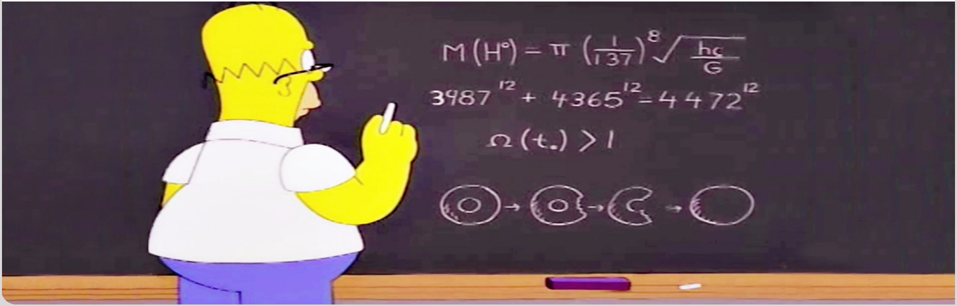
Karlsruhe Institute of Technology

Precision Higgs boson mass predictions in the (split) N/MSSM

Martin Gabelmann | 26.11.2020

TP2 WÜRZBURG - PARTICLE THEORY SEMINAR



Outline

1. Introduction to Supersymmetric Higgs Potentials
   - SUSY Motivations, Terminology and Examples
   - Higher-Order Corrections to m_h^2
2. Overview on Higher-Order Higgs-Mass Corrections
   - MSSM
   - NMSSM
3. Two-Loop fixed-order Corrections in the NMSSM
   - The Goldstone Boson ”Catastrophe”
   - Results

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m_h^{SM} VS. m_h^{SUSY}

Why is SUSY (*no longer?*) called a *beautiful* theory?

Consider loop correction to vector/fermion/Higgs mass:

- Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{Q^2}$

  protected by gauge symmetries, $m_V \to 0$

- Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{Q^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{Q^2}$

  which symmetry protects m_h?

  $m_{\text{heavy}} \to m_{\text{SUSY}}$

Other important buzzwords: GUT, Dark Matter, radiative EWSB, SUGRA...

Example: SUSY Toy-Model

see SUSY Primer by S.P. Martin

Consider the field content:

- gauge field G_μ^a
- scalar Φ

Needed SUSY partners:

- must be fermions: $n_F = n_B$
- Weyl fermions: $\tilde{G}^a, \tilde{\Phi}$
- $n_F^{Weyl} = 2$
- $\rightarrow \Phi$ must be complex

Gauge Superfield $\hat{G} = (G_\mu^a, \tilde{G}^a)$ and chiral superfield $\hat{\Phi} = (\Phi, \tilde{\Phi})$ share the same gauge/quantum numbers (respectively).

SUSY goes off-shell

- Higher-order corrections will involve off-shell states.
- $\overline{\sigma}^\mu \partial_\mu \tilde{\Phi} = 0 \rightarrow$ the SUSY algebra does not close off-shell!
- Idea: introduce auxiliary fields $\mathcal{L}_{aux} \supset FF^* + D^a D^a$ to match d.o.f.

	Φ	$\tilde{\Phi}$	F	G_μ^a	$\tilde{G}^a$	D^a
on-shell	2	2	0	2	2	0
off-shell	2	4	2	3	4	1

D-Terms

$$\mathcal{L}_{\text{gauge}} \supset g(\Phi^* T^a \Phi) D_a + \frac{1}{2} D^a D_a$$

$$\text{EOM} \rightarrow D^a = -g(\Phi^* T^a \Phi)$$

SUSY connects scalar with gauge sector

F-Terms

$$\mathcal{L}_{\text{chiral}} \supset -\frac{1}{2}\frac{\delta^2 W}{\delta \Phi^2} \tilde{\Phi}\tilde{\Phi} + \frac{\delta W}{\delta \Phi} F + F^* F$$

$$\text{EOM} \rightarrow F = -\frac{\delta W^*}{\delta \Phi}$$

SUSY connects Yukawa with scalar sector

The N/MSSM Scalar Potential

Encoded in the superpotential $W \equiv W(\Phi)$, $\dim W$=3:

$$W_{MSSM} = \mu \hat{H}_u \hat{H}_d + Y_L \hat{L} \hat{H}_u \hat{e} + Y_u \hat{Q} \hat{H}_u \hat{u} + Y_d \hat{U} \hat{H}_d \hat{d}$$
$$W_{NMSSM} = W_{MSSM}\big|_{\mu=\lambda\hat{S}} + \kappa \hat{S}^3$$

and constructed by:

$$V = \sum_{\Phi=H_{u,d},S,L,Q,u,d} \frac{\delta W}{\delta \Phi}\left(\frac{\delta W}{\delta \Phi}\right)^* + \frac{1}{2}(D_1 D_1 + D_2^a D_{2,a} + D_3^a D_{3,a})$$

with:

$$D_1 = \frac{g_1}{2}(|H_u|^2 - |H_d|^2) + \ldots$$
$$D_2^a = g_2(H_d^* \sigma^a H_d + H_u^* \sigma^a H_u) + \ldots$$
$$D_3^a = 0 \text{ (for this talk)}$$

Example: (N)MSSM Neutral Higgs Mass

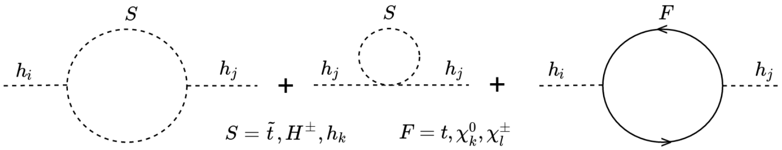
$$\left(m_h^{\text{tree}}\right)^2 = \underbrace{m_Z^2 \cos^2 2\beta}_{\text{D-Terms}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{F-Terms}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z$
- NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass.

At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ are:



- In the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$!

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Analytic structure of δm_h^{SUSY}

Perturbative series of δm_h will involve expansions in powers of:

- number of loops: $(4\pi)^{-2}$
- number of logs: $\log \frac{m_{\text{SUSY}}^2}{m_{\text{SM}}^2}$
- suppression by heavy scales: $\frac{m_{\text{SM}}^2}{m_{\text{SUSY}}^2}$
- coefficients/inputs: $C(m_t^{\overline{\text{DR}}}, X_t^{\overline{\text{DR}}}, ...) \leftrightarrow C(m_t^{\text{OS}}, X_t^{\text{OS}}, ...)$

Truncation at fixed-order leads to:

- dependence on renormalization scale (often m_h^{SUSY})
- schme dependence: only equivalent to all orders

However:

- If $m_{\text{SUSY}} \gg m_{\text{SM}}$, the expansion does not converge well.
  $\rightarrow$Need to resum large logs in EFT-framework.

Analytic structure of δm_h^{SUSY}: Fixed Order

A fixed-order *n*-loop result will incorporate the full logarithmic dependence $(4\pi)^{-2n}\sum_{k=0}^{n}\log^k$ (good and bad ones!) and constant $\frac{m_{\text{SM}}^2}{m_{\text{SUSY}}^2}$-terms:

- effective potential: $\partial^k_{h_{i_1},\dots,h_{i_k}} V_{eff}^{(n)} = G^{(n)}_{h_{i_1},\dots,h_{i_k}}(0)|_{\text{fin}}$
  - V_{eff} known up to 2- and 3-loops for general QFT [Martin, Patel, '18] [Martin, '17]
  - ∂V_{eff} numerically difficult
  - $p^2 = 0 \rightarrow$ massless particles are troublesome
- diagrammatic: calculate $\Sigma_{ij}^{(n)}(p^2)$ and $T_i^{(n)}$
  - computational expensive
  - mass hierarchies: expand loop-integrals
  - numerical evaluation: no access to logs, slow

Other ingredients:

- OS s/top sector: $(n-1)$-loop selfenergies
- OS v_{SM}: *n*-loop vector boson masses
- full *n*-loop SUSY RGEs from m_Z to $Q_{\text{Ren.}} \approx m_{\text{SUSY}}$ (thresholds!)

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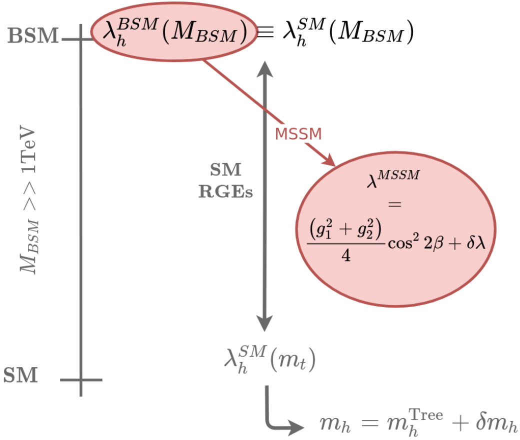
Analytic structure of δm_h^{SUSY}: EFT

Idea: avoid large logs by separation of SUSY/SM contributions.



missing: only resummation in leading logs, no p^2- and no $\frac{m^2_{\text{SM}}}{m^2_{\text{SUSY}}}$-dependence

attention: often $\lambda_h^{\text{BSM}}(\alpha_i^{\overline{\text{DR}}})$ but $\lambda_h^{\text{SM}}(\alpha_i^{\overline{\text{MS}}})$

Analytic structure of δm_h^{SUSY}: EFT vs. fixed-order

Common lore: *n-loop matching requires* $n+1$*-loop running*

0	α^0				$\alpha \propto (4\pi)^{-1}$
1	$\alpha \log$	α			
2	$\alpha^2 \log^2$	$\alpha^2 \log$	α^2		
	$\vdots$	$\vdots$	$\vdots$		
n	$\alpha^n \log^n$	$\alpha^n \log^{n-1}$	$\alpha^n \log^{n-2}$	$\dots$	α^n
	LL	NLL	NNLL	$\dots$	N^nLL

fixed-order (↓)

EFT (→)

However:
the two approaches are not as orthogonal as the table might suggest!

Hybrid Higgs Mass Calculations

→ combine FO and EFT results

- using pole mass matching [Athron, Park, Steudtner, Stöckinger, Voigt, '16] [Porod, Staub, '17]:
$\delta^{(1)}\lambda_{SM} = \frac{1}{v^2}\left(\delta^{(1)} m_h^{SUSY\,2} - \delta^{(1)} m_h^{SM\,2}\right)$
- combine $m^2_{\text{SM}}/m^2_{SUSY}$ from FO with resummed EFT results [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, '13] [Bahl, Hollik, '16] [Harlander, Klappert, Voigt, '19]
$m_h^2 \equiv \left(m_h^{\text{FO}}\right)^2 - \left(m_h^{\text{FO, large-logs}}\right)^2 + \left(m_h^{\text{EFT-resummed}}\right)^2$

[Harlander, Klappert, Voigt, '19]

Fixed Order (FO)	RGE Improved (EFT)	Ren. Conditions
Weak/TeV-scale SUSY	High/Split-scale SUSY	$\overline{\text{DR}}$: minimal subtraction
$m_{SUSY} \lesssim 1-2\,\text{TeV}$	$m_{SUSY} \gtrsim 1-2\,\text{TeV}$	OS: Express result through physical quantities
calculate full $\delta m_h^{SUSY}(m_{SUSY})$	matching, calculate $\delta\lambda_h^{\text{SUSY}}$	scheme dependence due to missing higher orders
full $\frac{m_{\text{SM}}}{m_{\text{SUSY}}}$ dependence	neglects $\frac{m_{\text{SM}}}{m_{\text{SUSY}}}$-terms	$m_h(m_t^{\overline{DR}}) \leftrightarrow m_h(m_t^{OS})$

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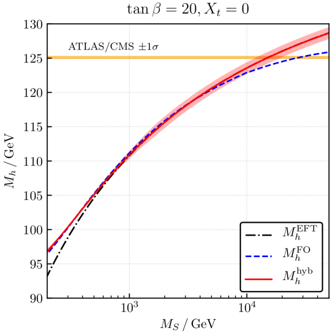

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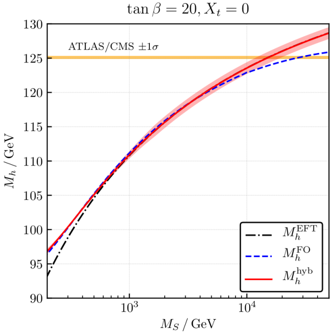

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Status in the MSSM: Fixed Order

Most precise results are based on **3-loop** self-energies&tadpoles: [Kant, Harlander, Mihaila, Steinhauser, '10]

- gaugeless limit $g_1, g_2 \to 0$ (no graphs involving EW vector bosons)
- vanishing external momenta: $p^2 \approx m_h^2 \approx v^2 g_{1,2}^2 \to 0$
- consider strong sector only, $t,\ \tilde{t},\ g,\ \tilde{g}$ (up to m_t^4-terms)
- assume hierarchies, e.g. (1) $m_{\tilde{g}} \gg m_{\tilde{t}}$, (2) $m_{\tilde{g}} \gg m_{\tilde{t}}$, etc.
- $\overline{\text{DR}}$ and $\overline{\text{MDR}}$
- new 3-loop results (semi-numerical) for general mass hierarchies [Reyes, Fazio, '19]

Two-loop self-energies (diagrammatic or effective potential): [Slavich, '01], [Martin, '01] , [Degrassi, Di Vita, Slavich, '14], [Borowka, Hahn, Heinemeyer, Heinrich, Hollik, '14], [...]

- $g_1, g_2, p^2 \to 0$
- $p^2 \neq 0$: $m_h^{p^2=0} - m_h^{p^2\neq 0} \approx 100 - 500\,\mathrm{MeV}$
- full mass hierarchies
- with CPV and RPV
- $\overline{\text{DR}}$ and OS conditions for $m_{\tilde{t}},\ m_t,\ X_{\tilde{t}}$ and $m_{H^\pm}$

One-loop self-energies:

- complete in all concerns

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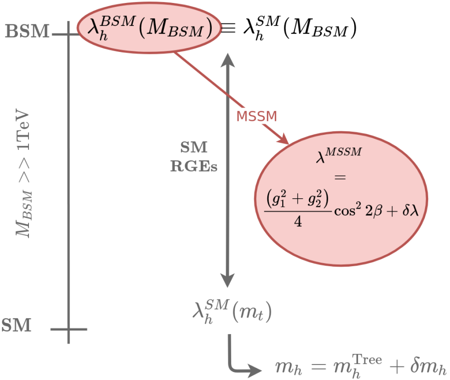
Status in the MSSM: EFT

Benefit from FO calculations:

- Pro: Hybrid approaches allow to "recycle" FO results
  $\rightarrow$ 3-loop pole-mass matching + 4-loop RGEs (N^{3}LL)
  [Harlander, Klappert, Ochoa, Voigt, '19]
- Con: applicable for high-scale SUSY
  i.e. one light Higgs but nothing else

Dedicated matching of scalar couplings:

- one-loop:
  - real 2HDM [Haber, Hempfling, '93]
  - complex 2HDM [Gorbahn, Jager, Nierste, Trine, '09 [Murphy, Rzehak, '19]
  - generic [MG, Muehlleitner, Staub, '18]
  - matching extended 2HDM Higgs-masses, Split-SUSY with light fermions, ...
- two-loop $\lambda_{\rm SM}$:
  - leading QCD [Bagnaschi, Slavich, '17]
  - mixed QCD-EW, combined with 3-loop hybrid [Bagnaschi, Degrassi, Passehr, Slavich, '19]
- three-loop: hopeless, but
  - $m_h^{\text{2L FO},p^2=0} - m_h^{\text{2L FO},p^2\neq 0} \approx 100 - 500\ \text{MeV}$
  - $m_h^{\text{3L, hybrid}} - m_h^{\text{2L, mixed QCD-EW}} \approx 10 - 100\ \text{MeV}$
  - $m_h^{\text{3L, hybrid}} - m_h^{\text{2L}} \approx 50 - 500\ \text{MeV}$
  - gauge/momentum-less approximation fully exploited(?)



Status in the MSSM: EFT

Benefit from FO calculations:

- Pro: Hybrid approaches allow to "recycle" FO results
  $\to$ 3-loop pole-mass matching + 4-loop RGEs (N^3LL)
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- Con: applicable for high-scale SUSY
  i.e. one light Higgs but nothing else

Dedicated matching of scalar couplings:

- one-loop:
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  - complex 2HDM [Gorbahn, Jager, Nierste, Trine, '09
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  - generic [MG, Muehlleitner, Staub, '18]
  - matching extended 2HDM Higgs-masses, Split-SUSY with light fermions, ...
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  - leading QCD [Bagnaschi, Slavich, '17]
  - mixed QCD-EW, combined with 3-loop hybrid [Bagnaschi, Degrassi, Passehr, Slavich, '19]
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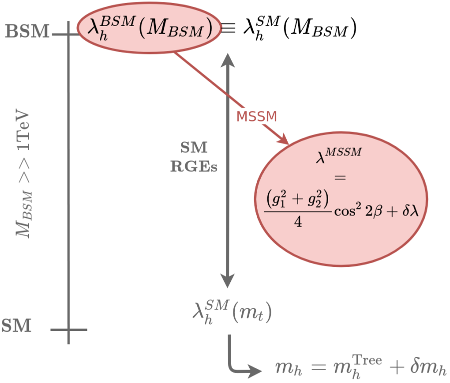

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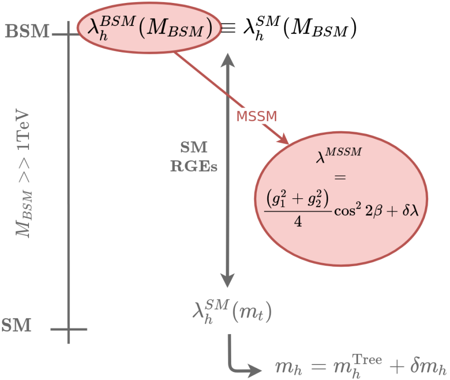

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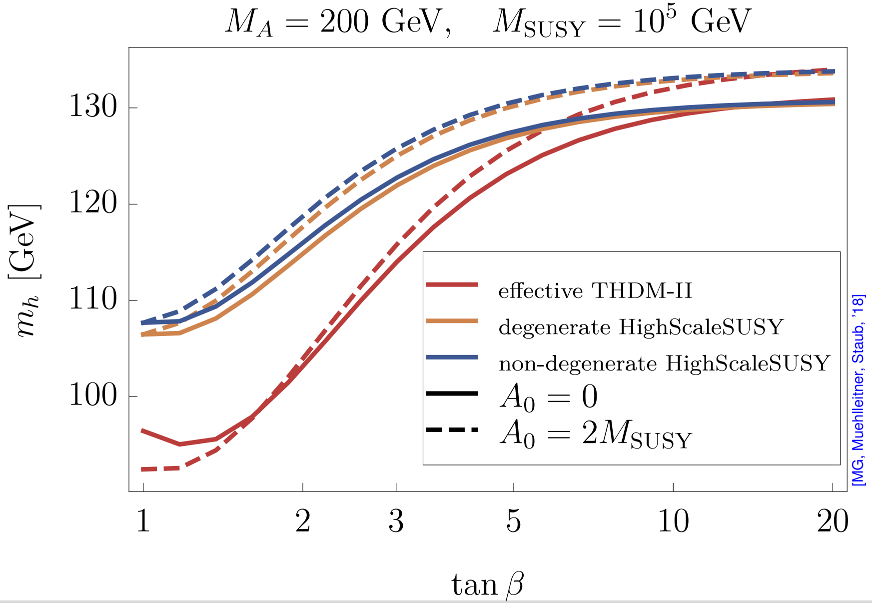

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Status in the NMSSM: Fixed Order

Full **one-loop** corrections are available in $\overline{\text{DR}}$ as well as OS scheme [Baglio, Grober, Muhlleitner, Dao, Rzehak, Spira, Streicher, Walz, '13].

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The NMSSM introduces many new couplings between S, H_u, H_d and the (s)fermion sector which are independent of gauge couplings!

$\to$ Many new **two-loop diagrams with Higgs-self-couplings**.
Leading contributions $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ are work-in-progress and subject to the s.c. *Goldstone Boson Catastrophe* (more later).

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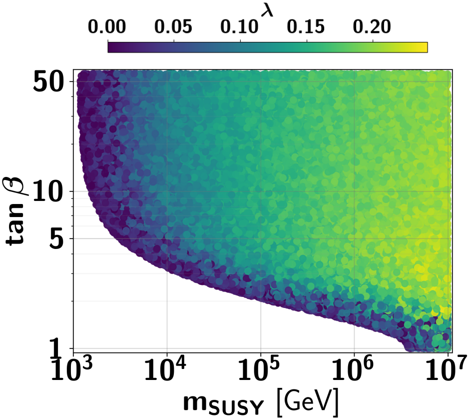

($\hat{X}_t = \sqrt{6}$, $\kappa = \frac{\lambda}{2}$ all SUSY masses degenerate)

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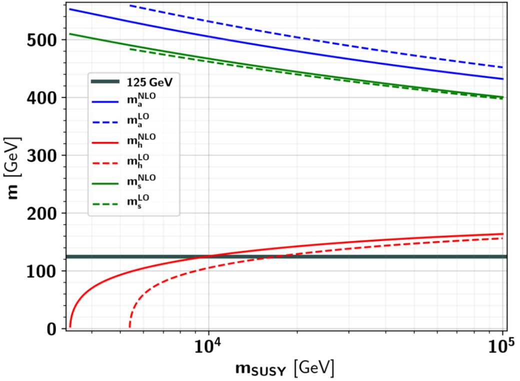

$(\lambda = \kappa/2 = 0.9, \tan\beta = 5, T_\lambda = -T_\kappa = 0.5\,\text{TeV}, A_0 = v_S = 0.1\,\text{TeV})$

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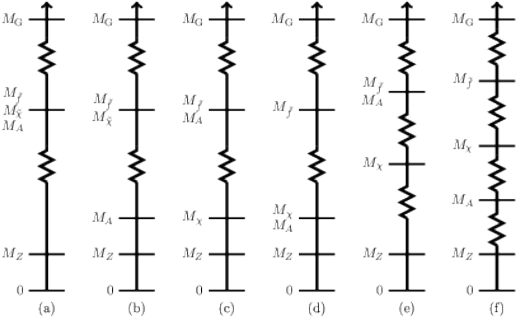

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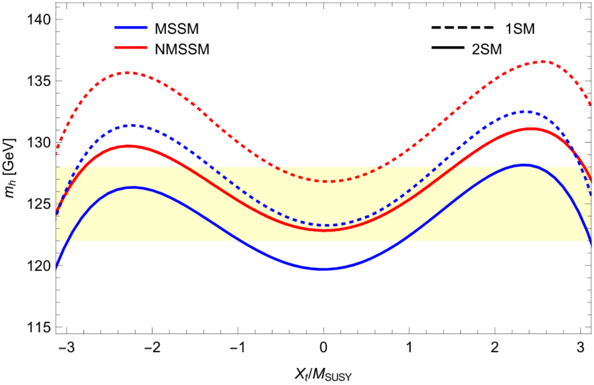


($m_{\text{SUSY}} = 5\,\text{TeV}$, $\lambda = 3\kappa = 0.2$, $\tan\beta = 2$)

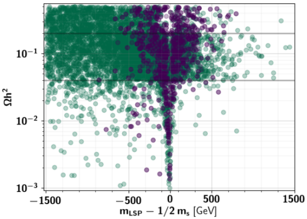
Intermezzo: EFT matchings beyond Higgs boson masses

Matchings can answer the question:

How does an EFT/simplified model compare to a given UV model?

- by how much does the parameter space differ?
- obeys UV symmetries?
  $\rightarrow$ radiately stable?

Example: Relic density in the singlet extended SM:



green/violet: with/without matching

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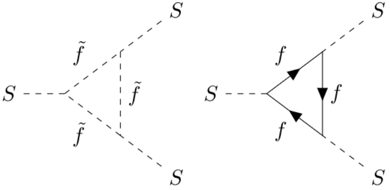

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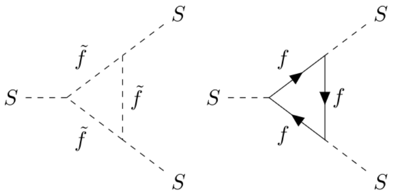

Decoupling heavy states also simplifies calculation of other observables:
Electroweak Phase Transitions: Boltzmann suppression
$\rightarrow$ efficiently calculate strength of phase transitions in (split-) SUSY
[Demidov, Gorbunov ’06], [Athron et. al ’19], [MG, Müller, Mühlleitner TBA]

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Two-Loop $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ Corrections

Ingredients:

- numerically solve:
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Most manpower required for (decreasing complexity):

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Two-Loop $\mathcal{O}\left((\alpha_\lambda + \alpha_\kappa + \alpha_t)^2\right)$ Corrections

Ingredients:

- numerically solve:
  $$det\left[p^2 \mathbb{1}_{5x5} - m_h^2 + \hat{\Sigma}_h^{(1)}(p^2) + \hat{\Sigma}_h^{(2)}(0)|_{g_{1,2}=0}\right] = 0$$
- renormalized self-energy:
  $$\hat{\Sigma}_h^{(2)}(p^2) = \Sigma_h^{(2)}(0) - \delta^{(2)} m_h^2$$
- two-loop mass matrix counter-term:
  $$\delta^{(2)} m_h^2 = \mathcal{O}\left((\delta^{(1)} Z_h)^2 + \delta^{(1)} m_h \delta^{(1)} Z_h\right) + \left(\delta^{(2)} Z_h^\dagger m_h^{(0)} + m_h^{(0)} \delta^{(2)} Z_h\right)$$
  $$+ m_h^{(0)}(\alpha \to \alpha + \delta^{(1)}\alpha + \delta^{(2)}\alpha)|_{\alpha=0}, \ \alpha = \{\lambda, \kappa, v, v_S, t_i, ...\}$$
- pure $\overline{\text{DR}}$: $\delta\alpha$ are purely divergent, canceled by wave-function ren. (SUSY-non ren.)
- OS quantities (e.g. $\delta v \leftrightarrow \delta m_{W,Z}$, or δt_i) : can yield extra finite contributions to $\delta^{(2)} m_h^2$

Most manpower required for (decreasing complexity):

- two-loop wave-function ren. constants $\delta^{(2)} Z_h = \left.\frac{\partial \Sigma_h(p^2)}{\partial p^2}\right|_{p^2=0}$
- two-loop vector self-energy diagrams $\delta^{(2)} \Sigma_V(0)$ (required for $\delta^{(2)} v$)
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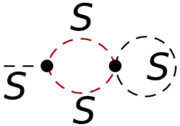
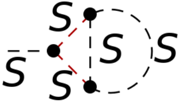
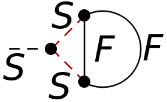
Generic Two-Loop Tadpoles I



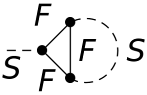
$$= -\frac{c_{s_0\bar{s}_1\bar{s}_2}\, c_{s_1 s_2 \bar{s}_3 s_3}}{1024\pi^4}\mathbf{A}_0(m^2_{s_3})\mathbf{B}_0(m^2_{s_1}, m^2_{s_2}, p^2 = 0) \xrightarrow{m^2_{s_{1,2}}\to 0} \lim_{\delta\to 0}\log\delta$$

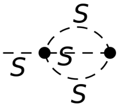


$$= \frac{i c_{s_0 s_1 s_4}\, c_{\bar{s}_1 s_2 s_3}\, c_{\bar{s}_2\bar{s}_3\bar{s}_4}}{1024\pi^4}\mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{s_3}, m^2_{s_2}, p^2 = 0) \xrightarrow{m^2_{s_{1,4}}\to 0} \lim_{\delta\to 0}\log\delta$$



$$\begin{aligned}
= \frac{i c_{s_0 s_1 s_4}}{1024\pi^4}\Big[&(c^L_{\bar{f}_3\bar{f}_2\bar{s}_4} c^R_{f_2 f_3 \bar{s}_1} + c^L_{f_2 f_3\bar{s}_1} c^R_{\bar{f}_3\bar{f}_2\bar{s}_4})[(\mathbf{A}_0(m^2_{f_2}) + \mathbf{A}_0(m^2_{f_3}))\mathbf{B}_0(m^2_{s_1}, m^2_{s_4}, 0)\\
&-\mathbf{S}(m^2_{f_3}, m^2_{f_2}, m^2_{s_1}, 0) + (m^2_{s_4} - m^2_{f_3})\mathbf{U}(m^2_{s_4}, m^2_{s_1}, m^2_{f_3}, m^2_{f_2}, 0)]\\
&-m^2_{f_2}\mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{f_3}, m^2_{f_2}, 0)(c^L_{\bar{f}_3\bar{f}_2\bar{s}_4} c^R_{f_2 f_3\bar{s}_1} + c^L_{f_2 f_3\bar{s}_1} c^R_{\bar{f}_3\bar{f}_2\bar{s}_4})\\
&-2 m_{f_3} m_{f_2}\mathbf{U}(m^2_{s_1}, m^2_{s_4}, m^2_{f_3}, m^2_{f_2}, 0)(c^R_{f_2 f_3\bar{s}_1} c^R_{\bar{f}_3\bar{f}_2\bar{s}_4} + c^L_{f_2 f_3\bar{s}_1} c^L_{\bar{f}_3\bar{f}_2\bar{s}_4})\Big]
\end{aligned}$$



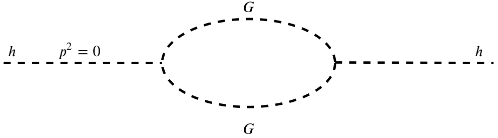


Intermezzo: Goldstone Boson Catastrophe in the Effective Potential

The effective potential needs to be summed over all possible masses (expressed as functions of the field or its VEV). E.g. at one-loop in the SM we have:

$$\delta^{(1)} m_h^2 = \frac{\partial^2}{\partial^2 v} \delta^{(1)} V_{eff} \propto \frac{\partial^2}{\partial^2 v} \sum_n m_n(v)^4 \log \frac{m_n(v)^2}{Q^2} \supset v^2 \lambda^2 \log \frac{m_G^2}{Q^2}$$

where m_G is the Goldstone mass. This is equivalent to the diagrammatic calculation:

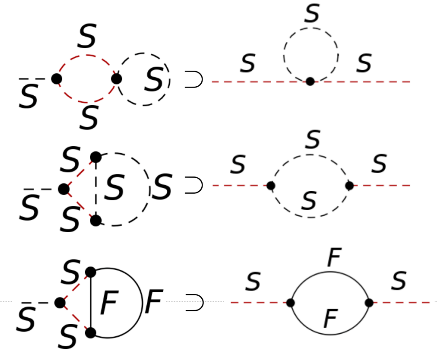


$$\lambda^2 v^2 \int dq^4 \frac{1}{\left(q^2 - m_G^2\right)^2} \equiv B_0(m_G^2, m_G^2, p^2 = 0) = v^2 \lambda^2 \log \frac{m_G^2}{Q^2} + \frac{1}{\epsilon}$$

Attention: For a vanishing $m_G \rightarrow 0$ this is IR-divergent (*"Goldstone Boson Catastrophe"*)!

Solutions:
- •Use external momentum [Braathen, Goodsell, '16] [Braathen Goodsell, Staub, '17]
- •OS Goldstone mass [Braathen, Goodsell, '16] [Braathen Goodsell, Staub, '17]
- •Resummation [Miro, Espinosa, Konstandin, '14] [Martin, '14] [Epsinosa, Garny, Konstandin, '16]

The Goldstone Boson Catastrophe in the Two-Loop Tadpoles



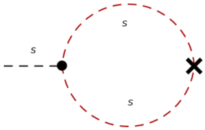
are all IR-divergent $\propto \log \frac{m_G^2}{Q^2}$, caused by Goldstone self-energy sub-graphs.

However: they are manifestly momentum-independent:

no momentum regularisation of IR-divergencies possible!

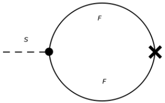
Generic Two-Loop Tadpoles II

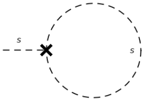
When working with **unrenormalised** loop-functions (in contrast to the renormalised V_{eff}), we also need to take into account:

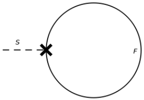


$$= -\frac{c_{s_0 s_1 s_2}}{32\pi^2}\Big[\Big(\mathbf{A}_0(m^2_{s_2}) - m^2_{s_1}\mathbf{B}_0(m^2_{s_1}, m^2_{s_2}, p^2 = 0)\Big)\delta^{(1)} Z_{\bar{s}_1\bar{s}_2} + \mathbf{B}_0(m^2_{s_2}, m^2_{s_1}, p^2 = 0)\delta^{(1)} c_{\bar{s}_1\bar{s}_2}\Big]$$

and



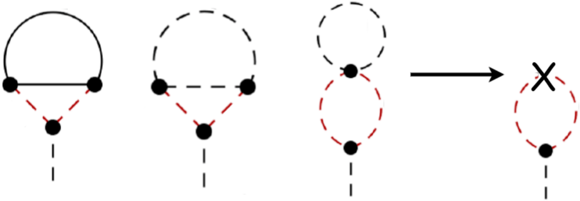




The first diagram is also IR-divergent.
Furthermore, it is conntected to the previous diagrams by the BPHZ formalism!

IR-Finite Two-Loop Tadpoles

Careful isolation of all IR-divergences and taking into account CT-diagrams leads to IR-finiteness!

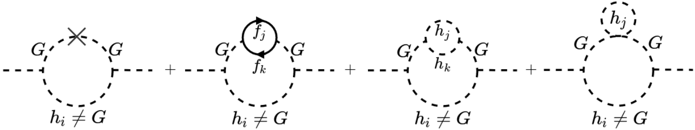


All two-loop IR-divergences are cancelled by they one-loop sub-graphs.

This mechanism is similar to the R-operation in the BPHZ formalism.

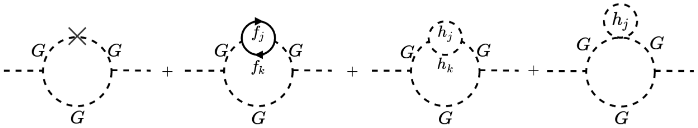
IR-Finite Two-Loop Selfenergies

Example of an IR-finite subset with intermediate IR-divergences:



IR-Divergent (!) Two-Loop Selfenergies

Example of an **IR-divergent** subset:



- $\propto A \cdot \overline{\log}\left(M^2_{\text{Regulator}}\right) + B \cdot \overline{\log}^2\left(M^2_{\text{Regulator}}\right)$
- $\rightarrow$ inclusion of finite external momentum required!
- **Solutions:**
  - use $p^2 \neq 0$ from the start $\rightarrow$ many scale problem
  - keep $p^2 = 0$, but expand IR-divergent integrals around small p^2 [Braathen, Goodsell, '16]

    $\rightarrow$ avoids numerical integration methods.

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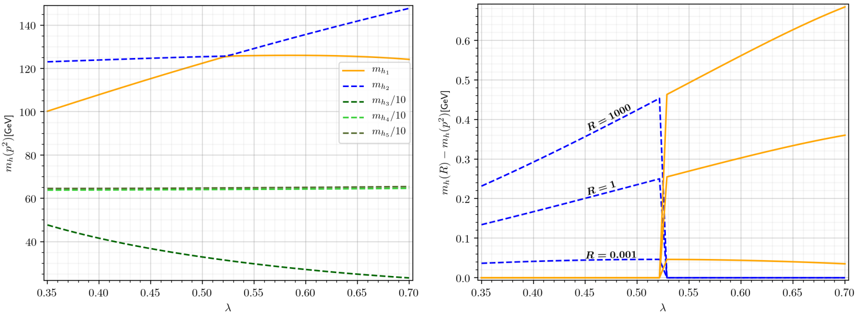
(Preliminary) Results: momentum dependence

Compare result obtained by small-momentum expansion with a numerical mass regulator $M_R^2 = RQ^2_{\text{Ren.}}$:



$Q = 1.3\,\text{TeV}, \kappa = 0.299, \tan\beta = 4.44, M_{H^\pm}^{\text{OS}} = 898\,\text{GeV}, A_t^{\text{OS}} = 3\,\text{TeV}$

Short Outlook

Full p^2- and Gauge Contributions

Were e.g. recently calculated for general QFTs in $\overline{\text{MS}}/\overline{\text{DR}}$ [Goodsell, Passehr, '19].
However, need p^2-dependence of loop integrals

- TSIL (integrates numerically): relatively slow

Hybrid Calculation

Use new results of FO calculation

- RGEs already (partially) implemented in NMSSMCalc
- needs SM Higgs pole-mass (rather straight forward)
- careful (re-)parametrisation of pole-masses and OS quantities

Precise prediction for stop-masses above $\approx 1 - 2\,\mathrm{TeV}$

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Summary

Higgs mass predictions being pushed by a very active community

Solved the Goldstone ”catastrophe” in NMSSMCalc:

- spurious IR-divergences cancel in sum of all graphs
- left-over divergences: require momentum regulation

Allow for precise Higgs mass predictions:

- MSSM: further accuracy requires momentum/gauge dependence
- NMSSM: mixed $\overline{\text{DR}}$-OS calculation on its way to full two-loop precision

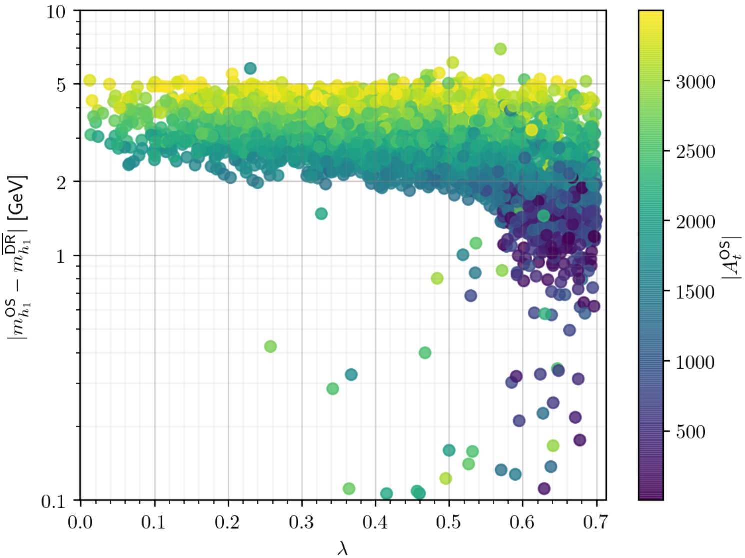
EFT $\leftrightarrow$ UV matching can be used beyond Higgs masses:

- Electroweak phase transitions in split-SUSY
- Benchmarking against simplified models

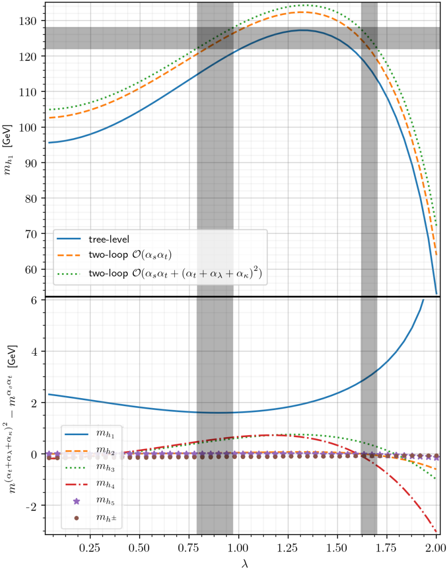
4 Backup

(Preliminary) Results: scheme dependence

Random scan yields:



(Preliminary) Results: size of new contributions



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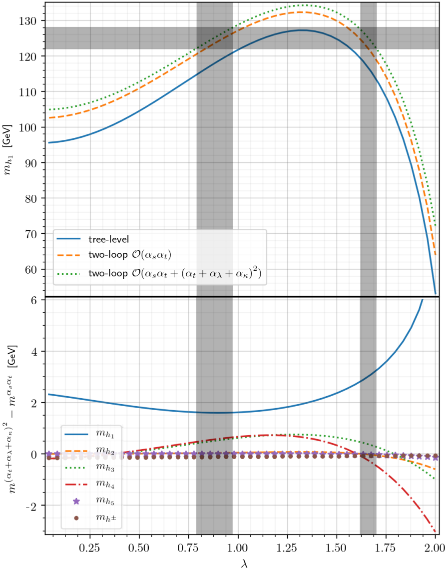


Two-Loop Diagrammatic n-Point Functions ($n \leq 2$)

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

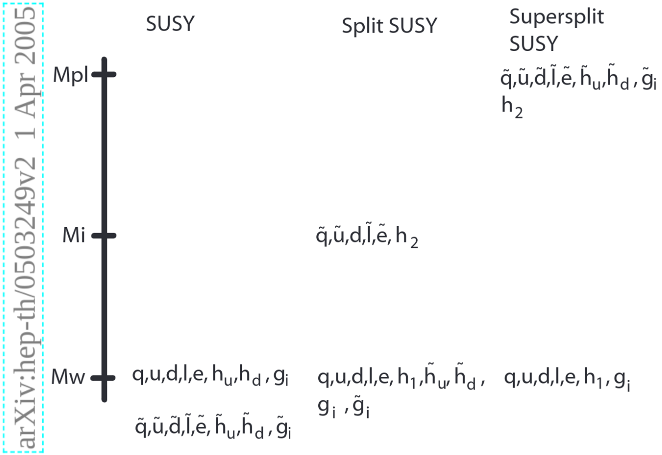
Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

Then:

- NMSSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules
- evaluate numerics

Terminology: EW/TeV- VS. split- VS. high-scale SUSY



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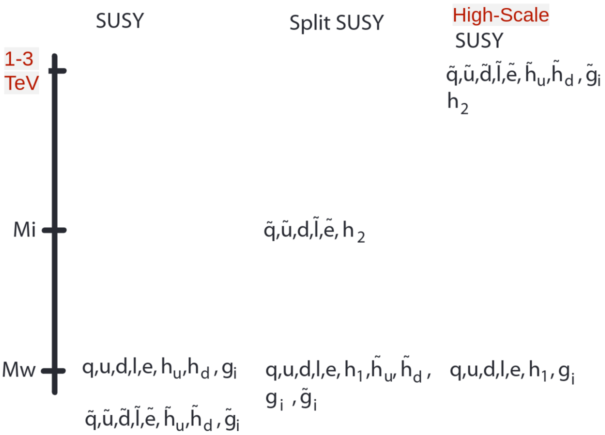

Effective Potential

No rigorous introduction here! (e.g. Peskin, Schroeder)

Consider quantum correction to classical action $S[\Phi_c]$:

$$W[J, \Phi_c = \Phi_0] = S[\Phi_0] + \underbrace{\int \frac{d^4k}{(2\pi)^4} \log\left[-k^2 + V''(\Phi_0)\right]}_{\delta^{(1)} V_{eff}\text{, one-loop Coleman-Weinberg effective potential}}$$

- $\frac{\partial^{(n)} \delta V_{eff}}{\partial \Phi_1 \ldots \partial \Phi_n}$ equivalent to scalar n-point function **with zero external momentum**
- typically calculated in minimal subtraction scheme
- e.g. $\delta V^{(1)} = \frac{1}{4} \sum_n (-1)^{2s_n}(2s_n + 1)(m_n^2)^2 \left(\log \frac{m_n^2}{Q^2} - k\right)$
  - already renormalized (UV-finite)!
  - k accounts for $\overline{\text{MS}}$ ($\frac{5}{6}$) or $\overline{\text{DR}}$ ($\frac{3}{2}$)
  - closed form encoding all loop-corrections
  - known up to three-loop order

Neat tool to calculate higher-order corrections to scalar quantities (?)

m_h^{SM} VS. m_h^{SUSY} II

Is there (still) an advantage w.r.t. Higgs mass?

Given that **all** other parameters of the theory are fixed by experiment:

any model	SUSY
m_h needs to be measured	m_h **predicted** perturbatively
unitarity: $m_h < 1\,\mathrm{TeV}$	MSSM: gauge sector only
$\Delta\rho_{SM} \propto \log\left(\frac{m_h^2}{m_Z^2}\right) + \frac{m_t^2}{m_Z^2}$	NMSSM: λ, κ constrained by SUSY non-renormalisation
$114 < m_h^{2009} < 154\,\mathrm{GeV}$ [Gfitter]	

SUSY is able to predict m_h^{theo} to *given* precision while the SM can at most extract m_h^{exp} from indirect fits. Thus, in general SUSY-relations can be falsified. Difficulty: many parameters to measure.

The N/MSSM Scalar Potential

Encoded in the superpotential $W \equiv W(\Phi)$, dim$W$=3:

$$W_{MSSM} = \mu \hat{H}_u \hat{H}_d + Y_L \hat{L} \hat{H}_u \hat{e} + Y_u \hat{Q} \hat{H}_u \hat{u} + Y_d \hat{U} \hat{H}_d \hat{d}$$
$$W_{NMSSM} = W_{MSSM}\big|_{\mu=\lambda\hat{S}} + \kappa \hat{S}^3$$

and constructed by:

$$V = \sum_{\Phi=H_{u,d},S,L,Q,u,d} \underbrace{\frac{\delta W}{\delta \Phi}\left(\frac{\delta W}{\delta \Phi}\right)^*}_{F-terms} + \underbrace{\frac{1}{2}\left(D_1 D_1 + D_2^a D_{2,a} + D_3^a D_{3,a}\right)}_{D-terms}$$

with:

$$D_1 = \frac{g_1}{2}\left(|H_u|^2 - |H_d|^2\right) + \dots$$
$$D_2^a = g_2\left(H_d^* \sigma^a H_d + H_u^* \sigma^a H_u\right) + \dots$$
$$D_3^a = 0 \text{ (for this talk)}$$