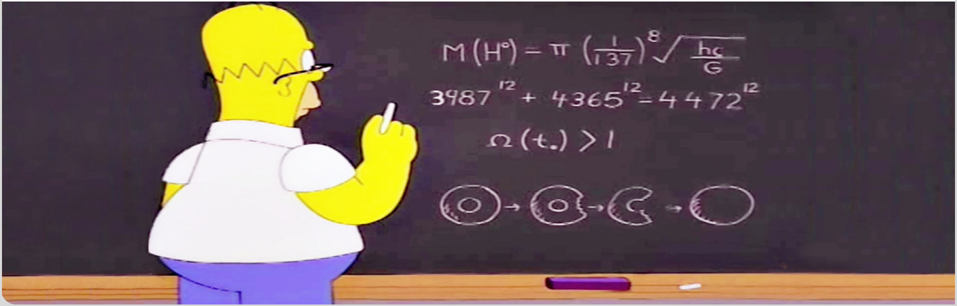
Karlsruhe Institute of Technology

Catastrophic Goldstones in Supersymmetric Higgs Boson Masses

Martin Gabelmann | 27.04.2020

INSTITUTE FOR THEORETICAL PHYSICS (ITP) - BSM SEMINAR



Outline

m_h^{SM} VS. m_h^{SUSY}

Why is SUSY (*no longer?*) called a *beautiful* theory?

Consider loop correction to vector/Higgs/fermion mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{Q^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{Q^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{Q^2}$

  which symmetry protects m_h?

  $m_{\text{heavy}} \to m_{\text{SUSY}}$

Other important buzzwords: GUT, Dark Matter, radiative EWSB, SUGRA...

Example: (N)MSSM Neutral Higgs Mass

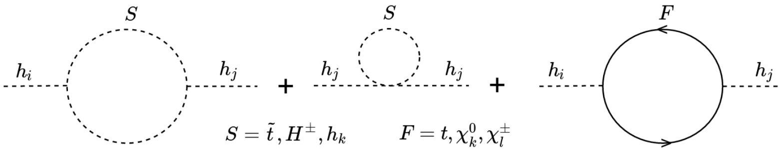
$$\left(m_h^{\text{tree}}\right)^2 = \underbrace{m_Z^2 \cos^2 2\beta}_{\text{D-Terms}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{F-Terms}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z$
- NMSSM: $\lambda < 0.7$ constrained by perturbative unitarity

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass.

At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ are:



- In the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$!

Analytic structure of δm_h^{SUSY}

Perturbative series of δm_h will involve expansions in powers of:

- number of loops: $(4\pi)^{-2}$
- number of logs: $\log \frac{m_{\text{SUSY}}^2}{m_{\text{SM}}^2}$
- suppression by heavy scales: $\frac{m_{\text{SM}}^2}{m_{\text{SUSY}}^2}$
- coefficients: $C(y_t^{\overline{\text{DR}}}, X_t^{\overline{\text{DR}}}, \ldots) \leftrightarrow C(y_t^{\text{OS}}, X_t^{\text{OS}}, \ldots)$

If $m_{\text{SUSY}} \gg m_{\text{SM}}$, the log-expansion at fixed-order might not converge well.
$\rightarrow$Need to resum large logs in EFT-framework.

Analytic structure of δm_h^{SUSY}: Fixed Order

A fixed-order n-loop result will incorporate the full logarithmic dependence $(4\pi)^{-2n}\sum_{k=0}^{n}\log^k$ (good and bad ones!) and constant $\frac{m^2_{\text{SUSY}}}{m^2_{\text{SM}}}$-terms:

- effective potential: $\partial^k_{h_{i_1},\ldots,h_{i_k}} V^{(n)}_{eff} = G^{(n)}_{h_{i_1},\ldots,h_{i_k}}(0)|_{\text{fin}}$
  - V_{eff} known up to 2- and 3-loops for general QFT [Martin, Patel, '18] [Martin, '17]
  - ∂V_{eff} numerically difficult
  - $p^2 = 0 \to$ massless particles are troublesome
- diagrammatic: calculate $\Sigma^{(n)}_{ij}(p^2)$ and $T^{(n)}_i$
  - computational expensive
  - mass hierarchies: expand loop-integrals
  - numerical evaluation: no access to logs, slow

Other ingredients:

- OS s/top sector: $(n-1)$-loop selfenergies
- OS v_{SM}: n-loop vector boson masses
- full n-loop SUSY RGEs from m_Z to $Q_{\text{Ren.}} \approx m_{\text{SUSY}}$ (thresholds!)

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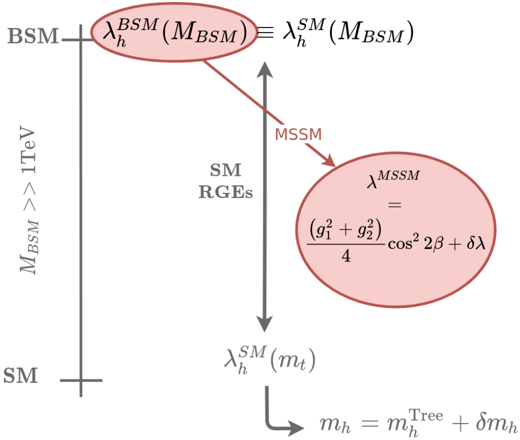
Analytic structure of δm_h^{SUSY}: EFT

Idea: avoid large logs by separation of SUSY/SM contributions.



missing: only resummation in leading logs, no p^2- and no $\frac{m^2_{\text{SM}}}{m^2_{\text{SUSY}}}$-dependence

The Three Frontiers in Higgs Mass Calculations

(Apart from increasing loops)

Fixed Order (FO)	RGE Improved (EFT)	Ren. Conditions

Fixed Order (FO)

- Weak/TeV-scale SUSY
- $m_{SUSY} \lesssim 1-2\,\mathrm{TeV}$
- calculate full $\delta m_h^{SUSY}(m_{SUSY})$
- full $\frac{m_{\rm SM}}{m_{\rm SUSY}}$ dependence

$$\delta m_h \propto \Sigma_{hh}^{(n)}(m_{\rm SM}, m_{\rm SUSY}, \ldots.)$$

All self-energies & tadpole contributions (SM+SUSY) computed at the scale m_{SUSY}.

RGE Improved (EFT)

- High/Split-scale SUSY
- $m_{SUSY} \gtrsim 1-2\,\mathrm{TeV}$
- matching, calculate $\delta\lambda_h^{\rm SUSY}$
- neglects $\frac{m_{\rm SM}}{m_{\rm SUSY}}$-terms

$$\lambda_h^{(n)SM}(m_{SUSY}) = \lambda_h^{(n)SUSY}(m_{SUSY})$$

$\downarrow$ $(n+1)$-RGEs

$$\delta m_h^{SM} = \Sigma_{hh}^{(n)}(\lambda_h^{SM}, \ldots)$$

Ren. Conditions

- $\overline{\rm DR}$: minimal subtraction
- OS: Express result through physical quantities
- scheme dependence due to missing higher orders

Scheme change is sensitive to higher-order corrections, e.g.:

$m_h(m_t^{\overline{DR}}) \leftrightarrow m_h(m_t^{OS})$

$m_h(m_{H^\pm}^{\overline{DR}}) \leftrightarrow m_h(m_{H^\pm}^{OS})$

Also "hybrid approaches" exist: combine FO and EFT results

- using pole mass matching [Athron, Park, Steudtner, Stöckinger, Voigt, '16] [Porod, Staub, '17]:
  $\delta^{(1)}\lambda_{SM} = \frac{1}{v^2}\left(\delta^{(1)} m_h^{SUSY\,2} - \delta^{(1)} m_h^{SM\,2}\right)$
- combine $m^2_{\rm SM}/m^2_{SUSY}$ from FO with resummed EFT results [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, '13] [Bahl, Hollik, '16] [Harlander, Klappert, Voigt, '19]
  $m_h^2 \equiv \left(m_h^{\rm FO}\right)^2 - \left(m_h^{\text{FO, large-logs}}\right)^2 + \left(m_h^{\text{EFT-resummed}}\right)^2$

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$m_{SUSY} \lesssim 1-2\,\text{TeV}$	$m_{SUSY} \gtrsim 1-2\,\text{TeV}$	OS: Express result through physical quantities
calculate full $\delta m_h^{SUSY}(m_{SUSY})$	matching, calculate $\delta\lambda_h^{\text{SUSY}}$	scheme dependence due to missing higher orders
full $\frac{m_{\text{SM}}}{m_{\text{SUSY}}}$ dependence	neglects $\frac{m_{\text{SM}}}{m_{\text{SUSY}}}$-terms	

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$$\lambda_h^{(n)SM}(m_{SUSY}) = \lambda_h^{(n)SUSY}(m_{SUSY})$$

$\downarrow$ $(n+1)$-RGEs

$$\delta m_h^{SM} = \Sigma_{hh}^{(n)}(\lambda_h^{SM}, \ldots)$$

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- full $\frac{m_{SM}}{m_{SUSY}}$ dependence

$\delta m_h \propto \Sigma_{hh}^{(n)}(m_{SM}, m_{SUSY}, \ldots.)$

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$\lambda_h^{(n)SM}(m_{SUSY}) = \lambda$

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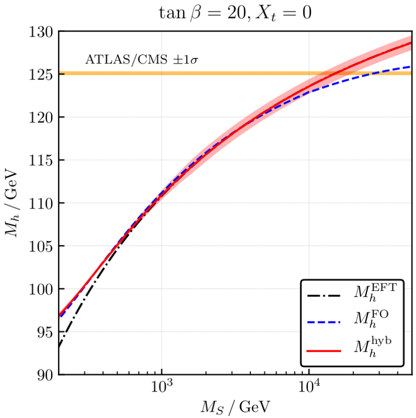

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1. Introduction to Supersymmetric Higgs Potentials
   - SUSY Motivations and Examples
   - Higher-Order Corrections to m_h^2
2. Overview on Higher-Order Higgs-Mass Corrections
   - MSSM
   - NMSSM
3. Catastrophic Two-Loop Corrections in the NMSSM
4. Outlook

Status in the MSSM: Fixed Order

Most precise results are based on **3-loop** self-energies&tadpoles: [Kant, Harlander, Mihaila, Steinhauser, '10]

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- consider strong sector only, $t,\ \tilde{t},\ g,\ \tilde{g}$ (up to m_t^4-terms)
- assume hierarchies, e.g. (1) $m_{\tilde{g}} \gg m_{\tilde{t}}$, (2) $m_{\tilde{g}} \gg m_{\tilde{t}}$, etc.
- $\overline{\text{DR}}$ and $\overline{\text{MDR}}$
- new 3-loop results (semi-numerical) for general mass hierarchies [Reyes, Fazio, '19]

Two-loop self-energies (diagrammatic or effective potential): [Slavich, '01], [Martin, '01] , [Degrassi, Di Vita, Slavich, '14], [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, '14], [...]

- $g_1, g_2, p^2 \to 0$
- $p^2 \neq 0$: $m_h^{p^2=0} - m_h^{p^2\neq 0} \approx 100 - 500\,\mathrm{MeV}$
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One-loop self-energies:

- complete in all concerns

[find t MSSM higgs mass]

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Status in the MSSM: EFT

Benefit from FO calculations:

- Pro: Hybrid approaches allow to "recycle" FO results
  $\rightarrow$ 3-loop pole-mass matching + 4-loop RGEs (N^3LL)
  [Harlander, Klappert, Ochoa, Voigt, '19]
- Con: applicable for high-scale SUSY
  i.e. one light Higgs but nothing else

Dedicated matching of scalar couplings:

- one-loop:
  - real 2HDM [Haber, Hempfling, '93]
  - complex 2HDM [Gorbahn, Jager, Nierste, Trine, '09 [Murphy, Rzehak, '19]
  - generic [Gabelmann, Muehlleitner, Staub, '18]
  - matching extended 2HDM Higgs-masses, Split-SUSY with light fermions, ...
- two-loop λ_{SM}:
  - leading QCD [Bagnaschi, Slavich, '17]
  - mixed QCD-EW, combined with 3-loop hybrid [Bagnaschi, Degrassi, Passehr, Slavich, '19]
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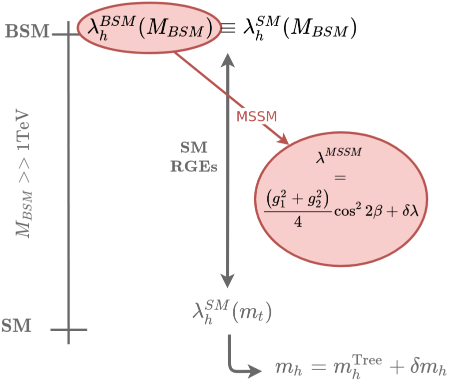

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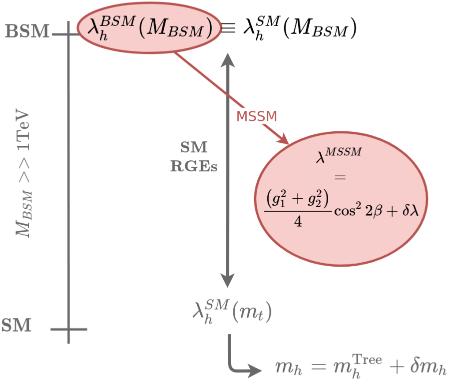

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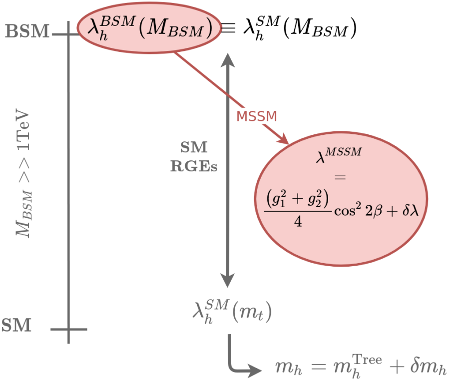

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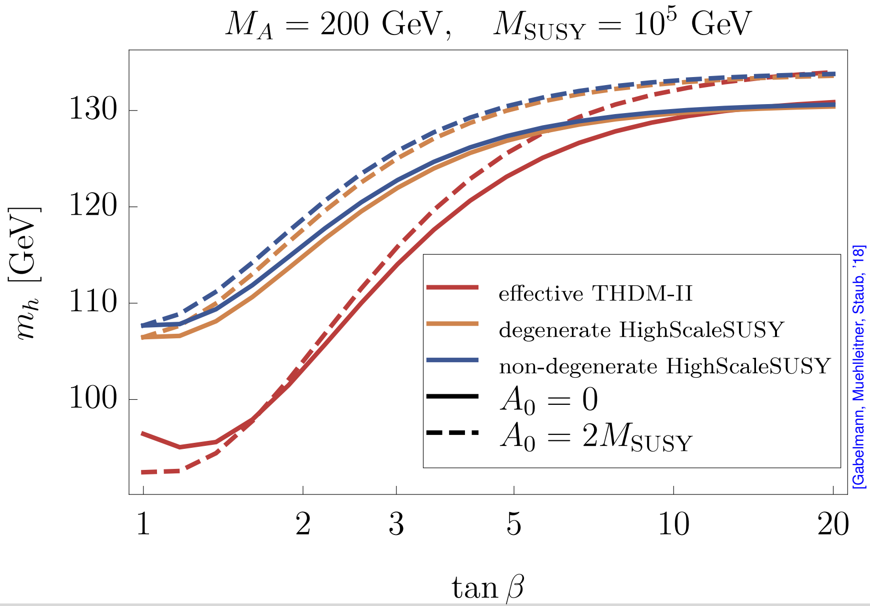
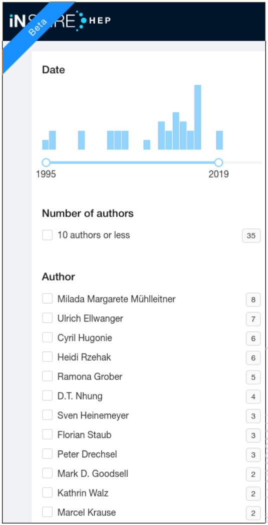


Status in the NMSSM: Fixed Order

Full one-loop corrections are available in $\overline{\text{DR}}$ as well as OS scheme [Baglio, Grober, Muhlleitner, Dao, Rzehak, Spira, Streicher, Walz, '13].
Two-loop corrections:

- again p^2, g_1, $g_2 \to 0$
- full $\overline{\text{DR}}$ with CPV and RPV [Goodsell, Staub, '16]
- partial OS: $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$ with CPV [Dao, Groeber, Krause, Muehlleitner, Rzehak, '19]



[find t NMSSM higgs mass]

Remember: In the **MSSM** Higgs-self couplings are given by D-terms.
Gaugeless-limit $\to$ **no two-loop diagrams with Higgs-self-couplings**!
In the NMSSM, we have additional F-terms:

$$W_{NMSSM} = W_{MSSM} + \lambda S H_u H_d + \kappa S^3$$

$$V_{NMSSM} \supset \sum_{\Phi = H_{u,d}, S} \left| \frac{\partial W}{\partial \Phi} \right|^2$$

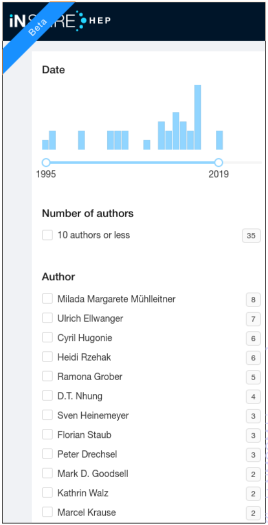
The NMSSM introduces many new couplings between S, H_u, H_d and the (s)fermion sector which are independent of gauge couplings!

$\to$ Many new **two-loop diagrams with Higgs-self-couplings**.
Leading contributions $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ are work-in-progress and subject to the s.c. *Goldstone Boson Catastrophe* (more later).

Status in the NMSSM: Fixed Order

Full one-loop corrections are available in $\overline{\text{DR}}$ as well as OS scheme [Baglio, Grober, Muhlleitner, Dao, Rzehak, Spira, Streicher, Walz, '13].
Two-loop corrections:

- again p^2, g_1, $g_2 \to 0$
- full $\overline{\text{DR}}$ with CPV and RPV [Goodsell, Staub, '16]
- partial OS: $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$ with CPV [Dao, Groeber, Krause, Muehlleitner, Rzehak, '19]



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Remember: In the **MSSM** Higgs-self couplings are given by D-terms.
Gaugeless-limit $\to$ **no two-loop diagrams with Higgs-self-couplings**!
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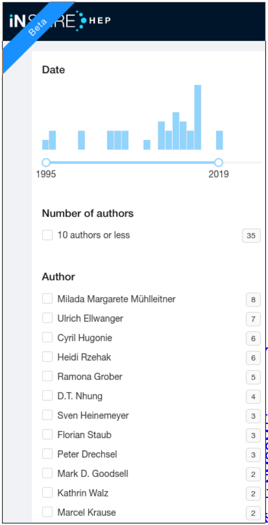

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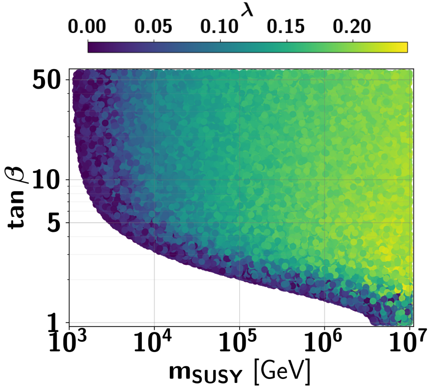

($\hat{X}_t = \sqrt{6}$, $\kappa = \frac{\lambda}{2}$ all other SUSY masses degenerate)

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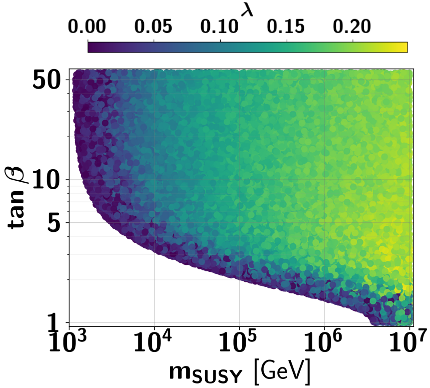

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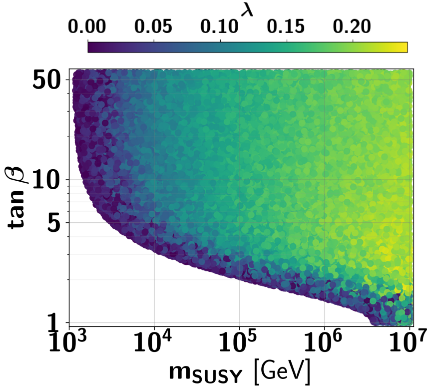

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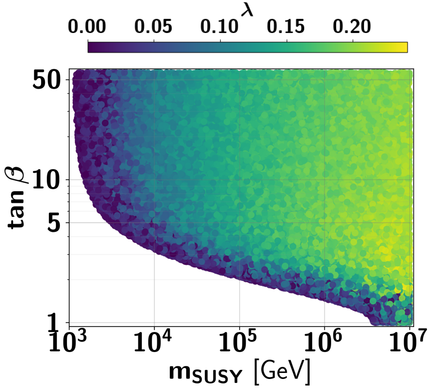

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Two-Loop Diagrammatic n-Point Functions ($n \leq 2$)

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

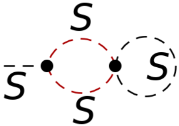
Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

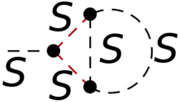
Then:

- NMSSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules
- evaluate numerics

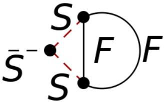
Generic Two-Loop Tadpoles I



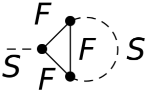
$$= -\frac{c_{s_0 \bar{s}_1 \bar{s}_2} c_{s_1 s_2 \bar{s}_3 s_3}}{1024\pi^4} \mathbf{A}_0(m^2_{s_3}) \mathbf{B}_0(m^2_{s_1}, m^2_{s_2}, p^2 = 0) \xrightarrow{m^2_{s_{1,2}} \to 0} \lim_{\delta \to 0} \log \delta$$

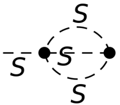


$$= \frac{i c_{s_0 s_1 s_4} c_{\bar{s}_1 s_2 s_3} c_{\bar{s}_2 \bar{s}_3 \bar{s}_4}}{1024\pi^4} \mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{s_3}, m^2_{s_2}, p^2 = 0) \xrightarrow{m^2_{s_{1,4}} \to 0} \lim_{\delta \to 0} \log \delta$$



$$\begin{aligned} = \frac{i c_{s_0 s_1 s_4}}{1024\pi^4} \Big[& (c^L_{\bar{f}_3 \bar{f}_2 \bar{s}_4} c^R_{f_2 f_3 \bar{s}_1} + c^L_{f_2 f_3 \bar{s}_1} c^R_{\bar{f}_3 \bar{f}_2 \bar{s}_4}) [(\mathbf{A}_0(m^2_{f_2}) + \mathbf{A}_0(m^2_{f_3})) \mathbf{B}_0(m^2_{s_1}, m^2_{s_4}, 0) \\ & - \mathbf{S}(m^2_{f_3}, m^2_{f_2}, m^2_{s_1}, 0) + (m^2_{s_4} - m^2_{f_3}) \mathbf{U}(m^2_{s_4}, m^2_{s_1}, m^2_{f_3}, m^2_{f_2}, 0)] \\ & - m^2_{f_2} \mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{f_3}, m^2_{f_2}, 0)(c^L_{\bar{f}_3 \bar{f}_2 \bar{s}_4} c^R_{f_2 f_3 \bar{s}_1} + c^L_{f_2 f_3 \bar{s}_1} c^R_{\bar{f}_3 \bar{f}_2 \bar{s}_4}) \\ & - 2 m_{f_3} m_{f_2} \mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{f_3}, m^2_{f_2}, 0)(c^R_{f_2 f_3 \bar{s}_1} c^R_{\bar{f}_3 \bar{f}_2 \bar{s}_4} + c^L_{f_2 f_3 \bar{s}_1} c^L_{\bar{f}_3 \bar{f}_2 \bar{s}_4}) \Big] \end{aligned}$$





Intermezzo: Effective Potential

No rigorous introduction here! (e.g. Peskin, Schroeder)

Consider quantum correction to classical action $S[\Phi_c]$:

$$W[J, \Phi_c = \Phi_0] = S[\Phi_0] + \underbrace{\int \frac{d^4 k}{(2\pi)^4} \log\left[-k^2 + V''(\Phi_0)\right]}_{\delta^{(1)} V_{eff}\text{, one-loop Coleman-Weinberg effective potential}}$$

- $\frac{\partial^{(n)} \delta V_{eff}}{\partial \Phi_1 \ldots \partial \Phi_n}$ equivalent to scalar n-point function **with zero external momentum**
- typically calculated in minimal subtraction scheme
- e.g. $\delta V^{(1)} = \frac{1}{4} \sum_n (-1)^{2s_n} (2s_n + 1)(m_n^2)^2 \left(\log \frac{m_n^2}{Q^2} - k\right)$
  - already renormalized (UV-finite)!
  - k accounts for $\overline{\text{MS}}$ ($\frac{5}{6}$) or $\overline{\text{DR}}$ ($\frac{3}{2}$)
  - closed form encoding all loop-corrections
  - known up to three-loop order

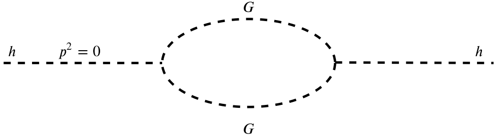
Neat tool to calculate higher-order corrections to scalar quantities (?)

Intermezzo: Goldstone Boson Catastrophe in the Effective Potential

The effective potential needs to be summed over all possible masses (expressed as functions of the field or its VEV). E.g. at one-loop in the SM we have:

$$\delta^{(1)} m_h^2 = \frac{\partial^2}{\partial^2 v}\delta^{(2)} V_{eff} \propto \frac{\partial^2}{\partial^2 v}\sum_n m_n(v)^4 \log \frac{m_n(v)^2}{Q^2} \supset v^2\lambda^2 \log \frac{m_G^2}{Q^2}$$

where m_G is the Goldstone mass. This is equivalent to the diagrammatic calculation:

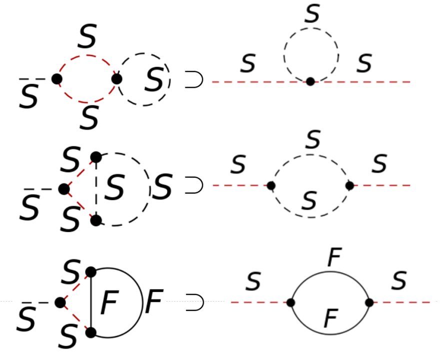


$$\lambda^2 v^2 \int dq^4 \frac{1}{\left(q^2 - m_G^2\right)^2} \equiv B_0(m_G^2, m_G^2, p^2 = 0) = v^2\lambda^2 \log \frac{m_G^2}{Q^2} + \frac{1}{\epsilon}$$

Attention: For a vanishing $m_G \to 0$ this is IR-divergent (*"Goldstone Boson Catastrophe"*)!

Solutions:
- Use external momentum [Braathen, Goodsell, '16] [Braathen Goodsell, Staub, '17]
- OS Goldstone mass [Braathen, Goodsell, '16] [Braathen Goodsell, Staub, '17]
- Resummation [Miro, Espinosa, Konstandin, '14] [Martin, '14] [Epsinosa, Garny, Konstandin, '16]

The Goldstone Boson Catastrophe in the Two-Loop Tadpoles



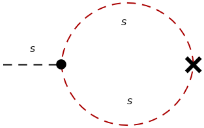
are all IR-divergent $\propto \log \frac{m_G^2}{Q^2}$, caused by Goldstone self-energy sub-graphs.

However: they are manifestly momentum-independent:

no momentum regularisation of IR-divergencies possible!

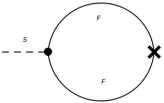
Generic Two-Loop Tadpoles II

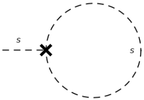
When working with **unrenormalised** loop-functions (in contrast to the renormalised V_{eff}), we also need to take into account:

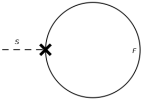


$$= -\frac{c_{s_0 s_1 s_2}}{32\pi^2}\Big[\Big(\mathbf{A}_0(m^2_{s_2}) - m^2_{s_1}\mathbf{B}_0(m^2_{s_1}, m^2_{s_2}, p^2 = 0)\Big)\delta^{(1)} Z_{\bar{s}_1 \bar{s}_2} + \mathbf{B}_0(m^2_{s_2}, m^2_{s_1}, p^2 = 0)\delta^{(1)} c_{\bar{s}_1 \bar{s}_2}\Big]$$

and



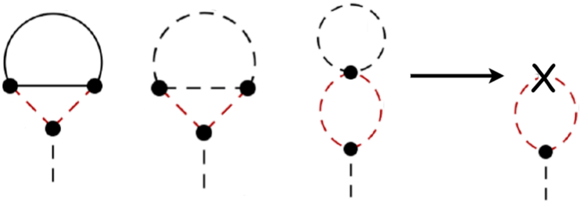




The first diagram is also IR-divergent.
Furthermore, it is conntected to the previous diagrams by the BPHZ formalism!

IR-Finite Two-Loop Tadpoles

Careful isolation of all IR-divergences and taking into account CT-diagrams leads to IR-finiteness!



All two-loop IR-divergences are cancelled by they one-loop sub-graphs.

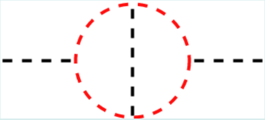
This mechanism is similar to the R-operation in the BPHZ formalism.

Outlook: Two-Loop Self-Energies

- Tadpoles: ✓
- Charged Higgs: proved cancellations within sub-graphs of the same topology ✓
- Neutral Higgs: *partially*
- use dim. reg. instead of mass regulator: check for BPHZ-IR

True IR-Divergences

Diagrams like:



are UV-finite (sub-graphs are three-point functions)
$\rightarrow$ no CT diagram to cancel!
Need (partial) momentum dependence (similar to one-loop case).

Far Outlook

p^2- and Gauge Contributions

Were e.g. recently calculated for general ren. theory in $\overline{\text{MS}}/\overline{\text{DR}}$ [Goodsell, Passehr, '19]. However, need p^2-dependence of loop integrals

- TSIL (integrates numerically): relatively slow
- maybe expansion in small p^2?

Hybrid Calculation

Use new results of FO calculation

- RGEs already (partially) implemented in NMSSMCalc
- needs SM Higgs pole-mass (rather straight forward)
- careful parametrisation of pole-masses

Precise prediction for stop-masses above $\approx 1 - 2\,\mathrm{TeV}$

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Summary

"New" solution to the Goldstone "Catastrophe":

- IR-divergences partially cured by BPHZ
- left-over divergences: requires momentum regulation

Allow for precise Higgs mass predictions:

- MSSM: further accuracy requires momentum/gauge dependence (GBC re-appears!)
- NMSSM: mixed $\overline{\text{DR}}$-OS calculation on its way to full two-loop precision
  - two-loop Tadpoles ✓
  - two-loop charged Higgs self-energy ✓
  - possible input for pole-mass matching