - m^2_{h^0}\right)}$$

$$\delta\beta = -\frac{\mathrm{Re}\Big[\left[\Sigma^{\mathrm{tad}}_{G^\pm H^\pm}(m^2_{H^\pm}) + \Sigma^{\mathrm{tad}}_{G^\pm H^\pm}(0)\right]_{\xi=1} + \Sigma^{\mathrm{add}}_{G^\pm H^\pm}(m^2_{H^\pm}) + \Sigma^{\mathrm{add}}_{G^\pm H^\pm}(0)\Big]}{2m^2_{H^\pm}}$$

"OS-pinched"

$$\delta\alpha = \frac{1}{m^2_{H^0} - m^2_{h^0}}\mathrm{Re}\left[\Sigma^{\mathrm{tad}}_{H^0h^0}\left(\frac{m^2_{H^0} + m^2_{h^0}}{2}\right)\right]_{\xi=1}$$

$$\delta\beta = -\frac{1}{m^2_{A^0}}\mathrm{Re}\left[\Sigma^{\mathrm{tad}}_{G^0A^0}\left(\frac{m^2_{A^0}}{2}\right)\right]_{\xi=1}$$

"p*-pinched"

- properties of the pinched scheme:
  - **process-independent** ✔
  - manifestly **gauge-independent** by construction ✔
  - unambiguous definition
  - numerically stable ?

Renormalization: Scalar Mixing Angles

- another approach: **process-dependent definition**
- use the decays $H^0/A^0 \longrightarrow \tau^+ \tau^-$ for defining $\delta\alpha$ and $\delta\beta$
- **renormalization condition** for the extraction of the angle counterterms:

$$\Gamma^{\mathrm{LO}} \stackrel{!}{=} \Gamma^{\mathrm{NLO,weak}}$$

(A. Freitas, D. Stöckinger: arXiv:hep-ph/0205281)

- exclusion of QED-type diagrams due to **IR divergences**
- properties:
  - the process-dependent $\delta\alpha$ and $\delta\beta$ lead to **gauge-independent S-matrix** elements **independently** of the tadpole scheme
  - the angle counterterms are **process-** and **flavor-dependent**

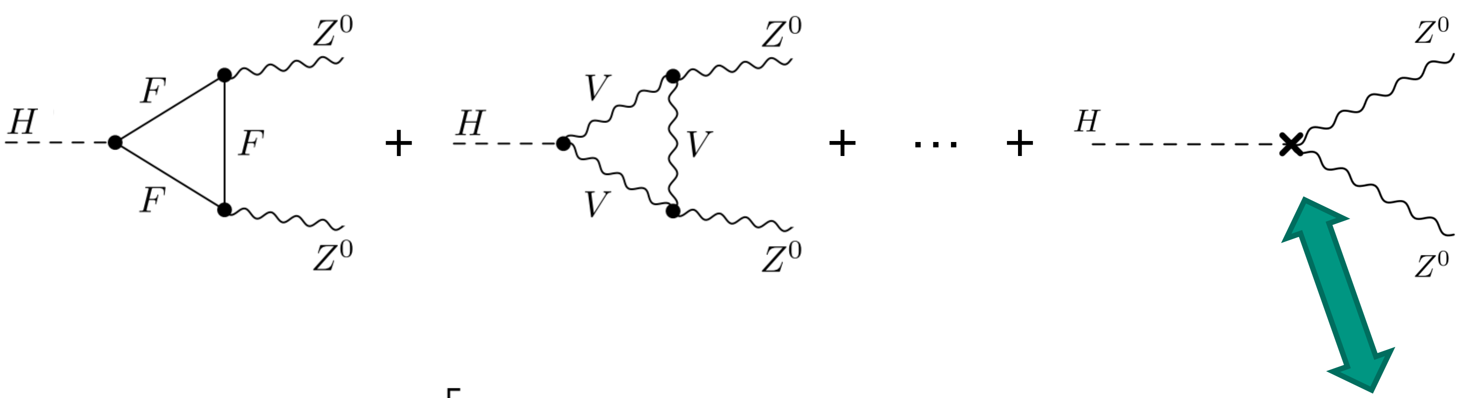
Renormalization: The Parameter m_{12}

- no obvious "on-shell" condition applicable for potential parameter m_{12}
- implementation of **two schemes**:
  - $\overline{\mathrm{MS}}$ condition for δm_{12}^2
  - process-dependent over $\Gamma^{\mathrm{LO}} \stackrel{!}{=} \Gamma^{\mathrm{NLO}}$ for the process $H^0 \longrightarrow A^0 A^0$
- disadvantage of process-dependent scheme: **kinematic constraint**
- both schemes are **gauge-independent**

➔ $\overline{\mathrm{MS}}$ scheme is the preferable scheme if **numerically stable**

Numerical Results: Considered Processes

- choose 2HDM-specific Higgs decays for analysis on **numerical stability**
- considered processes:
  - $H^+ \longrightarrow W^+ \, h/H$
  - $H \longrightarrow Z^0 \, Z^0$
  - $H \longrightarrow h \, h$
- **full** electroweak **one-loop** corrections considered



$$\mathcal{A}^{\mathrm{CT}}_{HZ^0Z^0} = \frac{g\,\mathrm{c}_{\beta-\alpha} m_W}{\mathrm{c}_W^2} \left(\varepsilon_2^* \cdot \varepsilon_3^*\right) \left[\frac{\delta g}{g} + \frac{\delta m_\mathrm{Z}^2}{m_\mathrm{Z}^2} - \frac{\delta m_\mathrm{W}^2}{2m_\mathrm{W}^2} + \frac{\delta Z_{HH}}{2} + \delta Z_{ZZ} + \frac{\mathrm{s}_{\beta-\alpha}}{\mathrm{c}_{\beta-\alpha}} \left(\frac{\delta Z_{hH}}{2} + \delta\alpha - \delta\beta \right) \right]$$

Numerical Results: Used Software

- development of the Python program 2HDMCalc
  - generates the (N)LO decay amplitudes **in R_ξ gauge** with FeynArts 3.9
  - calculates the amplitudes with FeynCalc 8.2.0
  - generates the NLO counterterm of the decay with Mathematica
  - checks automatically for **UV-finiteness** and **gauge-dependence**
  - evalutes the (N)LO decay widths numerically with LoopTools 2.12
- SM parameters are **fixed** for all parameter points
- 2HDM parameter set generation with ScannerS: (R. Coimbra, M.O.P. Sampaio, R. Santos: arXiv:1301.2599)
  variation of m_{H^0}, m_{A^0}, $m_{H^\pm}$, α, β, Λ_5
- constraints on the parameter sets: theoretically & experimentally **allowed**
  (**for details:** P. M. Ferreira, R. Guedes, M.O.P. Sampaio, and R. Santos: arXiv:1409.6723)
- additional constraints: all decays **happen on-shell**

Numerical Results: $H^+ \longrightarrow W^+ h^0$

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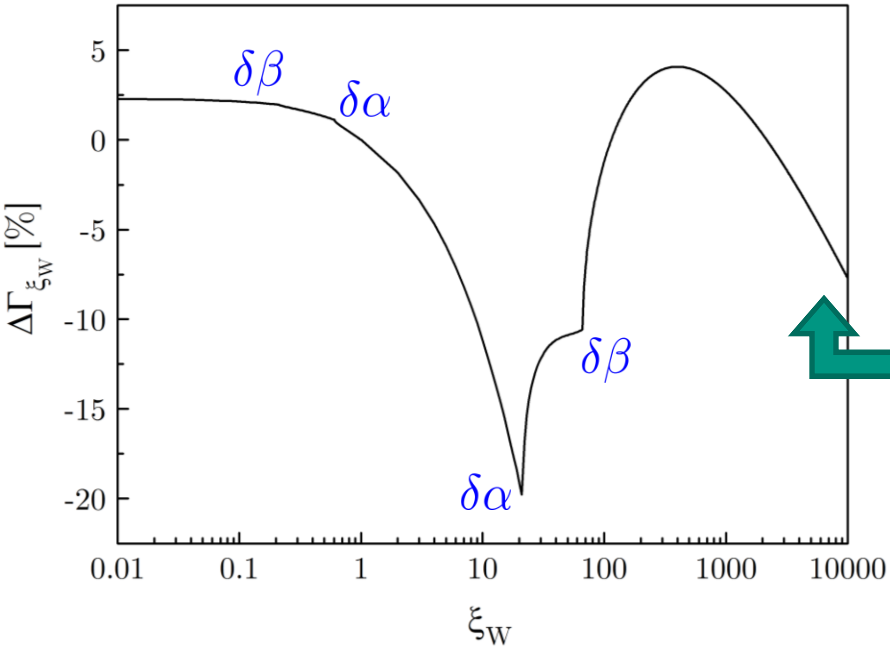
- numerical check of gauge-dependence for **Kanemura's scheme**

$$\Delta\Gamma_{\xi_W} := \frac{\left[\Gamma^{\text{NLO}}_{H^+W^+h^0}\right]_{\xi_W} - \left[\Gamma^{\text{NLO}}_{H^+W^+h^0}\right]_{\xi_W=1}}{\left[\Gamma^{\text{NLO}}_{H^+W^+h^0}\right]_{\xi_W=1}}$$

- fix $\xi_Z = 1$, vary ξ_W



$(m_{H^0} = 742.84$ GeV, $m_{A^0} = 700.13$ GeV, $m_{H^\pm} = 780.00$ GeV, $\alpha = -0.57$, $\tan\beta = 1.46$, $\Lambda_5 = 14.18$; type II 2HDM)

- kinks due to kinematic thresholds
- **strongly gauge-dependent**
- for large ξ_W: $\Delta\Gamma_{\xi_W}$ drops as

$$-(m_{H^0} - m_{h^0})\ln(\xi_W)$$

- Kanemura's scheme is not suitable for renormalizing the mixing angles

Numerical Results: $H^0 \longrightarrow Z^0\ Z^0$

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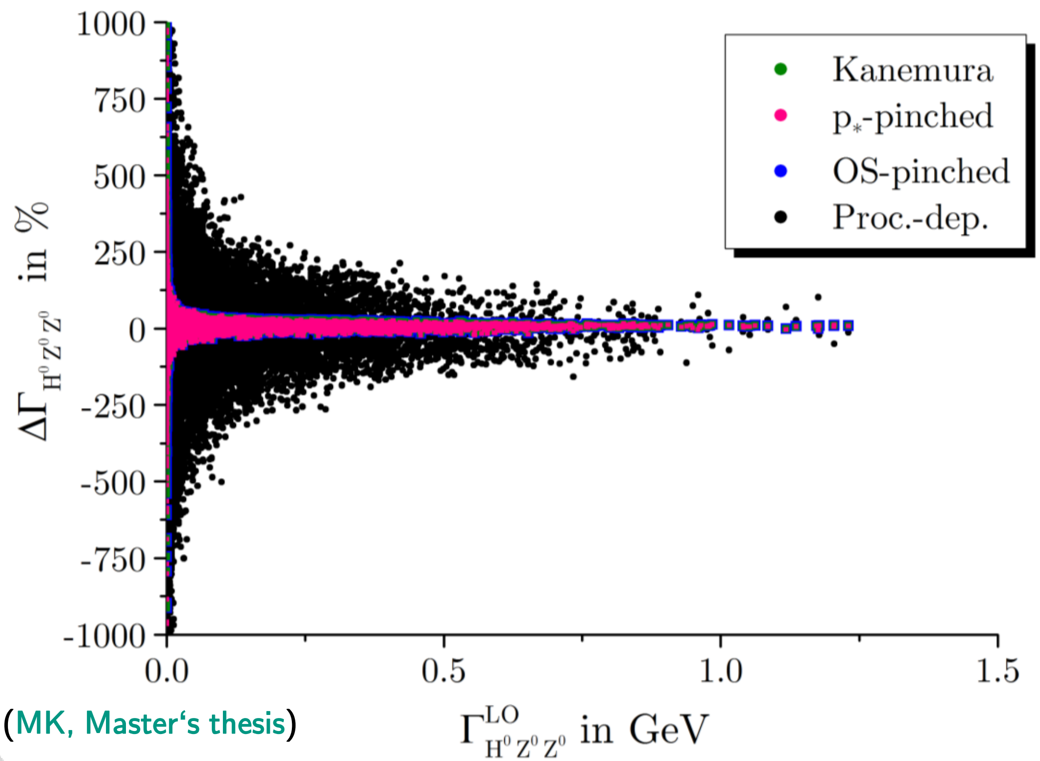
- check for **numerical stability**:

$$\Delta\Gamma_{H^0Z^0Z^0} := \frac{\Gamma^{\text{NLO}}_{H^0Z^0Z^0} - \Gamma^{\text{LO}}_{H^0Z^0Z^0}}{\Gamma^{\text{LO}}_{H^0Z^0Z^0}}$$

- all 70.000 points:



(MK, Master‘s thesis)

- Kanemura/pinched schemes: mostly within $\pm$ **30%**
- process-dependent scheme: mostly up to $\pm$ **250%**

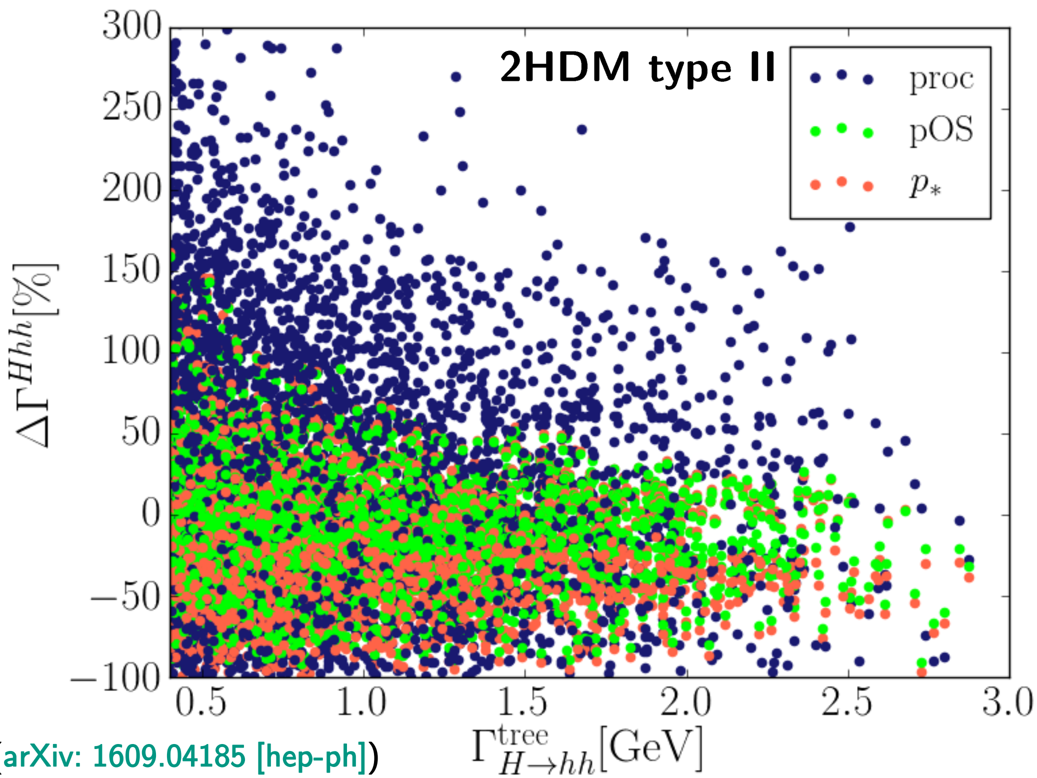
- **process-dependent** scheme: numerically **unstable**
- **pinched** schemes: numerically **stable**

Numerical Results: $H \longrightarrow h\, h$

- check for **numerical stability**:

$$\Delta\Gamma_{Hhh} := \frac{\Gamma^{\rm NLO}_{Hhh} - \Gamma^{\rm LO}_{Hhh}}{\Gamma^{\rm LO}_{Hhh}}$$

- points in parameter space still allowed by experiment and theory



(arXiv: 1609.04185 [hep-ph])

- up to few hundreds of percents for **all schemes**
- process-dependent scheme: slightly **larger deviations**
- numerical **instability** of all schemes?

Numerical Results: $H \longrightarrow h\, h$

- large electroweak corrections to **trilinear Higgs couplings** observed before (S. Kanemura, Y. Okada, E. Senaha, and C. P. Yuan: arXiv:hep-ph/0408364)
- scalar masses m_ϕ^2 depend on 2HDM potential parameters λ_i
- decoupling limit:
  - m_ϕ^2 can be large **without** λ_i becoming large
  - loop effects of heavy scalars are **suppressed**

  ➔ small NLO corrections possible

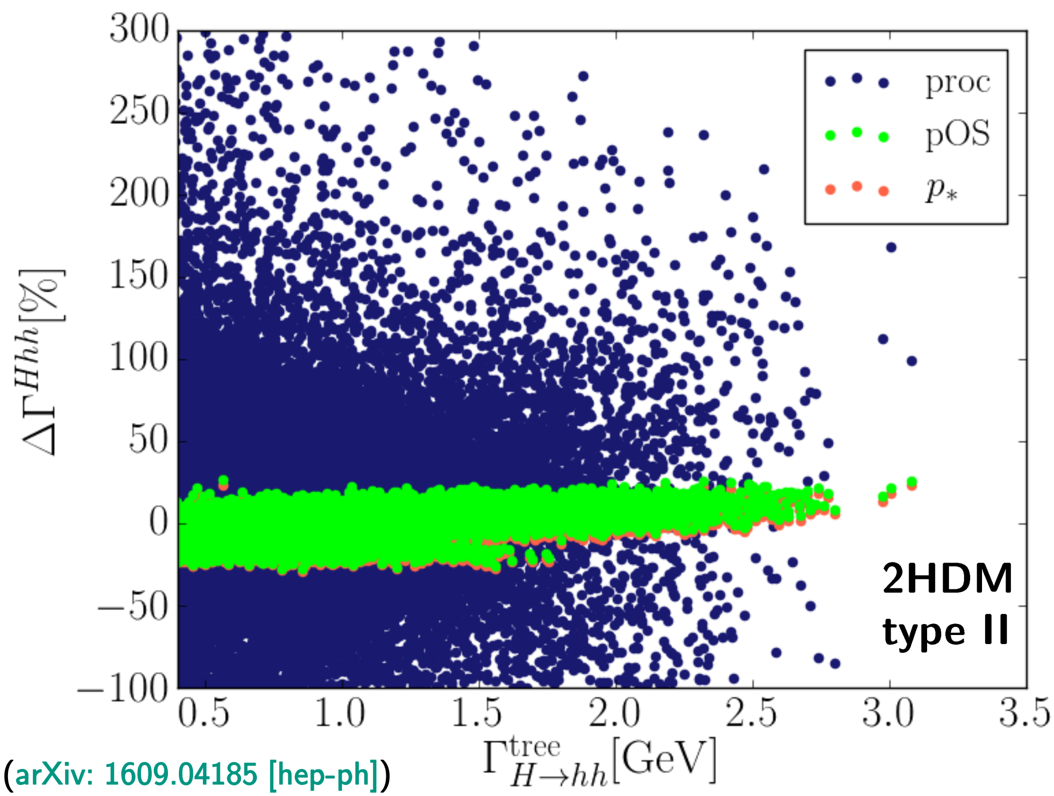
  ➔ "MSSM-like"
- strong coupling limit:
  - large masses m_ϕ^2 imply large λ_i
  - NLO corrections are a function of m_H^4

  ➔ NLO vertex corrections can become large

  ➔ "2HDM-exclusive"

Numerical Results: $H \longrightarrow h\,h$

- analyze points in the **decoupling limit only**
- exclude points from the **wrong-sign regime** where $\sin(\alpha) < 0$



(arXiv: 1609.04185 [hep-ph])

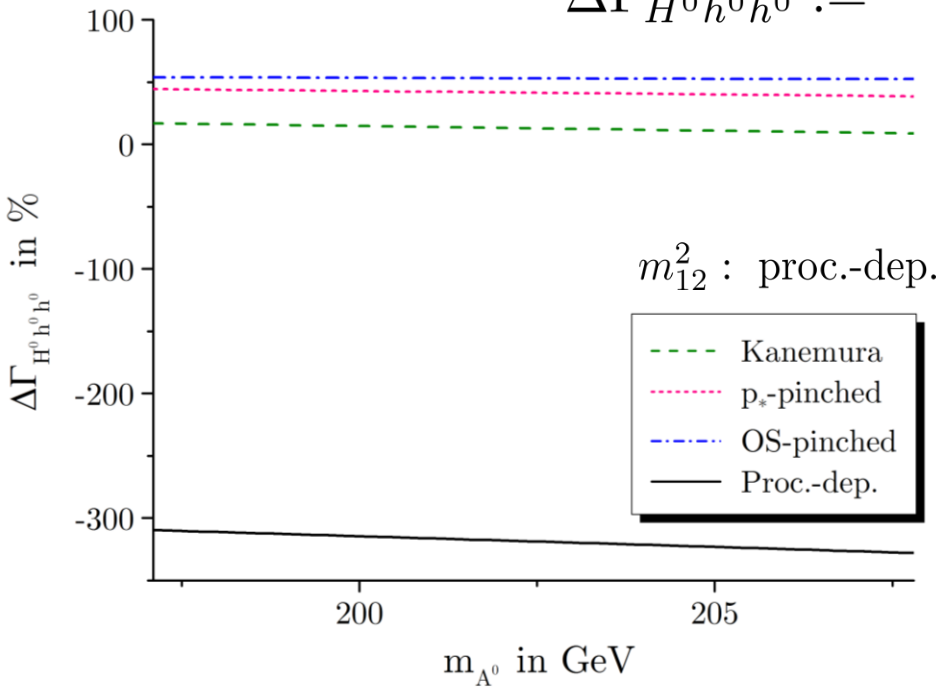
- pinched schemes: mostly within $\pm$ **30%**
- process-dependent scheme: mostly up to $\pm$ **250%**
- Higgs-to-Higgs decay features numerically stable results
- numerical instability: **freedom of parameter choice** in the 2HDM

Numerical Results: $H^0 \longrightarrow h^0\, h^0$

- alternative scheme for m_{12}^2 : process-dependent via $H^0 \longrightarrow A^0\, A^0$

- check for **numerical stability**:

$$\Delta\Gamma_{H^0 h^0 h^0} := \frac{\Gamma^{\rm NLO}_{H^0 h^0 h^0} - \Gamma^{\rm LO}_{H^0 h^0 h^0}}{\Gamma^{\rm LO}_{H^0 h^0 h^0}}$$



($m_{H^0} = 607.46$ GeV, m_{A^0} varied, $m_{H^\pm} = 617.94$ GeV, $\alpha = -0.78$, $\tan\beta = 1.15$, $\Lambda_5 = 1.03$; type II 2HDM)

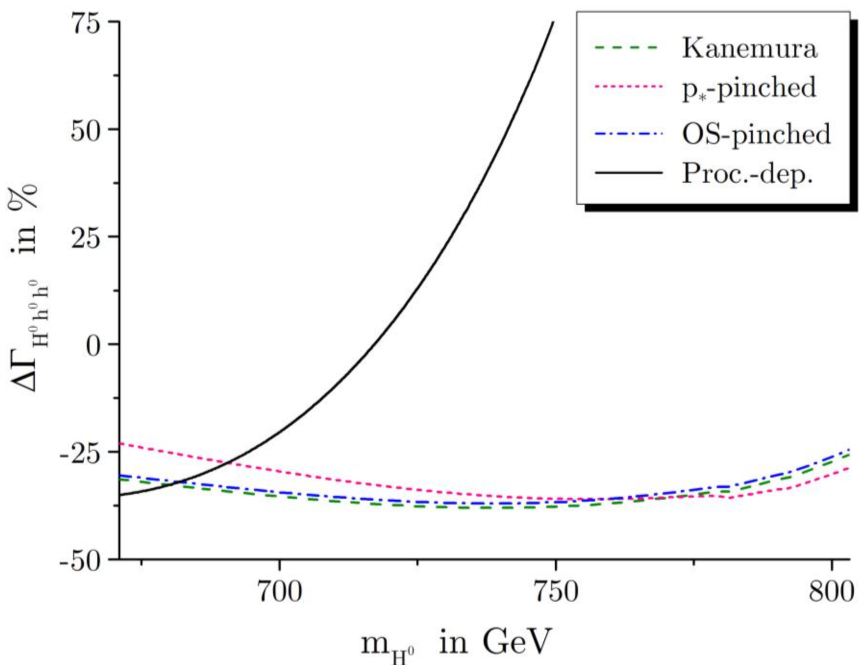
- for most sets: **negative** NLO partial decay width
  ➔ unsuitable scheme?

- proc.-dep. scheme for mixing angles **unsuitable** in general

- more parameter sets needed for higher statistics

Numerical Results: $H \longrightarrow h\, h$

- consistency check: choose a parameter set in the decoupling limit
- Kanemura & pinched schemes numerically **stable**; proc.-dep.: **unstable**



(m_{H^0} varied, $m_{A^0} = 700.13$ GeV, $m_{H^\pm} = 700.35$ GeV,
$\alpha = -0.57$, $\tan\beta = 1.46$, $\Lambda_5 = 14.18$; type II 2HDM)

- the Higgs-to-Higgs decay features numerically stable results as well

- numerical instability:

  consequence of **freedom of parameter choice** in the 2HDM

➔ choose parameters in the decoupling limit for instability check

Numerical Results: Used Software

- development of the Python program 2HDMCalc
  - generates the (N)LO decay amplitudes **in R_ξ gauge** with FeynArts 3.9
  - calculates the amplitudes with FeynCalc 8.2.0
  - generates the NLO counterterm of the decay with Mathematica
  - checks automatically for **UV-finiteness** and **gauge-dependence**
  - evalutes the (N)LO decay widths numerically with LoopTools 2.12
- **current status**: works automatically (i.e. tested) for certain decay topologies
- possible future work:
  - **fully automate** 2HDMCalc
  - calculate **all** decays within the 2HDM @NLO

Numerical Results: Input Parameters

- SM parameters are **fixed** for all parameter points
- 2HDM parameter set generation with ScannerS: (R. Coimbra, M.O.P. Sampaio, R. Santos: arXiv:1301.2599)
  variation of $m_{H^0},\ m_{A^0},\ m_{H^\pm},\ \alpha,\ \beta,\ \Lambda_5$
- constraints on the parameter sets:
  - VEVs represent the global minimum of the potential
  - tree-level unitarity & boundedness from below
  - **electroweak precision measurements** (95% consistency with S, T, U)
  - LEP and recent LHC results (interfaced with SusHi and HDECAY)

  (for details: P. M. Ferreira, R. Guedes, M.O.P. Sampaio, and R. Santos: arXiv:1409.6723)
- additional **kinematic constraints**:
  - 70.000 points with $m_{H^\pm} \geq m_{h^0} + m_W \quad \wedge \quad m_{H^0} \geq 2m_{h^0}$
  - 20.000 points with $m_{H^0} \geq 2m_{h^0} \quad \wedge \quad m_{H^0} \geq 2m_{A^0}$