New model for transmission probabilities of membrane bellows

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Outline

- Principle of vacuum simulations
- Membrane bellows in vacuum simulations
- Membrane bellow analysis
- Variations of bellow parameters
Principle of vacuum simulations: method

- test particle Monte Carlo simulation for free molecular flow
  - Molflow+ and ProVac3D

- surfaces approximated by mesh
  - individual properties of each surface element:
    - sticking coefficient $\alpha_i$
    - desorption probability and angular distribution $\cos^n(\Theta)$
    - diffuse (Lambertian) reflection

- particle tracking produces for each surface
  - number of desorptions $D_i$
  - number of hits $H_i$
  - number of adsorptions $A_i$
**Principle of vacuum simulations: results**

- **conductance**
  - desorbing and adsorbing surface \( (\alpha_1 = 100\%) \) at entrance \( D_1, A_1 \)
  - adsorbing surface \( (\alpha_E = 100\%) \) at exit \( A_E \)
  - conductance: flow times **transmission probability** \( w = A_E/D_1 \)

- **ratio of pressures**
  - number of hits \( H_i \) *normalized to surface area* \( F_i \)
  - **pressure ratio** of two surfaces: \( p_1/p_2 = H_1 \cdot F_2/H_2 \cdot F_1 \)
Membrane bellows in vacuum simulations

- Implementation of membrane bellows: membrane elements approximated by mesh
- Entrance and exit tube surfaces: 30 each
- Surfaces for n membrane elements: $60 \cdot n$
  - Large increase in number of surface elements
  - Increase in simulation time by factors up to 500
  - Is it possible to replace the bellow with a straight tube?
Membrane bellow analysis: parameters

**Simulation parameters:**
- Tube diameter: $d = 100$ mm
- Tube length / tube diameter: **variable**
- Bellow length / tube length: **variable**
- Bellow height / tube diameter: **fixed**
- Width of single bellow element: **fixed**

**Monte Carlo data:**
- Number of desorptions $D_1$
- Number of adsorptions $A_E$

**Analysis of simulation data:**
- Transmission probability
  $$w(L_B, L_T) = \frac{A_E}{D_1}$$
Membrane bellow analysis: results

- plot the transmission probability $w$ over $L_T / d$ and $L_B / L_T$ (3D plot)

- $L_B / L_T$ dependence seems to be linear:
  \[ w \left( \frac{L_B}{L_T} \right) = c_1 + c_2 \cdot \frac{L_B}{L_T} \]

- $L_T / d$ dependence seems to follow a form given by K. Jousten:
  \[ w \left( \frac{L_T}{d} \right) = c_1 \cdot \frac{1+c_2 \cdot \frac{L_T}{d}}{1+c_3 \cdot \frac{L_T}{d}+c_4 \cdot \left( \frac{L_T}{d} \right)^2} \]

  (note: the original form of the curve has different parameters, but the same functional dependence. See K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details)

- the $c_i$ are the parameters of the regressions
Membrane bellow analysis: results

- **combine both formulas** for the two degrees of freedom:

\[
\begin{align*}
    w \left( \frac{L_T}{d}, \frac{L_B}{L_T} \right) &= \left( A + B \cdot \frac{L_B}{L_T} \right) \cdot \frac{1 + \left( C + D \cdot \frac{L_B}{L_T} \right) \cdot \frac{L_T}{d}}{1 + \left( E + F \cdot \frac{L_B}{L_T} \right) \cdot \frac{L_T}{d} + \left( G + H \cdot \frac{L_B}{L_T} \right) \cdot \left( \frac{L_T}{d} \right)^2}
\end{align*}
\]

- \{A, ..., H\} are the **parameters of the regression** given by:

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<th>$\sigma$</th>
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<tr>
<td>H</td>
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</table>
Analytical analysis of transmission probability

- consider the case without bellows, i.e. straight tubes:

\[ w \left( \frac{L_T}{d}, 0 \right) = A \cdot \frac{1 + C \cdot \frac{L_T}{d}}{1 + E \cdot \frac{L_T}{d} + G \cdot \left( \frac{L_T}{d} \right)^2} \]

- for very long tubes, this becomes:

\[ w \left( \frac{L_T}{d}, 0 \right) \bigg|_{(L_T/d) \to \infty} \rightarrow \frac{A \cdot C}{G} \cdot \left( \frac{L_T}{d} \right)^{-1} \approx (1.306 \pm 0.074) \cdot \left( \frac{L_T}{d} \right)^{-1} \]

- note that this is in accordance with the formula given by K. Jousten (see K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details), where we have for a straight circular tube:

\[ w \left( \frac{L_T}{d} \right) \bigg|_{(L_T/d) \to \infty} \rightarrow \frac{4}{3} \cdot \left( \frac{L_T}{d} \right)^{-1} \]

- for any arbitrary bellow, the transmission probability will tend to zero with increasing length:

\[ w \left( \frac{L_T}{d}, \frac{L_B}{L_T} \right) \bigg|_{(L_T/d) \to \infty} \rightarrow 0 \]
Analytical analysis of transmission probability

- for very short tubes, the following limit applies:

\[
  w \left( \frac{L_T}{d}, 0 \right) \bigg|_{(L_T/d) \to 0} \longrightarrow A \cdot \left[ 1 - (E - C) \cdot \frac{L_T}{d} \right] \\
  \approx (0.997 \pm 0.002) \cdot \left[ 1 - (1.010 \pm 0.026) \cdot \frac{L_T}{d} \right]
\]

- note that this is in accordance with the formula given by K. Jousten (see K. Jousten et al, Handbook of Vacuum Technology, p. 136 for details), where we have for a straight circular tube:

\[
  w \left( \frac{L_T}{d} \right) \bigg|_{(L_T/d) \to 0} \longrightarrow 1 - \frac{L_T}{d}
\]
Replacement of bellows

- **aim:** replace a **bellow** with fixed design parameters with a **straight tube** with different design parameters, but with the **same** transmission probability

- for a **bellow** with fixed $L_B / L_T$ and $L_T / d$, the transmission probability can be calculated with the **regression results:**

$$ w_B \left( \frac{L_T}{d}, \frac{L_B}{L_T} \right) = \left( A + B \cdot \frac{L_B}{L_T} \right) \cdot \frac{1+\left(C+D \cdot \frac{L_B}{L_T}\right) \cdot \frac{L_T}{d}}{1+(E+F \cdot \frac{L_B}{L_T}) \cdot \frac{L_T}{d}+(G+H \cdot \frac{L_B}{L_T}) \cdot \left(\frac{L_T}{d}\right)^2} $$ (1)

- the formula for straight tubes can be solved for $L_T / d$:

$$ w_0 \left( \frac{L_T}{d}, 0 \right) = A \cdot \frac{1+C \cdot \frac{L_T}{d}}{1+E \cdot \frac{L_T}{d}+G \cdot \left(\frac{L_T}{d}\right)^2} $$

$$ \Leftrightarrow $$

$$ \frac{L_T}{d} = \frac{A \cdot C - w_0 \cdot E + \sqrt{(A \cdot C - w_0 \cdot E)^2 + 4w_0 \cdot G \cdot (A - w_0)}}{2 \cdot w_0 \cdot G} $$

$$ \Rightarrow \left( \frac{L_T}{d} \right)^* = \frac{A \cdot C - w_B \cdot E + \sqrt{(A \cdot C - w_B \cdot E)^2 + 4w_B \cdot G \cdot (A - w_B)}}{2 \cdot w_B \cdot G} $$ (2)
Replacement of bellows

- procedure for replacing bellows with straight tubes:
  - calculate the transmission probability $w_B$ for the bellow with design parameters $L_B / L_T$ and $L_T / d$ with Eq. (1)
  - with this $w_B$, calculate the new length $(L_T / d)^*$ with help of Eq. (2)
  - replace the bellow in the simulation with a straight tube with design parameter $(L_T / d)^*$

- this procedure has been tested for a few bellows with different design parameters
  - error below few percent
Bellow parameter variations: parameters

- **simulation parameters:**
  - tube diameter: $d = 100$ mm
  - tube length / tube diameter: fixed
  - bellow length / tube length: fixed
  - bellow height / tube diameter: variable
  - width of single bellow element: variable

![Diagram of bellow parameters](image-url)
Bellow parameter variations: $h / d$ results

- variation of $h / d$ changes the transmission probability **considerably** when the height tends to zero (i.e. the bellow becomes a straight tube)
- in the **regime of normal bellow heights** (i.e. industrial ones), the transmission probability changes only **in the order of a few percent**
  - $h / d$ variance is negligible for industrial bellow designs
one finds a **minimum** in the transm. prob. $p$ for angles $\alpha$ **between $50^\circ$-60^\circ**

- for **small angles**: bellow acts as nearly straight tube $\Rightarrow w$ increases
- for **large angles**: multiple reflections in one bellow element; overall particle reflection becomes more uniform $\Rightarrow w$ increases
- for angles **between $50^\circ$-60^\circ**: bellow surfaces facing the desorption area have perfect angle for direct reflection of the particles to the source $\Rightarrow w$ decreases to a minimum
Conclusion

- a **wide range** of bellow design parameters have been simulated

- a **numerical model for transmission probabilities** has been found and is **in concordance with analytical and numerical solutions** in standard text books

- a **procedure for replacing bellows with straight tubes** for simulations has been worked out

- **variations** in bellow widths and heights **have no significant effect** (for standard bellow parameters)
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Back-up slides
Transport and Pumping Sections

Differential Pumping Section (DPS)
- active pumping: 4 TMPs
- tritium retention: $10^5$
- magnetic field: 5.6 T

Cryogenic Pumping Section (CPS)
- based on cryo-sorption
- tritium retention: $>10^7$
- magnetic field: 5.6 T

O. Kazachenko et al., NIM A 587 (2008) 136
F. Eichelhardt et al, Fusion Science and Technology 54 (2008) 615
The Karlsruhe Tritium Neutrino Experiment

**Sensitivity on $m(\nu_e)$:**

$2 \text{ eV/c}^2 \rightarrow 0.2 \text{ eV/c}^2$

The KATRIN collaboration

- **objective:** measure eff. neutrino mass with electrons from tritium $\beta$-decay
- **international collaboration** from 5 countries (D, US, CZ, RUS, UK)
- ~ 130 scientists
The KATRIN Setup - Overview

Tritium source

$^3\text{H}$

$\beta$-decay

$e^-$

$10^{10} \text{ e}^-/\text{s}$

$E = 18.6 \text{ keV}$

$^3\text{He}$

Pre spectrometer

$E > 18.3 \text{ keV}$

$\Delta E = 0.93 \text{ eV}$

$e^-$

$10^3 \text{ e}^-/\text{s}$

Spectrometer

$e^-$

$1 \text{ e}^-/\text{s}$

Detector

$e^-$

$T_2$ flow:

pressure:

$1.9 \text{ mbar} \cdot \ell/\text{s}$

$3.4 \times 10^{-3} \text{ mbar} \ (T_2)$

$<10^{-14} \text{ mbar} \cdot \ell/\text{s}$

$\sim 10^{-11} \text{ mbar} \ (H_2)$
Membrane bellows in the KATRIN experiment

Differential Pumping Section (DPS)
- active pumping: 4 TMPs
- tritium retention: $10^5$
- magnetic field: 5.6 T

Cryogenic Pumping Section (CPS)
- based on cryosorption
- tritium retention: $>10^7$
- magnetic field: 5.6 T

O. Kazachenko et al., NIM A 587 (2008) 136
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Bellow parameter variations: \( b / d \) results

- variation of bellow width \( b / d \) changes the transmission probability only in the order of a few percent
  \[ \Rightarrow b / d \text{ variance is negligible} \]