Karlsruhe Institute of Technology

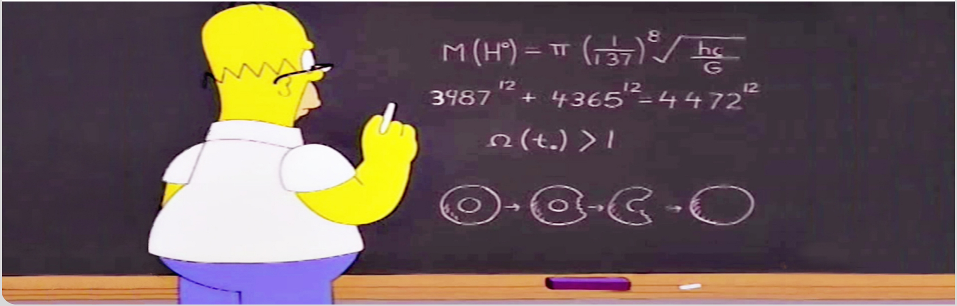
Two-Loop Higgs Boson Mass Corrections

in the CP-Violating Next-to-Minimal Supersymmetric Standard Model (NMSSM)

arxiv:2106.06990

Nhung Dao, **Martin Gabelmann**, Margarete Mühlleitner, and Heidi Rzehak | September 29th 2021

8TH KSETA PLENARY WORKSHOP



Outline

SUSY and the Higgs Boson Mass I

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu^2}$

  which symmetry protects m_h?

  $\to$ SUSY

Later: in SUSY $m_{\text{heavy}} \to m_{\text{SUSY}}$ (scale where SUSY is broken) and $\delta m_h \xrightarrow{m_{\text{SUSY}} \to 0} 0$.

SUSY and the Higgs Boson Mass II

How is this different compared to non-SUSY BSM?

SM / non-SUSY BSM	SUSY
m_h needs to be measured	m_h **predicted** perturbatively
unitarity: $m_h < 1\,\mathrm{TeV}$	$m_h(m_{\text{SUSY}})$ function of SUSY particle masses and couplings
$\Delta\rho_{SM} \propto \log\left(\frac{m_h^2}{m_Z^2}\right) + \frac{m_t^2}{m_Z^2}$	constraints on SUSY particle masses/decays translate to Higgs boson mass prediction (and vice versa)
$114 < m_h^{2009} < 154\,\mathrm{GeV}$ [Gfitter]	
$m_h^{2012} \approx 125\,\mathrm{GeV}$	

Test SUSY-relations at colliders:

$m_h^{\text{LHC}} \approx 125 \pm 0.2\,\mathrm{GeV}$ [ATLAS, CMS] $\quad ⚡ \quad m_h(m_{\text{SUSY}} \approx 1-2\,\mathrm{TeV}) \approx 125 \pm 3\,\mathrm{GeV}$

Precision predictions required in order to study SUSY parameter space.

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The CP-Violating NMSSM

The Complex Next-to-Minimal Supersymmetric Standard Model

- Singlet extension of the MSSM.
- Theoretically well-motivated (solves μ- and little-hierarchy-problem).
- Rich phenomenology in the Higgs boson sector:

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \; H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \; S = \frac{e^{i\varphi_s}}{\sqrt{2}} (v_s + h_s + i a_s)$$

mix to

$h_1, h_2, h_3, h_4, h_5, G^0$ (mass ordered) and $h^\pm, G^\pm$

LHC measurements: h_1 **or** h_2 play the role of the Higgs boson h measured at LHC (h_1 or h_2 are "SM-like"). MSSM: no CPV at tree-level and always $h_1 = h$.

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The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z < 125\,\mathrm{GeV}$ ↯
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass.
At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ from the top/stop sector are:

- $M_{\tilde{t}} = m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\mathrm{GeV})$! $\rightarrow$ higher-orders required

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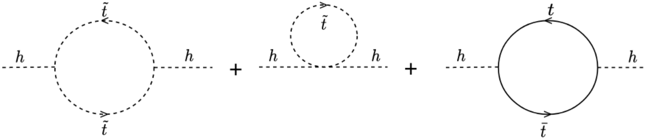

$$\approx \frac{1}{(4\pi)^2} \frac{m_t^4}{v^2} \log \frac{m_t^2}{M_{\tilde{t}}^2}$$

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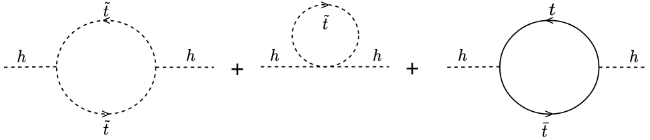

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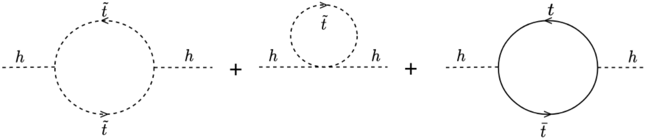

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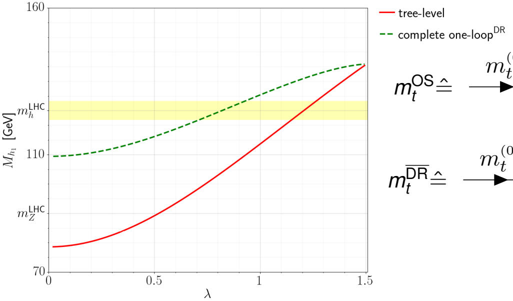
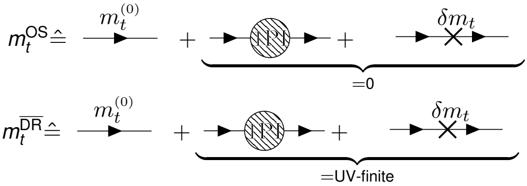
Example of Higher-Order Corrections: One-Loop

One-Loop corrections:

- all one-loop contributions are well-known
- how to estimate the uncertainty without calculating higher-orders?
  $\rightarrow$ using different renormalization conditions [cf. talk by Gudrun Heinrich]
- $\overline{\text{DR}}$ or OS renormalization of top & stop sector [Graf et al. '12]
  $\rightarrow$ estimate uncertainty due to missing higher-orders





$$m_h(m_t^{\text{OS}}) \overset{\text{all-orders}}{=} m_h(m_t^{\overline{\text{DR}}})$$

$\mu_{\text{Ren.}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} = 1.3\,\text{TeV}$, $\kappa = 0.43$, $\tan\beta = 3.7$, $M_{H^\pm}^{\text{OS}} = 640\,\text{GeV}$, $A_t^{\text{OS}} = 2\,\text{TeV}$

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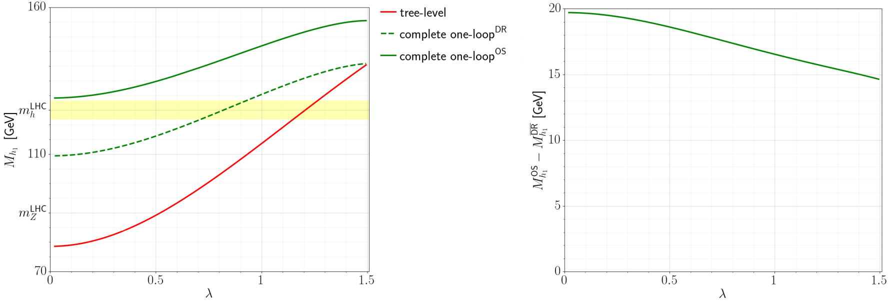

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Example of Higher-Order Corrections: Two-Loop

- gaugeless limit $g_1, g_2 \to 0$
- vanishing external momentum:
  $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s \alpha_t)$ mixed OS/$\overline{\text{DR}}$ [Dao et al. 14']
- $\mathcal{O}(\alpha_t^2)$ very sensitive to top mass
  $\to$ intruduces larger theoretical uncertainty [Dao et al. '19]
- $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$ can be large [Goodsell et al. '16]
  $\to$ uncertainty reduced?
- **this work**: mixed OS/$\overline{\text{DR}}$ renormalization [Dao et al. '21]
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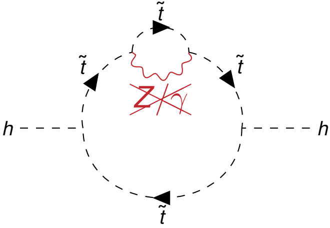

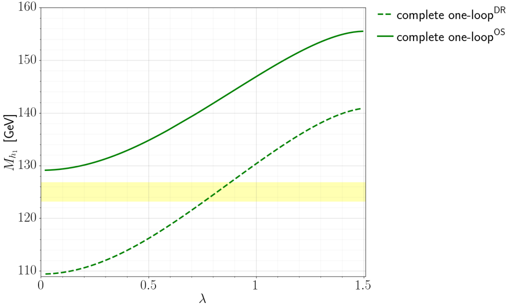

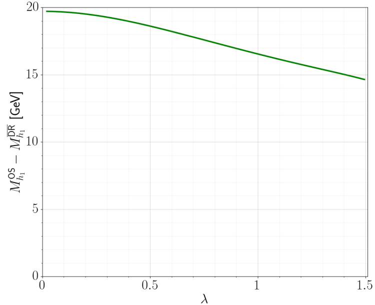

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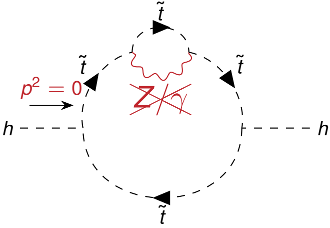

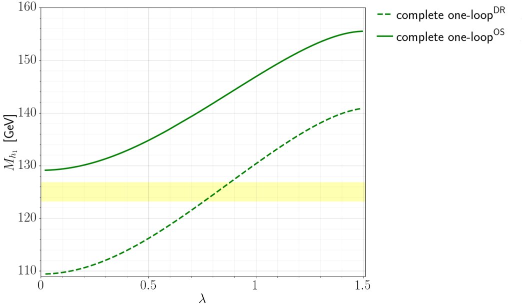

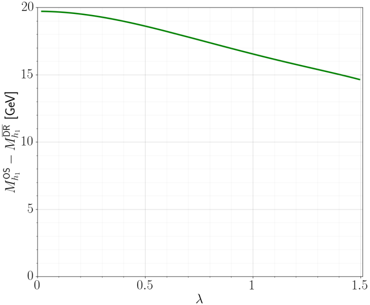

Example of Higher-Order Corrections: Two-Loop $\alpha_s\alpha_t \propto \frac{g_3^2 y_t^2}{16\pi^2}$

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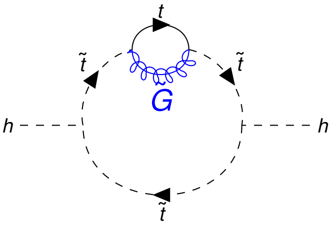

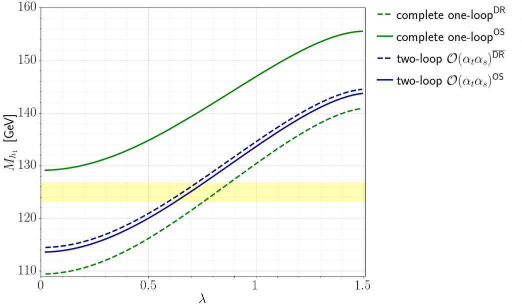

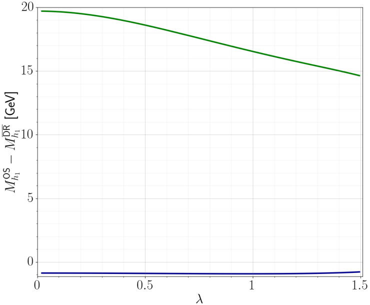

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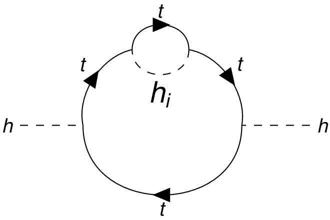

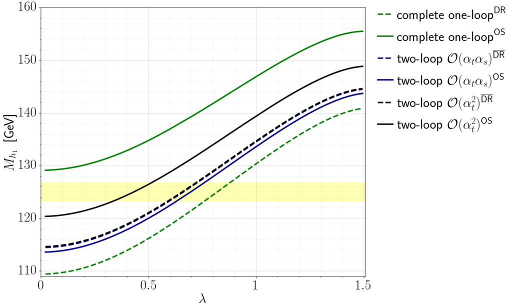

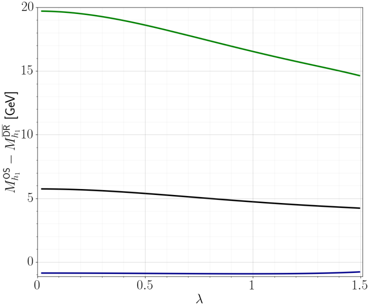

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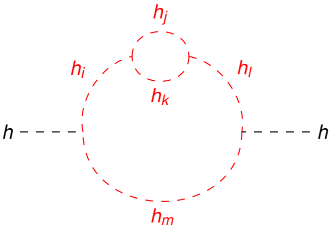

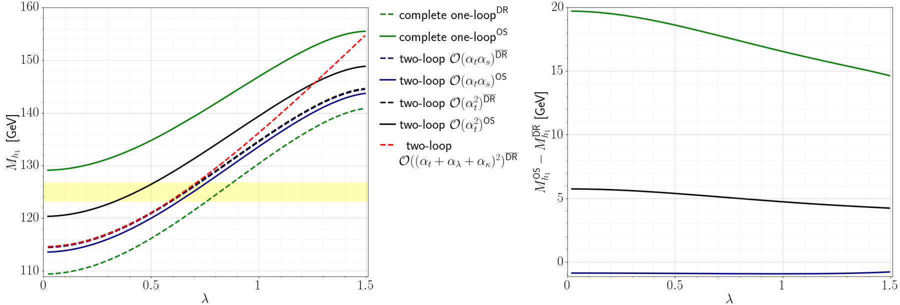

Example of Higher-Order Corrections: Two-Loop $\alpha_t\alpha_\lambda \propto \frac{y_t^2\lambda^2}{16\pi^2}$

- gaugeless limit $g_1, g_2 \to 0$
- vanishing external momentum:
  $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$ [Dao et al. 14']
- $\mathcal{O}(\alpha_t^2)$ very sensitive to top mass
  $\to$ intruduces larger theoretical uncertainty [Dao et al. '19]
- $\boldsymbol{\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)}$ can be large [Goodsell et al. '16]
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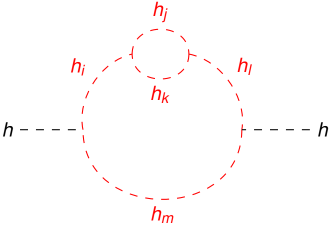

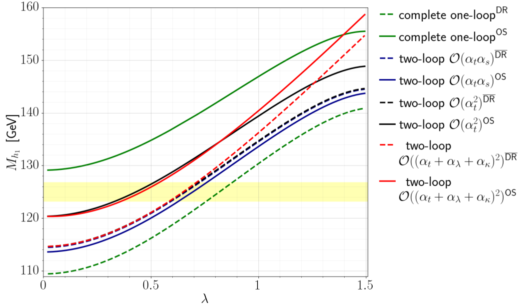

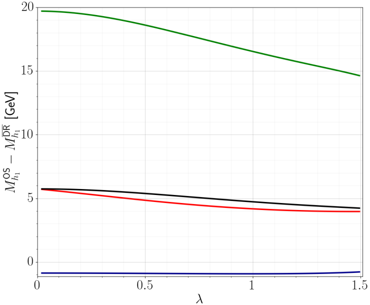

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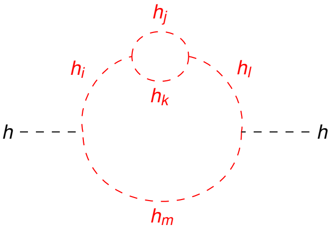
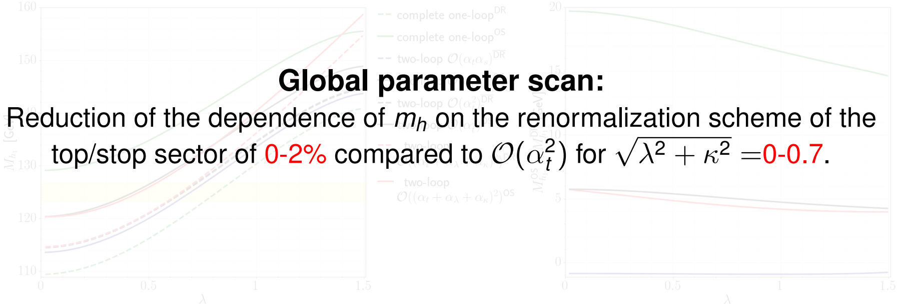
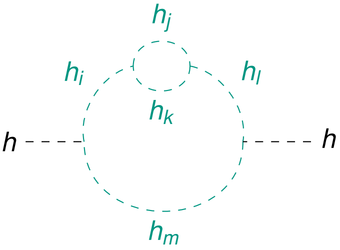




Global parameter scan:

Reduction of the dependence of m_h on the renormalization scheme of the top/stop sector of 0-2% compared to $\mathcal{O}(\alpha_t^2)$ for $\sqrt{\lambda^2 + \kappa^2}$ =0-0.7.

Comparison with MSSM/previous Results



$\xrightarrow{g_1, g_2 \to 0} 0$

In the MSSM, Higgs-self couplings are given by gauge couplings:

$$V^{\text{quartic}}_{\text{MSSM}} \propto \boldsymbol{g_1}^2(|H_u|^2 - |H_d|^2)^2 + \boldsymbol{g_2}^2(H_u\sigma_a H_u + H_d\sigma_a H_d)^2 \xrightarrow{g_1, g_2 \to 0} 0$$

In the NMSSM, there are additional non-zero self-couplings:

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$\to$ Many new **two-loop diagrams with Higgs-self-couplings**.
Massless Goldstones $\to$ appearance of IR divergences.

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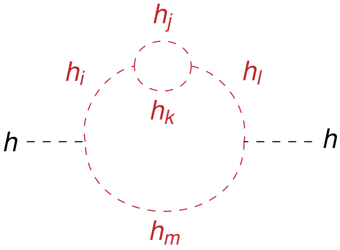

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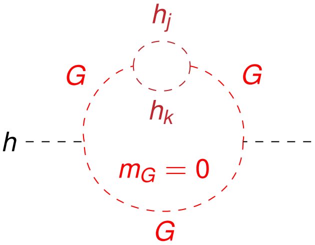

$$\propto \log p^2 \xrightarrow{p^2 \to 0} -\infty \qquad \lightning$$

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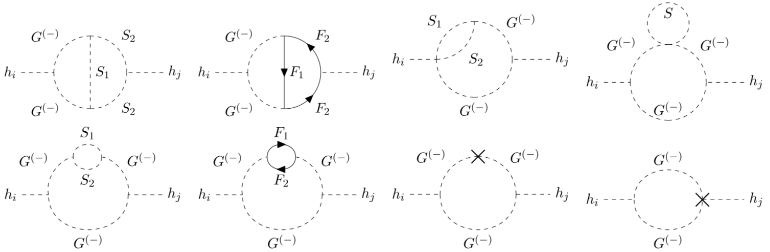
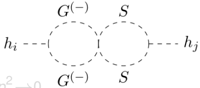
IR-Divergent Two-Loop Selfenergies





$$\{S_1,S_2\} = \{h,h\},\{\tilde{t},\tilde{t}\}(,\{\tilde{d},\tilde{t}\},\{h,h^-\},\{h^-,h\})$$
$$\{F_1,F_2\} = \{\chi,\chi\},\{t,t\}(,\{d,t\},\{\chi,\chi^-\},\{\chi^-,\chi\})$$
$$S = \{h,h^-,\tilde{t},\tilde{d}\}$$

- $\propto \log p^2 \xrightarrow{p^2\to 0} -\infty$
- $\to$ inclusion of finite external momentum required!
- **Solutions:**
  - assume $p^2 \neq 0 \to$ multi-scale problem (numerical integration required)
  - use $M_{\text{Goldstone}} = 0 \to M_{\text{Regulator}}$; test if $\partial m_h / \partial M^2_{\text{Regulator}}$ is small
  - assume partial $p^2 \neq 0$; only in IR-divergent diagrams [Braathen, Goodsell, '16] $\to$ avoids numerical integration methods.

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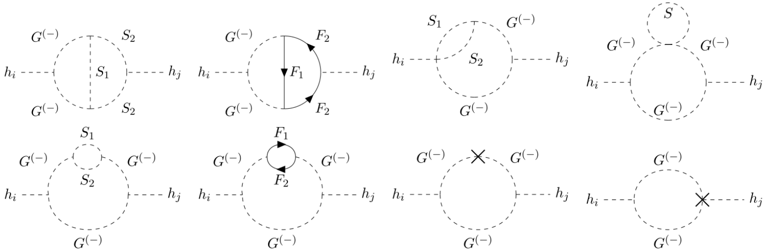

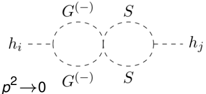

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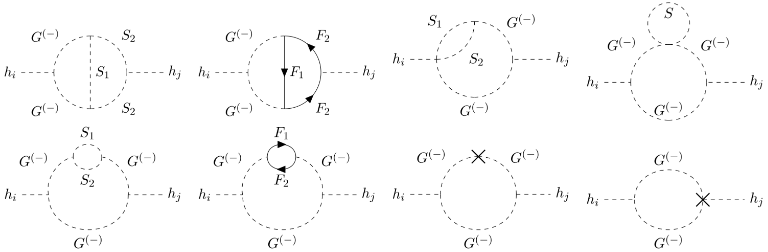

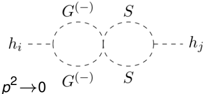

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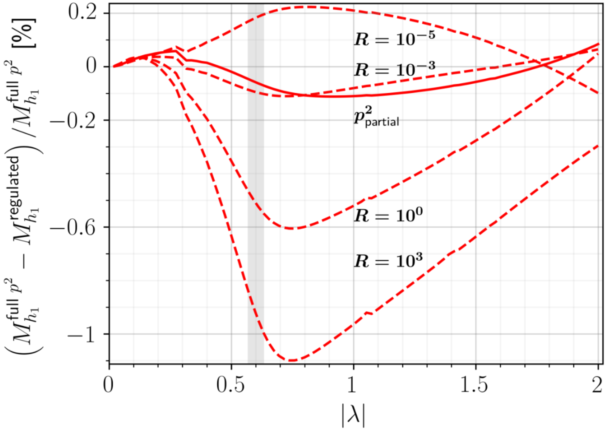
Example: momentum dependence

Compare **full-momentum** result with **partial-momentum** (solid) and **mass-regulator** (dashed, $M^2_{\text{Regulator}} = R\mu^2_{\text{Ren.}}$):



Short Outlook

Higgs mass predictions being pushed by a very active community

- full p^2- **and** gauge-contributions [Goodsell, Passehr, '19]
- combine with 3-loop MSSM results [Kant, Harlander, Mihaila, Steinhauser, '10] $\rightarrow$ reduce uncertainty further (?)
- better treatment of m_t
- need to resum large logs if $m_{\text{SUSY}} > 1 - 2\,\mathrm{TeV}$

Summary

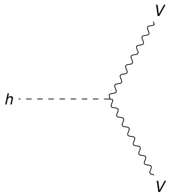
Solved the issue of IR-divergent self-energies and implemented them in the code `NMSSMCALC`:

- three different solutions to IR divergences
- impact much smaller than renormalization-scheme uncertainty and the overall two-loop corrections

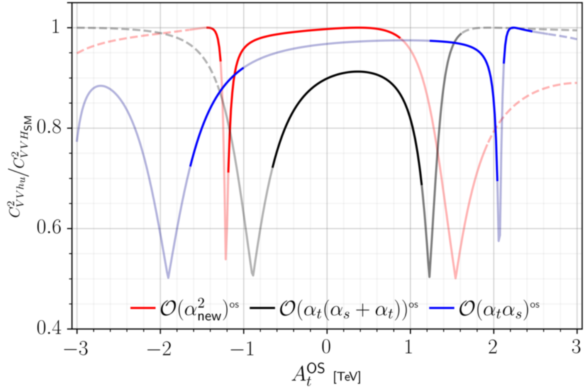
Allows for precise Higgs mass predictions:

- mixed $\overline{\text{DR}}$-OS calculation on its way to full two-loop precision
- further accuracy requires momentum/gauge dependence and more loops

Influence of Higgs Mass Prediction onto Collider Phenomenology



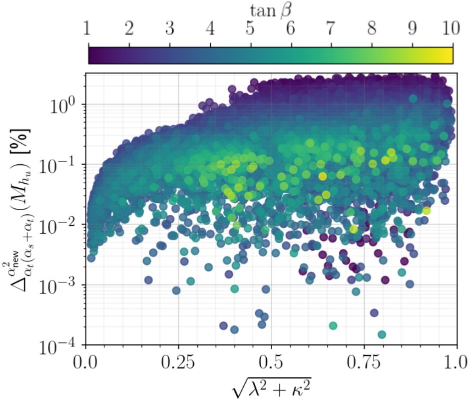
$= C_{VVh}, \quad V = Z, W^{\pm}$



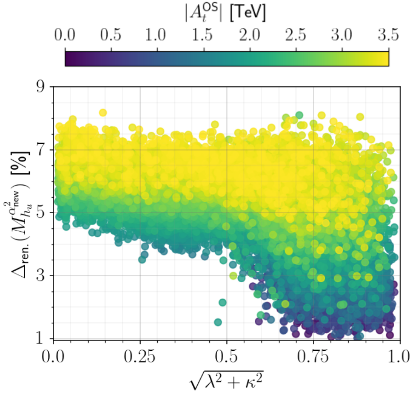
- transparent: either Higgs signal rates or Higgs boson mass not reproduced
- bold: all theoretical and experimental constraints fulfilled

Results: Global Scan

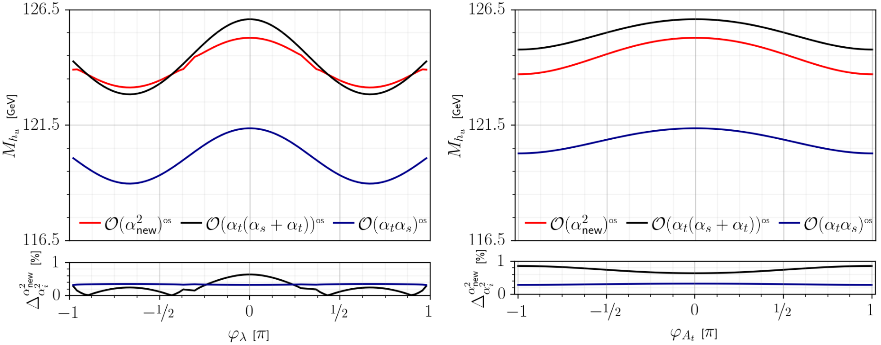
relative size



renormalization scheme dependence

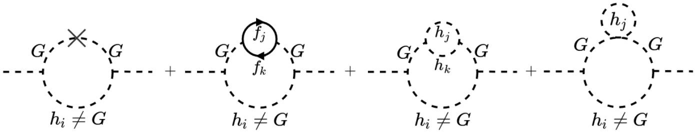


Results: CPV Phases



IR-Finite Two-Loop Selfenergies

Example of an IR-finite subset with intermediate IR-divergences:



Careful isolation of divergences using mass regulator or dimensional regularisation shows:

- IR-divergence of first diagram cancels against the other three
- cancellation happens only if $M^{\text{1-loop}}_{\text{Goldstone}} \equiv 0$
- $\rightarrow$ working at the *tree-level* minimum is sufficient [this work] or alternatively using an OS-condition for the Goldstone mass [Braathen, Goodsell, '16]

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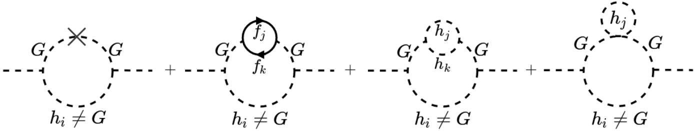

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Two-Loop Diagrammatic n-Point Functions ($n \leq 2$)

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

Then:

- NMSSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules
- evaluate numerics

Two-Loop $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ Corrections

Ingredients:

- numerically solve:
  $$det\left[p^2 \mathbb{1}_{5x5} - m_h^2 + \hat{\Sigma}_h^{(1)}(p^2) + \hat{\Sigma}_h^{(2)}(0)|_{g_{1,2}=0}\right] = 0$$
- renormalized self-energy:
  $$\hat{\Sigma}_h^{(2)}(p^2) = \Sigma_h^{(2)}(0) - \delta^{(2)} m_h^2$$
- two-loop mass matrix counter-term:
  $$\delta^{(2)} m_h^2 = \mathcal{O}\left((\delta^{(1)} Z_h)^2 + \delta^{(1)} m_h \delta^{(1)} Z_h\right) + \left(\delta^{(2)} Z_h^\dagger m_h^{(0)} + m_h^{(0)} \delta^{(2)} Z_h\right)$$
  $$+ m_h^{(0)}(\alpha \to \alpha + \delta^{(1)}\alpha + \delta^{(2)}\alpha)|_{\alpha=0}, \ \alpha = \{\lambda, \kappa, v, v_S, t_i, ...\}$$
- pure $\overline{\text{DR}}$: $\delta\alpha$ are purely divergent, canceled by wave-function ren. (SUSY-non ren.)
- OS quantities (e.g. $\delta v \leftrightarrow \delta m_{W,Z}$, or δt_i) : can yield extra finite contributions to $\delta^{(2)} m_h^2$

Most manpower required for (decreasing complexity):

- two-loop wave-function ren. constants $\delta^{(2)} Z_h = \left.\frac{\partial \Sigma_h(p^2)}{\partial p^2}\right|_{p^2=0}$
- two-loop vector self-energy diagrams $\delta^{(2)} \Sigma_V(0)$ (required for $\delta^{(2)} v$)
- two-loop scalar self-energy diagrams $\delta^{(2)} \Sigma_h(0)$
- two-loop tadpole diagrams $\delta^{(2)} t_i$

The same discussion applies for the charged Higgs bosons, $h \to H^\pm$.

Status in the MSSM (fixed-order)

Most precise results are based on **3-loop** self-energies&tadpoles: [Kant, Harlander, Mihaila, Steinhauser, '10]

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- consider strong sector only, $t,\ \tilde{t},\ g,\ \tilde{g}$ (up to m_t^4-terms)
- assume hierarchies, e.g. (1) $m_{\tilde{g}} \gg m_{\tilde{t}}$, (2) $m_{\tilde{g}} \gg m_{\tilde{t}}$, etc.
- $\overline{\text{DR}}$ and $\overline{\text{MDR}}$
- new 3-loop results (semi-numerical) for general mass hierarchies [Reyes, Fazio, '19]

Two-loop self-energies (diagrammatic or effective potential): [Slavich, '01], [Martin, '01] , [Degrassi, Di Vita, Slavich, '14], [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, '14], [...]

- $g_1, g_2, p^2 \to 0$
- $p^2 \neq 0$: $m_h^{p^2=0} - m_h^{p^2\neq 0} \approx 100 - 500\,\mathrm{MeV}$
- full mass hierarchies
- with CPV and RPV
- $\overline{\text{DR}}$ and OS conditions for $m_{\tilde{t}},\ m_t$ and $m_{H^\pm}$