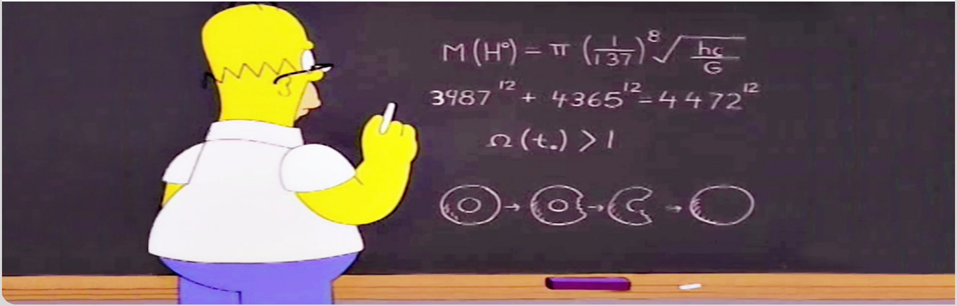
Karlsruhe Institute of Technology

Two-Loop Higgs Boson Masses in the CP-Violating NMSSM

Nhung Dao, **Martin Gabelmann**, Margarete Mühlleitner, and Heidi Rzehak | 16.03.2021

DPG DORTMUND



Why SUSY?

Consider heavy BSM loop corrections to vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  which symmetry protects m_h?

  $\to$ SUSY

Other important buzzwords: GUT, Dark Matter, radiative EWSB, SUGRA...

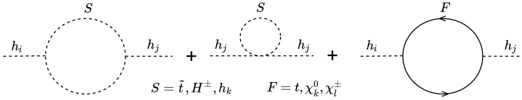
Introduction: (N)MSSM Lightest Neutral Higgs Mass

$$\left(m_h^{\text{tree}}\right)^2 = \underbrace{m_Z^2 \cos^2 2\beta}_{\text{(N)MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z$
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass. At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ are:



$$\approx \frac{1}{(4\pi)^2} \frac{m_t^4}{v^2} \log \frac{m_t^2}{m_{\text{SUSY}}^2 + m_t^2}$$

- In the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$! $\rightarrow$ higher-orders required

Status in the NMSSM (fixed-order)

Two-loop corrections:

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- full $\overline{\text{DR}}$ with CPV and RPV (SARAH) [Goodsell, Staub, '16]
- mixed $\overline{\text{DR}}$−OS top/stop sector and charged Higgs boson and OS VEV: $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$ with CPV (NMSSMCALC) [Dao, Groeber, Krause, Muehlleitner, Rzehak, '19]
- $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ with non-zero p^2 work-in-progress (NMSSMCALC, this talk).

Attention: In the **MSSM**, Higgs-self couplings are given by gauge couplings (D-terms).
In the NMSSM, we have additional non-zero couplings (F-terms):

$$V_{\text{MSSM}}^{\text{quartic}} \propto g_1^2(|H_u|^2 - |H_d|^2)^2 + g_2^2(H_u\sigma_a H_u + H_d\sigma_a H_d)^2 \to 0$$
$$V_{\text{NMSSM}}^{\text{quartic}} \propto V_{\text{MSSM}}^{\text{quartic}} + |\lambda H_u H_d + \kappa S^2|^2$$

→ Many new **two-loop diagrams with Higgs-self-couplings**.
Massless Goldstones → appearance of IR-divergences (*Goldstone Boson Catastrophe*).

One-Loop corrections:

- all coupling and mass combinations, full p^2
- $\overline{\text{DR}}$ as well as mixed $\overline{\text{DR}}$−OS schemes [T. Graf, R. Grober, M. Muhlleitner, H. Rzehak, K. Walz, '12]

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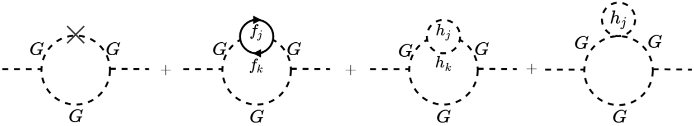
IR-Divergent (!) Two-Loop Selfenergies

Example of an **IR-divergent** subset:



- $\propto \log p^2 \xrightarrow{p^2 \to 0} \infty$
- $\rightarrow$ inclusion of finite external momentum required!
- **Solutions:**
  - use $M_{\text{Goldstone}} = 0 \rightarrow M_{\text{Regulator}}$ and check that $\partial m_h / \partial M^2_{\text{Regulator}}$ is small
  - use $p^2 \neq 0$ only in IR-divergent integrals and expand around small p^2

    [Braathen, Goodsell, '16]

    $\rightarrow$ avoids numerical integration methods.
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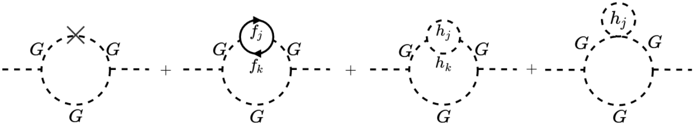

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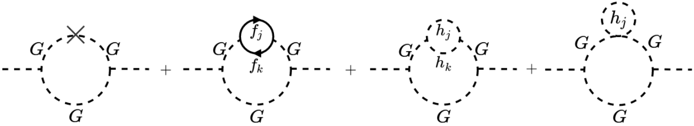

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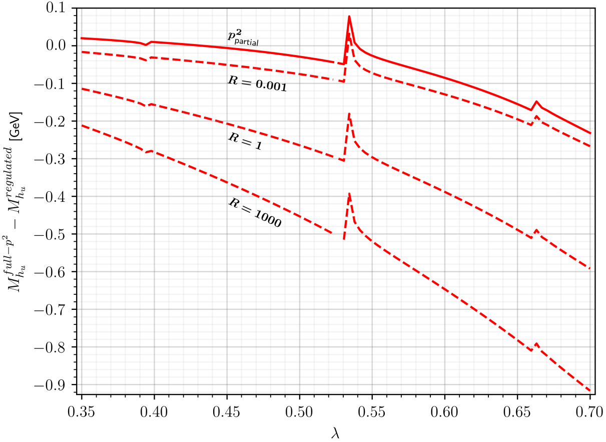
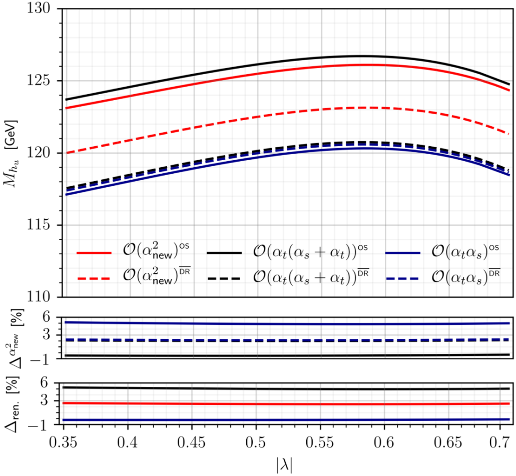
(Preliminary) Results: momentum dependence

Compare full-momentum result with small-momentum expansion (solid) and mass regulator (dashed, $M_R^2 = R\mu^2_{\text{Ren.}}$):



$\mu_{\text{Ren.}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 1.3\,\text{TeV}$, $\kappa = 0.43$, $\tan\beta = 3.7$, $M_{H^\pm}^{\text{OS}} = 640\,\text{GeV}$, $A_t^{\text{OS}} = 2\,\text{TeV}$

(Preliminary) Results: size of the corrections



(same parameter point as before)

$$\Delta^{\alpha_{\text{new}}} = \frac{M_{H_u}^{\alpha_{\text{new}}} - M_{H_u}^{\alpha_t(\alpha_s+\alpha_t)}}{M_{H_u}^{\alpha_{\text{new}}}} \approx -1\%$$

$$\Delta_{\text{ren.}} = \frac{M_{H_u}^{\text{OS}} - M_{H_u}^{\overline{\text{DR}}}}{M_{H_u}^{\overline{\text{DR}}}} \approx 3\%\ (6\%)\ (-1\%)$$

Short Outlook (wishlist)

Fixed-Order

- full p^2- **and** gauge contributions [Goodsell, Passehr, '19]
- combine with 3-loop MSSM results [Kant, Harlander, Mihaila, Steinhauser, '10]

EFT/Hybrid Calculation

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Summary

Higgs mass predictions being pushed by a very active community

Solved the Goldstone "catastrophe" in `NMSSMCALC`:

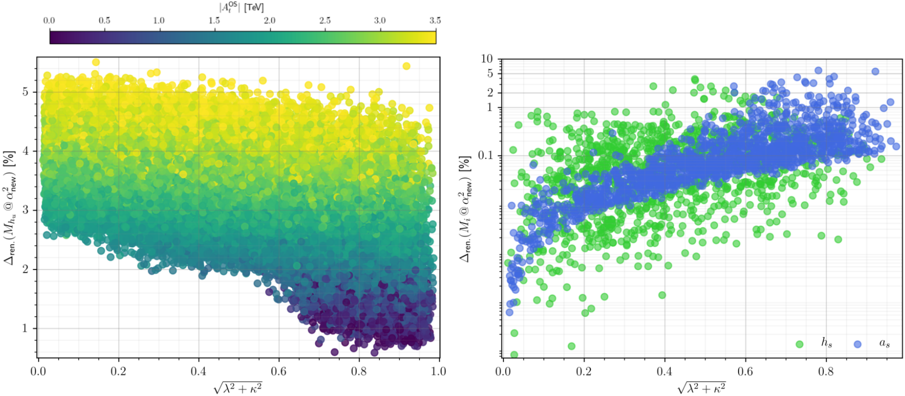
- two different IR-regulators
- impact much smaller than renormalization-scheme uncertainty and overall two-loop corrections

Allows for precise Higgs mass predictions:

- mixed $\overline{\text{DR}}$-OS calculation on its way to full two-loop precision
- further accuracy requires momentum/gauge dependence

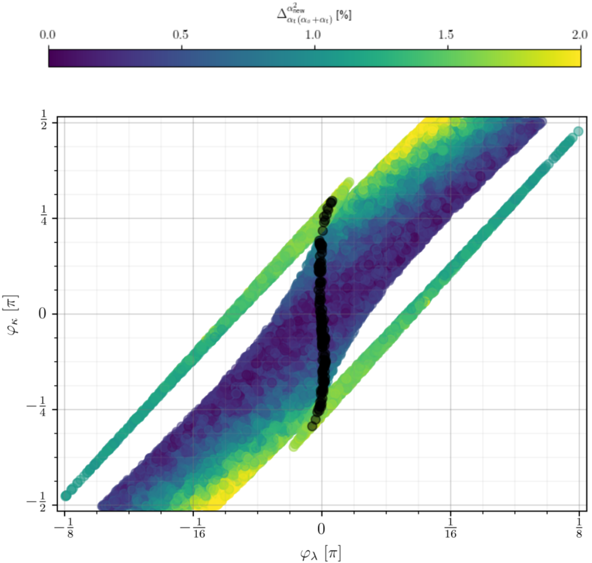
(Preliminary) Results: scheme dependence

random scan yields:



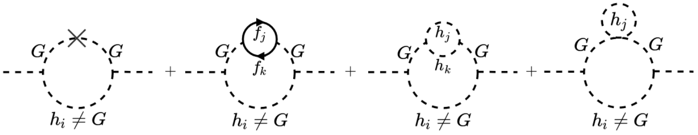
(Preliminary) Results: CPV Phases

Random scan over phases of previous parameter point:



IR-Finite Two-Loop Selfenergies

Example of an IR-finite subset with intermediate IR-divergences:



Careful isolation of divergences using mass regulator or dimensional regularisation shows:

- IR-divergence of first diagram cancels against the other three
- cancellation happens only if $M^{\text{1-loop}}_{\text{Goldstone}} \equiv 0$
- $\rightarrow$ working at the *tree-level* minimum is sufficient [this work] or alternatively using an OS-condition for the Goldstone mass [Braathen, Goodsell, '16]

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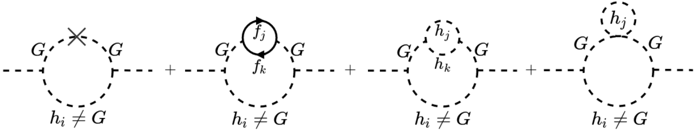

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Two-Loop Diagrammatic n-Point Functions ($n \leq 2$)

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

Then:

- NMSSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules
- evaluate numerics

Two-Loop $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ Corrections

Ingredients:

- numerically solve:
  $$det\left[p^2 \mathbb{1}_{5x5} - m_h^2 + \hat{\Sigma}_h^{(1)}(p^2) + \hat{\Sigma}_h^{(2)}(0)|_{g_{1,2}=0}\right] = 0$$
- renormalized self-energy:
  $$\hat{\Sigma}_h^{(2)}(p^2) = \Sigma_h^{(2)}(0) - \delta^{(2)} m_h^2$$
- two-loop mass matrix counter-term:
  $$\delta^{(2)} m_h^2 = \mathcal{O}\left((\delta^{(1)} Z_h)^2 + \delta^{(1)} m_h \delta^{(1)} Z_h\right) + \left(\delta^{(2)} Z_h^\dagger m_h^{(0)} + m_h^{(0)} \delta^{(2)} Z_h\right)$$
  $$+ m_h^{(0)}(\alpha \to \alpha + \delta^{(1)}\alpha + \delta^{(2)}\alpha)|_{\alpha=0}, \ \alpha = \{\lambda, \kappa, v, v_S, t_i, ...\}$$
- pure $\overline{\text{DR}}$: $\delta\alpha$ are purely divergent, canceled by wave-function ren. (SUSY-non ren.)
- OS quantities (e.g. $\delta v \leftrightarrow \delta m_{W,Z}$, or δt_i) : can yield extra finite contributions to $\delta^{(2)} m_h^2$

Most manpower required for (decreasing complexity):

- two-loop wave-function ren. constants $\delta^{(2)} Z_h = \left.\frac{\partial \Sigma_h(p^2)}{\partial p^2}\right|_{p^2=0}$
- two-loop vector self-energy diagrams $\delta^{(2)}\Sigma_V(0)$ (required for $\delta^{(2)} v$)
- two-loop scalar self-energy diagrams $\delta^{(2)}\Sigma_h(0)$
- two-loop tadpole diagrams $\delta^{(2)} t_i$

The same discussion applies for the charged Higgs bosons, $h \to H^\pm$.

The N/MSSM Scalar Potential

...is encoded in the superpotential $W \equiv W(\Phi)$, dim$W$=3:

$$W_{MSSM} = \mu \hat{H}_u \hat{H}_d + Y_L \hat{L} \hat{H}_u \hat{e} + Y_u \hat{Q} \hat{H}_u \hat{u} + Y_d \hat{U} \hat{H}_d \hat{d}$$
$$W_{NMSSM} = W_{MSSM}\big|_{\mu=\lambda\hat{S}} + \kappa \hat{S}^3$$

and constructed by:

$$V = \sum_{\Phi=H_{u,d},S,L,Q,u,d} \frac{\delta W}{\delta \Phi}\left(\frac{\delta W}{\delta \Phi}\right)^* + \frac{1}{2}(D_1 D_1 + D_2^a D_{2,a} + D_3^a D_{3,a})$$

with the $SU(3)xSU(2)xU(1)$ D-Terms:

$$D_1 = \frac{g_1}{2}(|H_u|^2 - |H_d|^2 + ...)$$
$$D_2^a = g_2(H_d^* \sigma^a H_d + H_u^* \sigma^a H_u + ...)$$
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Status in the MSSM (fixed-order)

Most precise results are based on **3-loop** self-energies&tadpoles: [Kant, Harlander, Mihaila, Steinhauser, '10]

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- consider strong sector only, $t,\ \tilde{t},\ g,\ \tilde{g}$ (up to m_t^4-terms)
- assume hierarchies, e.g. (1) $m_{\tilde{g}} \gg m_{\tilde{t}}$, (2) $m_{\tilde{g}} \gg m_{\tilde{t}}$, etc.
- $\overline{\text{DR}}$ and $\overline{\text{MDR}}$
- new 3-loop results (semi-numerical) for general mass hierarchies [Reyes, Fazio, '19]

Two-loop self-energies (diagrammatic or effective potential): [Slavich, '01], [Martin, '01] , [Degrassi, Di Vita, Slavich, '14], [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, '14], [...]

- $g_1, g_2, p^2 \to 0$
- $p^2 \neq 0$: $m_h^{p^2=0} - m_h^{p^2\neq 0} \approx 100 - 500\,\mathrm{MeV}$
- full mass hierarchies
- with CPV and RPV
- $\overline{\text{DR}}$ and OS conditions for $m_{\tilde{t}},\ m_t$ and $m_{H^\pm}$

m_h^{SM} VS. m_h^{SUSY} II

Is there (still) an advantage w.r.t. Higgs mass?

Given that **all** other parameters of the theory are fixed by experiment:

any model	SUSY
- m_h needs to be measured - unitarity: $m_h < 1\,\mathrm{TeV}$ - $\Delta\rho_{SM} \propto \log\left(\frac{m_h^2}{m_Z^2}\right) + \frac{m_t^2}{m_Z^2}$ - $114 < m_h^{2009} < 154\,\mathrm{GeV}$ [Gfitter]	- m_h **predicted** perturbatively - MSSM: gauge sector only - NMSSM: λ, κ constrained by SUSY non-renormalisation

SUSY is able to predict m_h^{theo} to *given* precision while the SM can at most extract m_h^{exp} from indirect fits. Thus, in general SUSY-relations can be falsified. Difficulty: many parameters to measure.