Karlsruhe Institute of Technology

New model for transmission probabilities of membrane bellows

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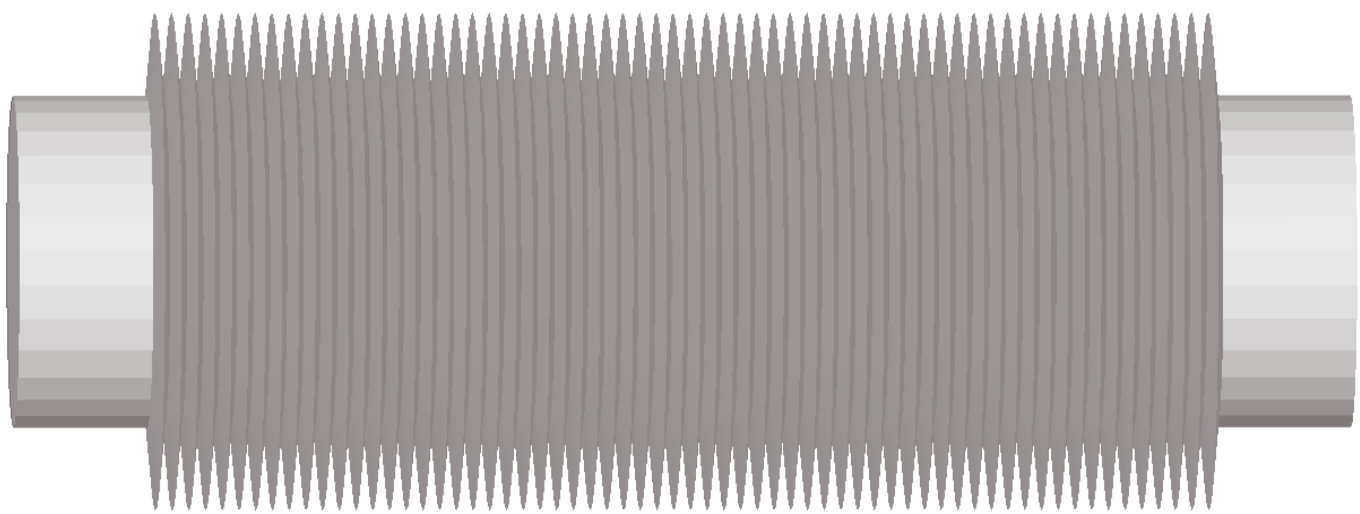


Outline

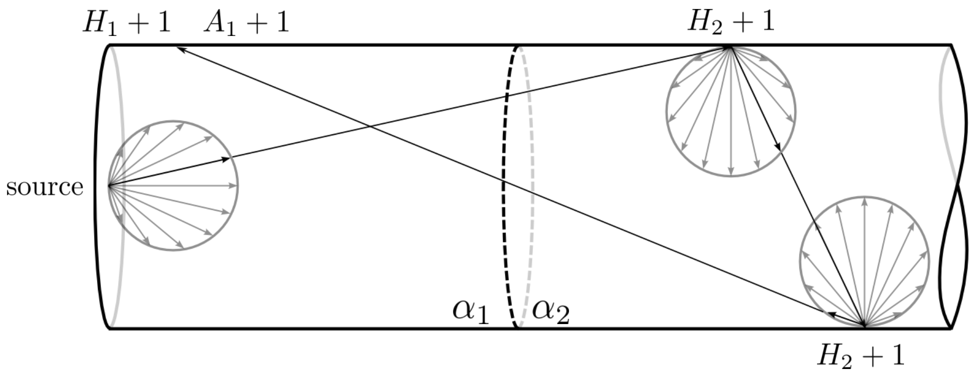
- Principle of vacuum simulations
- Membrane bellows in vacuum simulations
- Membrane bellow analysis
- Variations of bellow parameters

Principle of vacuum simulations: method

- **test particle Monte Carlo simulation for free molecular flow**
  - **Molflow+** and ProVac3D

- **surfaces approximated by mesh**
  - individual properties of each surface element:
    - sticking coefficient α_i
    - desorption probability and angular distribution $\cos^n(\Theta)$
    - diffuse (Lambertian) reflection

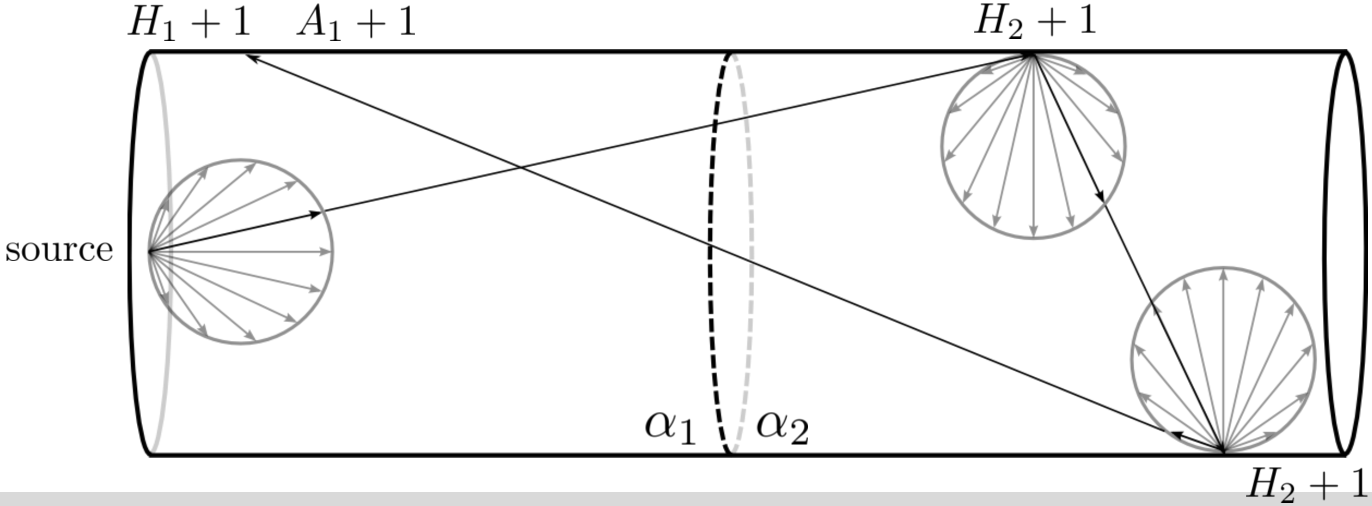
- **particle tracking produces for each surface**
  - number of desorptions D_i
  - number of hits H_i
  - number of adsorptions A_i



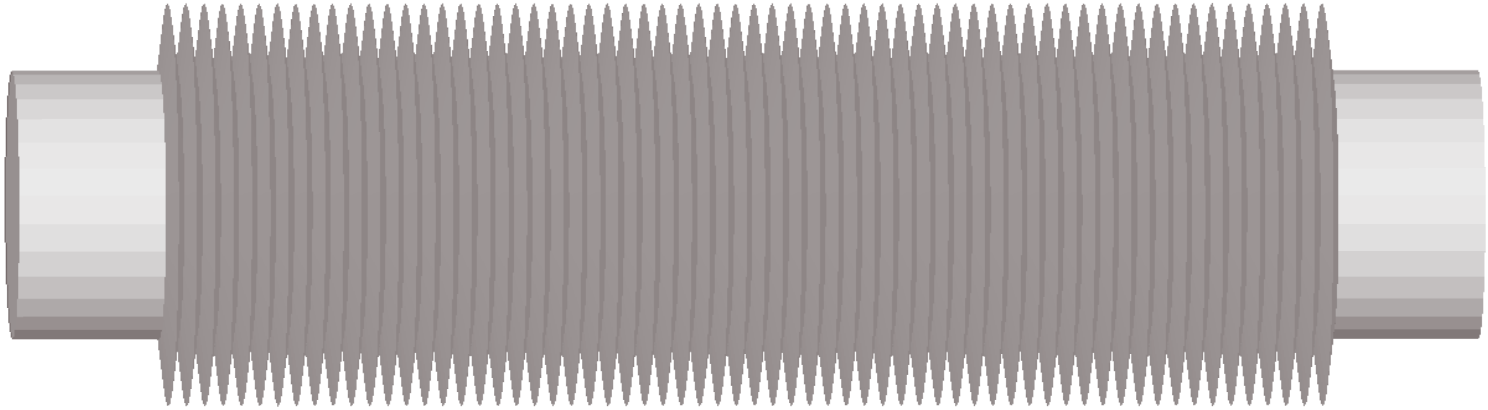
Principle of vacuum simulations: results

- **conductance**
  - desorbing and adsorbing surface (α_1 = 100%) at entrance D_1, A_1
  - adsorbing surface (α_E = 100%) at exit A_E
  - conductance: flow times **transmission probability *w*** = A_E/D_1

- **ratio of pressures**
  - number of hits H_i *normalized to surface area* F_i
  - **pressure ratio** of two surfaces: $p_1/p_2 = H_1 \cdot F_2 / H_2 \cdot F_1$



Membrane bellows in vacuum simulations

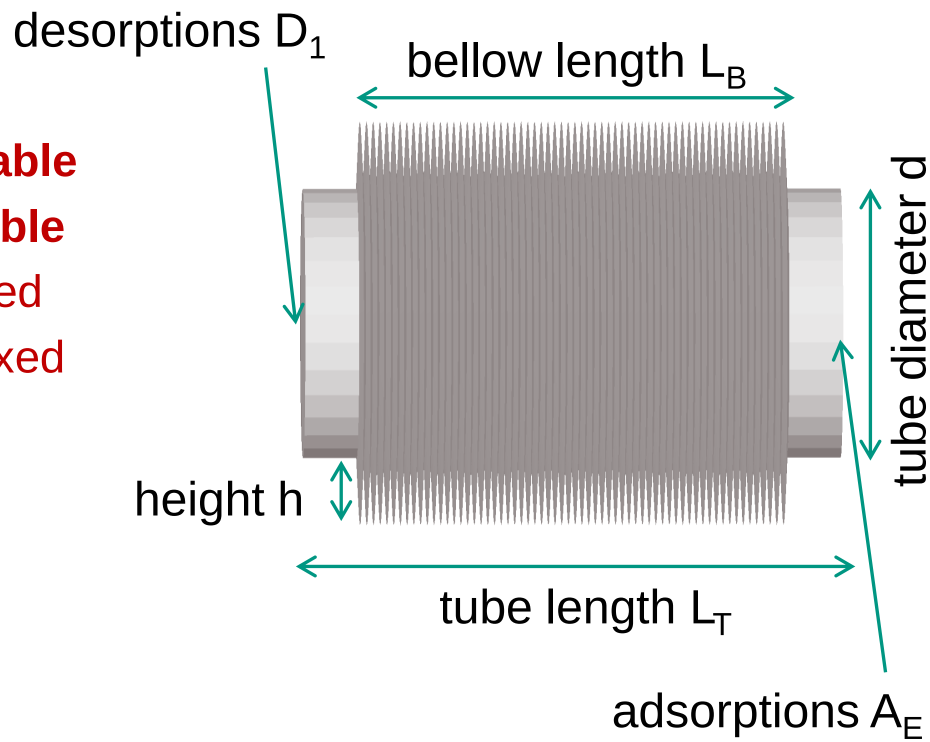


- implementation of membrane bellows: membrane elements approximated by mesh
- entrance and exit tube surfaces: 30 each
- surfaces for n membrane elements: $60 \cdot n$

  ➔ large increase in number of surface elements
  ➔ increase in simulation time by **factors up to 500**
  ➔ is it possible to **replace the bellow** with a straight tube?

Membrane bellow analysis: parameters

- **simulation parameters:**
  - tube diameter: d = 100 mm
  - tube length / tube diameter: **variable**
  - bellow length / tube length: **variable**
  - bellow height / tube diameter: fixed
  - width of single bellow element: fixed



- **Monte Carlo data:**
  - number of desorptions D_1
  - number of adsorptions A_E

- **analysis of simulation data:**
  - transmission probability $w(L_\mathrm{B},\ L_\mathrm{T}) = \frac{A_\mathrm{E}}{D_1}$

Membrane bellow analysis: results

- plot the transmission probability w over L_T / d and L_B / L_T (3D plot)

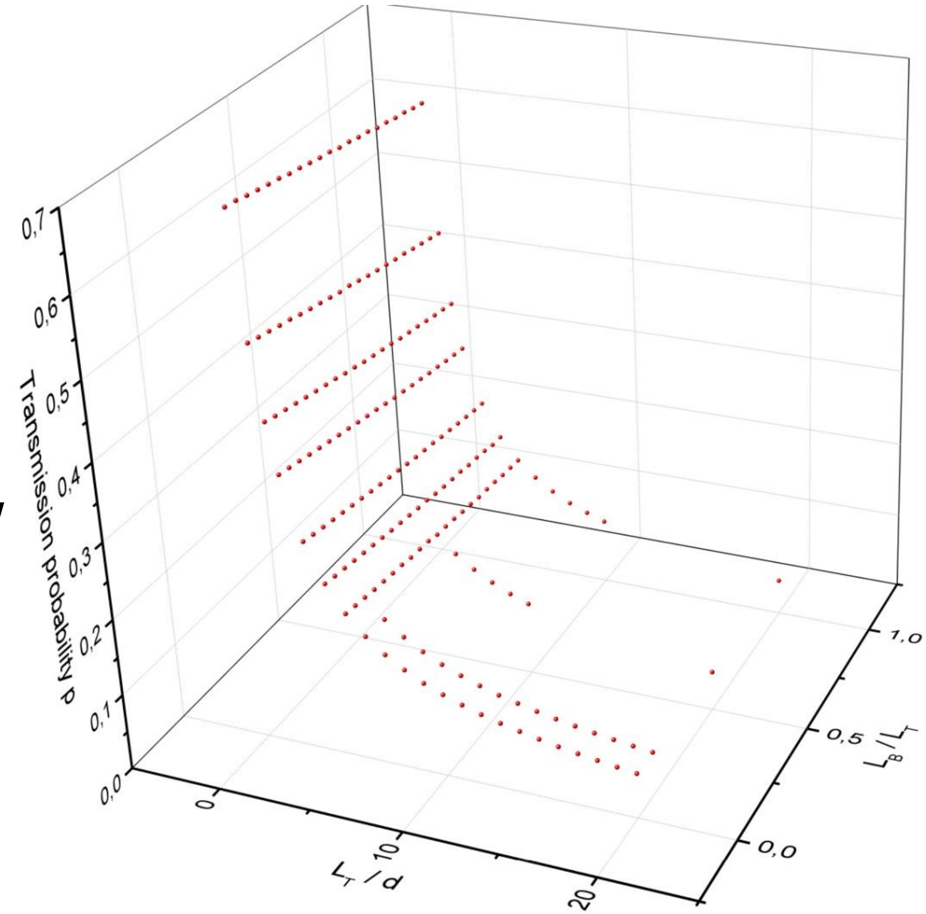
- **L_B / L_T** dependence seems to be **linear**:

$$w\left(\frac{L_B}{L_T}\right) = c_1 + c_2 \cdot \frac{L_B}{L_T}$$

- **L_T / d** dependence seems to follow a form given by **K. Jousten**:

$$w\left(\frac{L_T}{d}\right) = c_1 \cdot \frac{1+c_2\cdot\frac{L_T}{d}}{1+c_3\cdot\frac{L_T}{d}+c_4\cdot\left(\frac{L_T}{d}\right)^2}$$

(note: the original form of the curve has different parameters, but the same functional dependence. See K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details)



- the c_i are the **parameters of the regressions**

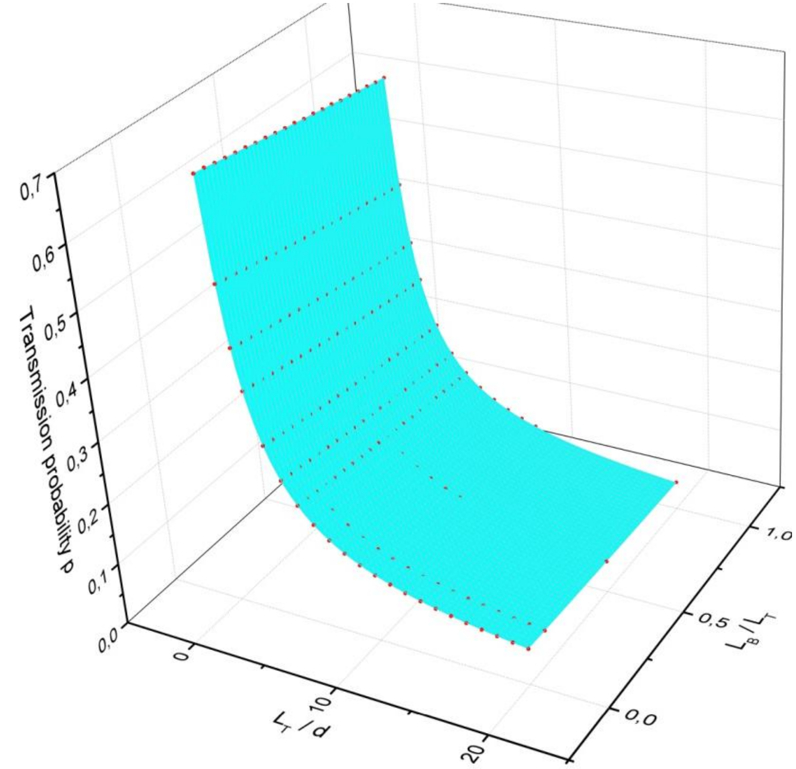
Membrane bellow analysis: results

- **combine both formulas** for the two degrees of freedom:

$$w\left(\frac{L_T}{d},\frac{L_B}{L_T}\right)=\left(A+B\cdot\frac{L_B}{L_T}\right)\cdot\frac{1+\left(C+D\cdot\frac{L_B}{L_T}\right)\cdot\frac{L_T}{d}}{1+\left(E+F\cdot\frac{L_B}{L_T}\right)\cdot\frac{L_T}{d}+\left(G+H\cdot\frac{L_B}{L_T}\right)\cdot\left(\frac{L_T}{d}\right)^2}$$

- {A, …, H} are the **parameters of the regression** given by:

parameter	value	σ
A	0,9974	0,0016
B	0,0343	0,0017
C	0,3898	0,0151
D	-0,0131	0,0043
E	1,3994	0,0209
F	0,3419	0,0077
G	0,2976	0,0124
H	0,0936	0,0068



Analytical analysis of transmission probability

- consider the case without bellows, i.e. **straight tubes**:

$$w\left(\frac{L_T}{d},0\right)=A\cdot\frac{1+C\cdot\frac{L_T}{d}}{1+E\cdot\frac{L_T}{d}+G\cdot\left(\frac{L_T}{d}\right)^2}$$

- for **very long tubes**, this becomes:

$$w\left(\frac{L_T}{d},0\right)\Bigg|_{(L_T/d)\to\infty}\longrightarrow\ \frac{A\cdot C}{G}\cdot\left(\frac{L_T}{d}\right)^{-1}\approx(1.306\pm0.074)\cdot\left(\frac{L_T}{d}\right)^{-1}$$

- note that this is **in accordance** with the formula given by K. Jousten (see K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details), where we have for a **straight circular tube**:

$$w\left(\frac{L_T}{d}\right)\Bigg|_{(L_T/d)\to\infty}\longrightarrow\frac{4}{3}\cdot\left(\frac{L_T}{d}\right)^{-1}$$

- for **any arbitrary bellow**, the transmission probability will **tend to zero** with increasing length:

$$w\left(\frac{L_T}{d},\frac{L_B}{L_T}\right)\Bigg|_{(L_T/d)\to\infty}\longrightarrow\ 0$$

Analytical analysis of transmission probability

- for **very short tubes**, the following limit applies:

$$w\left(\tfrac{L_T}{d},0\right)\Big|_{(L_T/d)\to 0} \longrightarrow A\cdot\left[1-(E-C)\cdot\tfrac{L_T}{d}\right]$$
$$\approx (0.997\pm 0.002)\cdot\left[1-(1.010\pm 0.026)\cdot\tfrac{L_T}{d}\right]$$

- note that this is **in accordance** with the formula given by K. Jousten (see K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details), where we have for a **straight circular tube**:

$$w\left(\tfrac{L_T}{d}\right)\Big|_{(L_T/d)\to 0} \longrightarrow 1-\tfrac{L_T}{d}$$

Replacement of bellows

- **aim:** replace a **bellow** with fixed design parameters with a **straight tube** with different design parameters, but with **the same transmission probability**

- for a **bellow** with fixed L_B / L_T and L_T / d, the transmission probability can be calculated with the **regression results**:

parameter	value	σ
A	0,9974	0,0016
B	0,0343	0,0017
C	0,3898	0,0151
D	-0,0131	0,0043
E	1,3994	0,0209
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G	0,2976	0,0124
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$$w_{\mathrm{B}}\left(\frac{L_T}{d},\frac{L_B}{L_T}\right)=\left(A+B\cdot\frac{L_B}{L_T}\right)\cdot\frac{1+\left(C+D\cdot\frac{L_B}{L_T}\right)\cdot\frac{L_T}{d}}{1+\left(E+F\cdot\frac{L_B}{L_T}\right)\cdot\frac{L_T}{d}+\left(G+H\cdot\frac{L_B}{L_T}\right)\cdot\left(\frac{L_T}{d}\right)^2}\qquad(1)$$

- the formula for straight tubes can be solved for L_T / d:

$$w_0\left(\frac{L_T}{d},0\right)=A\cdot\frac{1+C\cdot\frac{L_T}{d}}{1+E\cdot\frac{L_T}{d}+G\cdot\left(\frac{L_T}{d}\right)^2}$$

$$\Leftrightarrow$$

$$\frac{L_T}{d}=\frac{A\cdot C-w_0\cdot E+\sqrt{(A\cdot C-w_0\cdot E)^2+4w_0\cdot G\cdot(A-w_0)}}{2\cdot w_0\cdot G}$$

$$\Rightarrow\quad\left(\frac{L_T}{d}\right)^*=\frac{A\cdot C-w_{\mathrm{B}}\cdot E+\sqrt{(A\cdot C-w_{\mathrm{B}}\cdot E)^2+4w_{\mathrm{B}}\cdot G\cdot(A-w_{\mathrm{B}})}}{2\cdot w_{\mathrm{B}}\cdot G}\qquad(2)$$

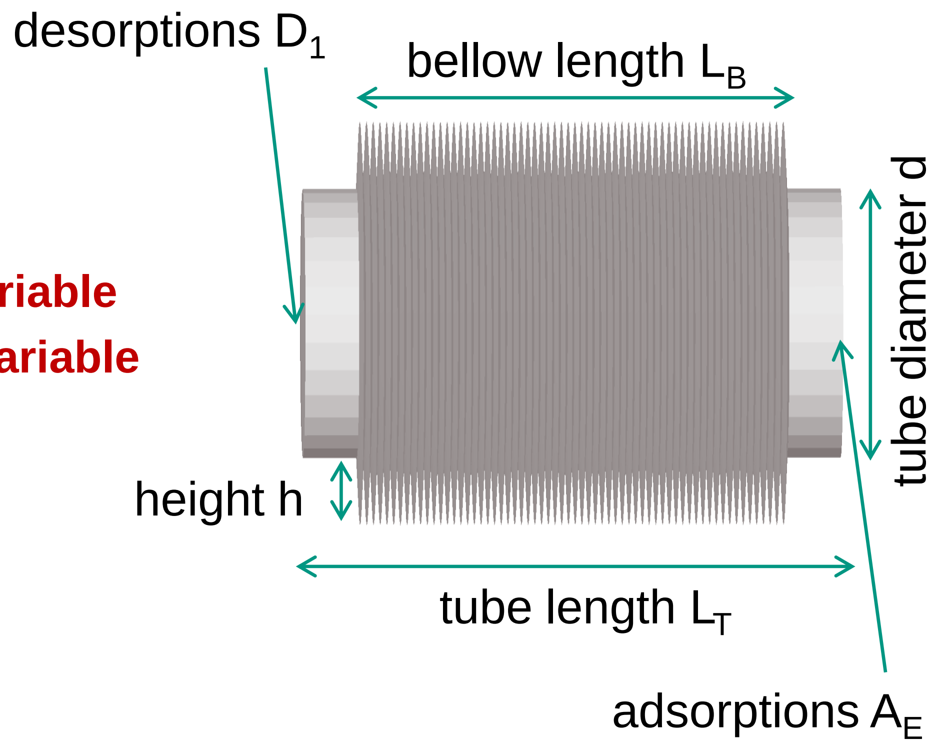
Replacement of bellows

- procedure for replacing bellows with straight tubes:
  - **calculate** the **transmission probability w_B** for the bellow with design parameters L_B / L_T and L_T / d with **Eq. (1)**
  - with this w_B, **calculate** the new length **(L_T / d)*** with help of **Eq. (2)**
  - **replace the bellow** in the simulation with a **straight tube** with design parameter **(L_T / d)***
- this procedure **has been tested** for a few bellows with different design parameters

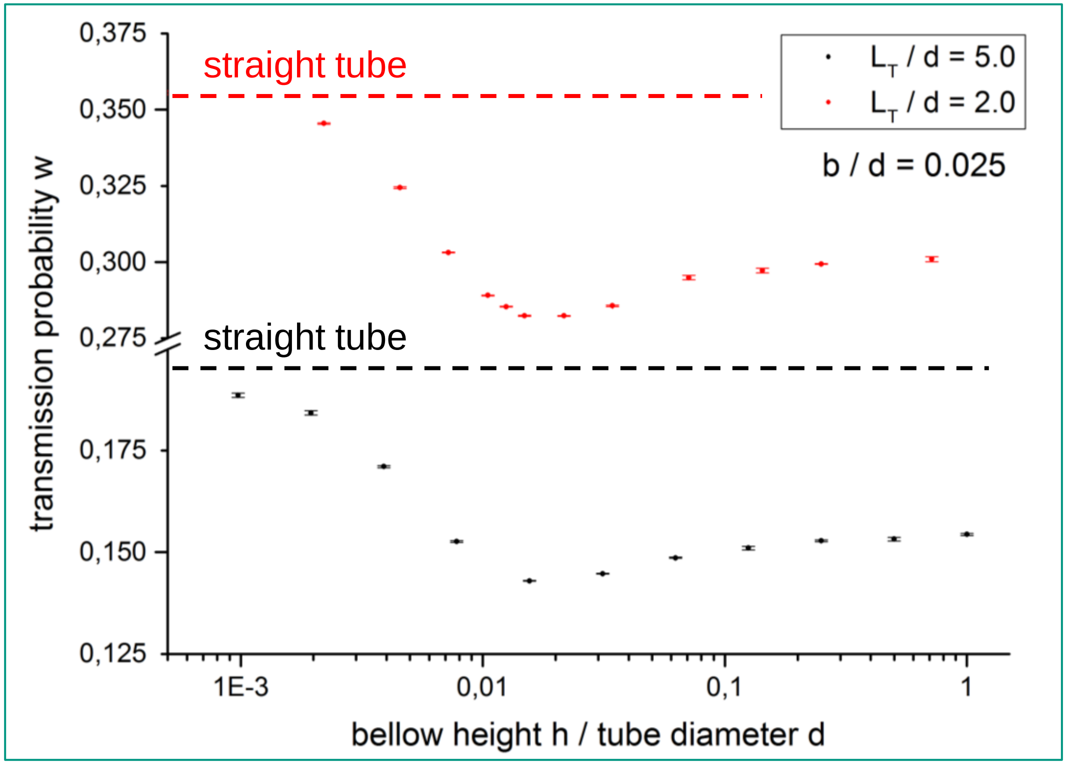
  ➔ **error below few percent**

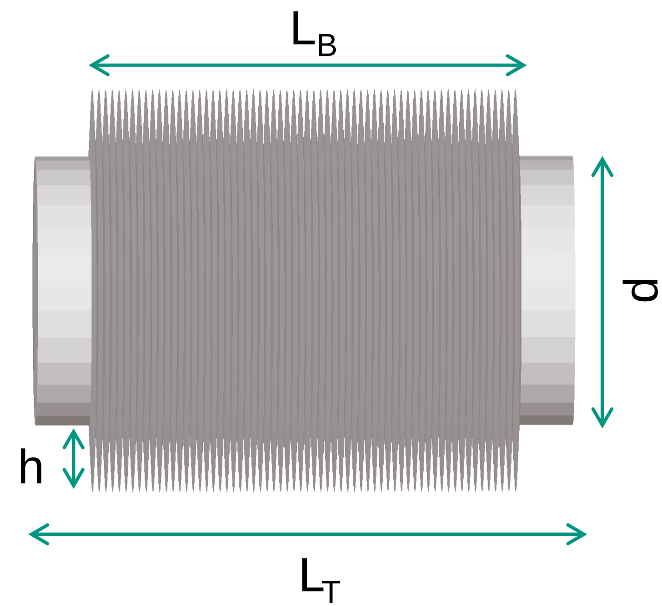
Bellow parameter variations: parameters

- **simulation parameters:**
  - tube diameter: d = 100 mm
  - tube length / tube diameter: fixed
  - bellow length / tube length: fixed
  - bellow height / tube diameter: **variable**
  - width of single bellow element: **variable**



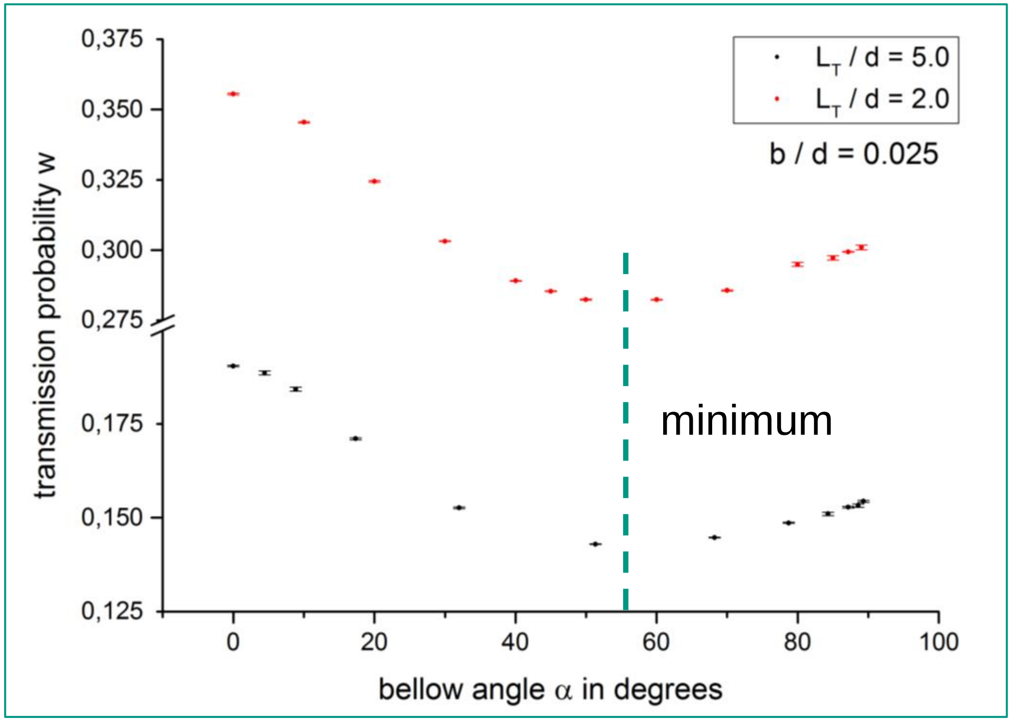
Bellow parameter variations: h / d results

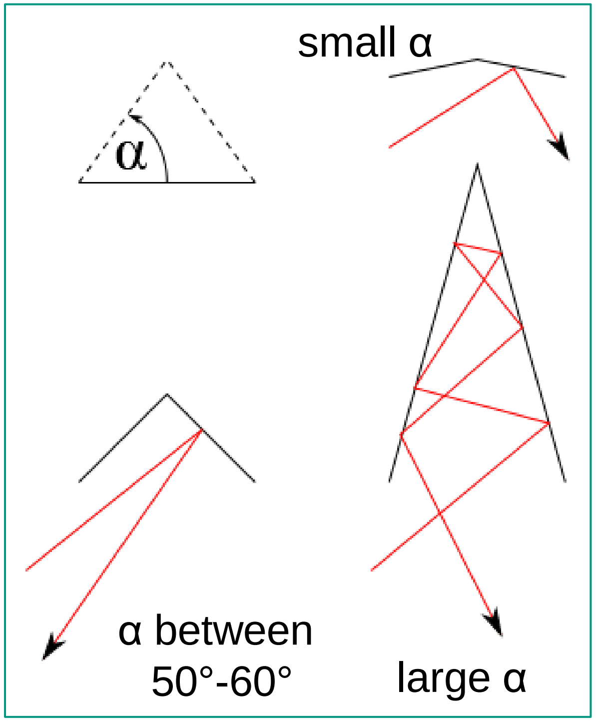




- variation of **h / d** changes the transmission probability **considerably** when the **height tends to zero** (i.e. the bellow becomes a straight tube)
- in the **regime of normal bellow heights** (i.e. industrial ones), the transmission probability changes only **in the order of a few percent**
  ➔ h / d variance is negligible for industrial bellow designs

Bellow parameter variations: h / d results





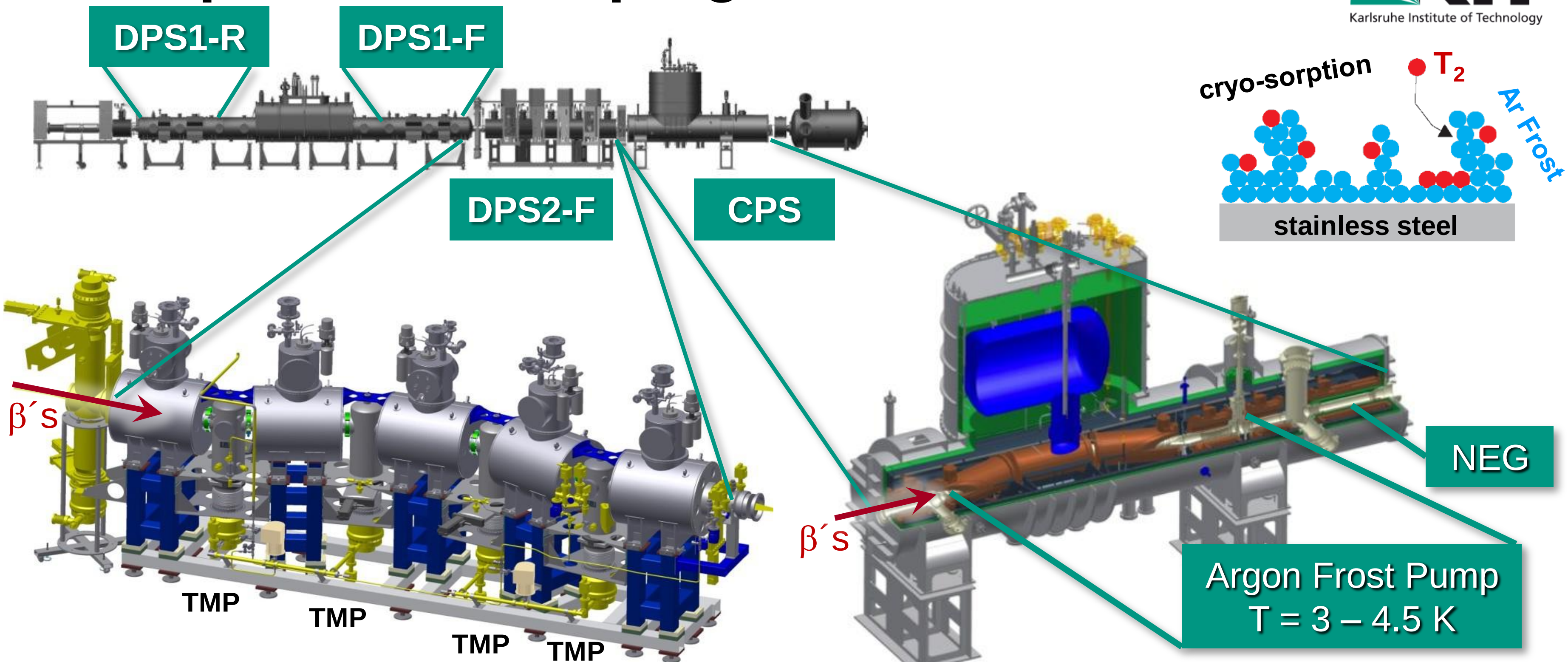
- one finds a **minimum** in the transm. prob. p for angles **α between 50°-60°**
- physical reasons:
  - for **small angles**: bellow acts as **nearly straight tube** → w increases
  - for **large angles:** multiple reflections in one bellow element; overall particle reflection becomes more **uniform** → w increases
  - for **angles between 50°-60°:** bellow surfaces facing the desorption area have perfect angle for **direct reflection** of the particles to the source → w decreases to a minimum

Conclusion

- a **wide range** of bellow design parameters have been simulated
- a **numerical model for transmission probabilities** has been found and is **in concordance with analytical and numerical solutions** in standard text books
- a **procedure for replacing bellows with straight tubes** for simulations has been worked out
- **variations** in bellow widths and heights **have no significant effect** (for standard bellow parameters)

Back-up slides

Transport and Pumping Sections



Differential Pumping Section (DPS)

- active pumping: 4 TMPs
- tritium retention: 10^5
- magnetic field: 5.6 T

Cryogenic Pumping Section (CPS)

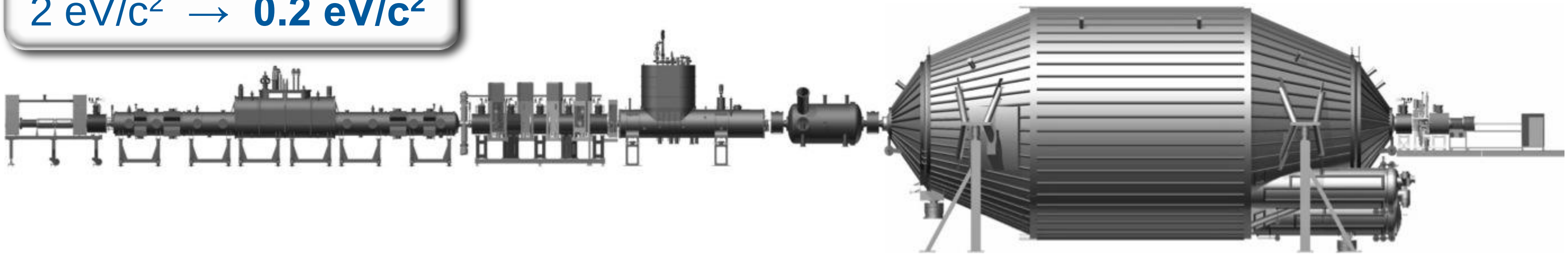
- based on by cryo-sorption
- tritium retention: $>10^7$
- magnetic field: 5.6 T

O. Kazachenko et al., NIM A 587 (2008) 136

F. Eichelhardt et al, Fusion Science and Technology 54 (2008) 615

The **Ka**rlsruhe **Tri**tium **N**eutrino Experiment

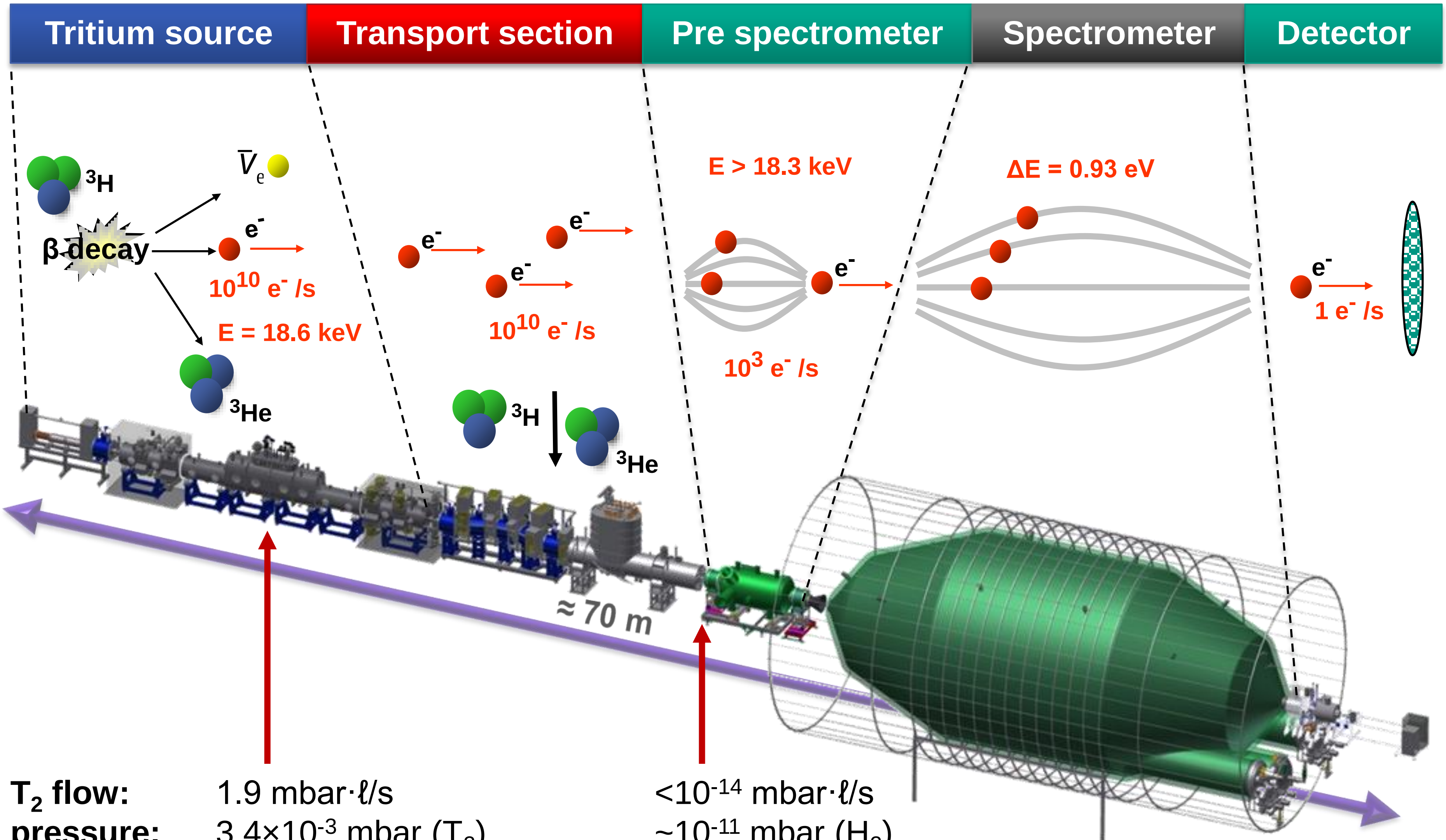
Sensitivity on $m(\nu_e)$:
$2\ eV/c^2 \rightarrow$ **$0.2\ eV/c^2$**



The KATRIN collaboration

- objective: measure eff. neutrino mass with electrons from tritium β-decay
- international collaboration from 5 countries (D, US, CZ, RUS, UK)
- ~ 130 scientists

The KATRIN Setup - Overview



T_2 flow:	1.9 mbar·ℓ/s	<10^{-14} mbar·ℓ/s
pressure:	3.4×10^{-3} mbar (T_2)	~10^{-11} mbar (H_2)

Membrane bellows in the KATRIN experiment

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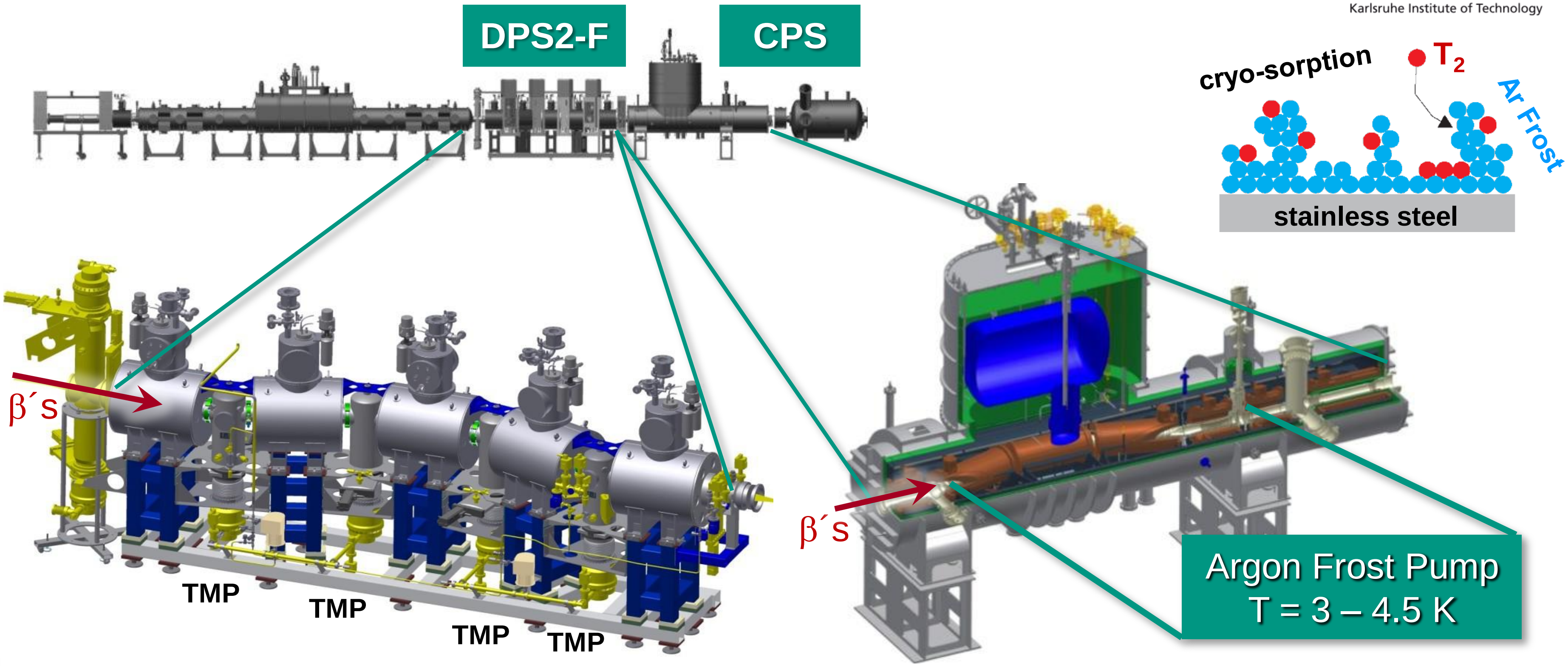

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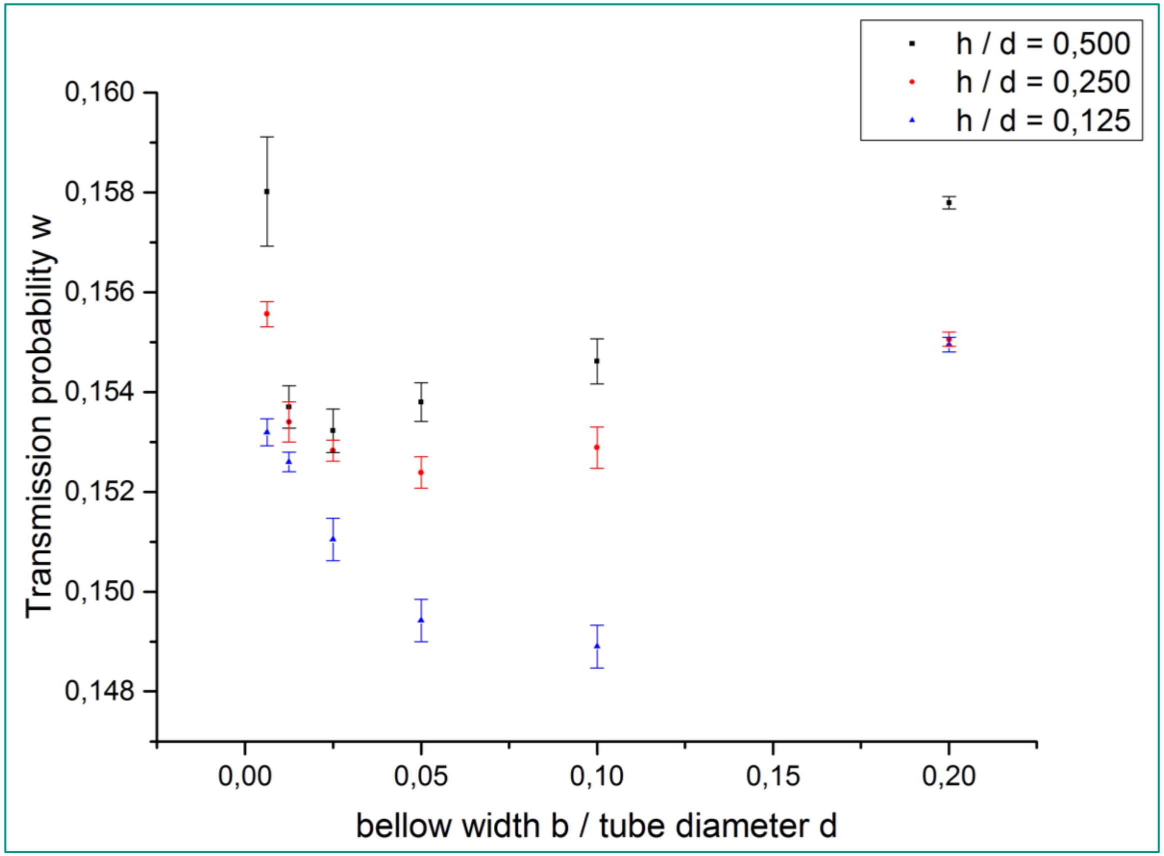
Bellow parameter variations: b / d results



- variation of bellow width **b / d** changes the transmission probability only in the **order of a few percent**
  ➔ b / d variance is negligible