New constraints from vacuum stability on MSSM parameters

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Markus Bobrowski[†], Guillaume Chalons[*],
Wolfgang Gregor Hollik[†], Ulrich Nierste[†]

[†]Institut für Theoretische Teilchenphysik (TTP)
Karlsruher Institut für Technologie (KIT)
[*] LPSC Grenoble

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- discovery of Higgs boson [ATLAS, CMS: 4^{th} July 2012]
- $m_h = 126\,\mathrm{GeV}$: light SUSY (MSSM) Higgs

Stability of the electroweak vacuum

- ground state of the theory (global minimum)
- *unbounded from below* (UFB) limits
- further minima $\hookrightarrow$ vacuum decay or stable vacuum

This talk

- UFB bounds for effective 2HDM
- UFB direction $\hookrightarrow$ deeper minimum
- new class of constraints from vacuum stability on SUSY (Higgs) parameters using 1-loop effective potential

What has been done and what will be new

Already done

- effective potential for lightest SM-like Higgs mass [Okada, Yamaguchi, Yanagida 1991; Ellis, Ridolfi, Zwirner 1991; Casas, Espinosa, Quiros 1995]
- charge and color breaking minima [Frère, Jones, Raby 1983; Casas, Lleyda, Muños 1995, Casas, Dimopoulos 1996]
- ...
- computer code: finding all tree-level minima and perturbe them by one loop [`Vevacious`: Camargo-Molina, O'Leary, Porod, Staub]

Not yet

- stability conditions for the effective 2HDM
- charge and color conserving deeper minima by one loop
- lightest Higgs mass in the effective 2HDM (not covered)

Higgs potential of 2HDM type II

$$
\begin{aligned}
V = {} & m_{11}^2 \; H_d^\dagger H_d + m_{22}^2 \; H_u^\dagger H_u + \left(m_{12}^2 \; H_u \cdot H_d + \text{h.c.} \right) \\
& + \frac{\lambda_1}{2} \left(H_d^\dagger H_d \right)^2 + \frac{\lambda_2}{2} \left(H_u^\dagger H_u \right)^2 \\
& + \lambda_3 \left(H_u^\dagger H_u \right) \left(H_d^\dagger H_d \right) + \lambda_4 \left(H_u^\dagger H_d \right) \left(H_d^\dagger H_u \right) + \{ \lambda_5, \lambda_6, \lambda_7 \}
\end{aligned}
$$

In the MSSM: tree potential calculated from D-terms and $\mathcal{L}_{\text{soft}}$

$$
\begin{aligned}
m_{11}^{2\,\text{tree}} &= |\mu|^2 + m_{H_d}^2, & \lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = -\lambda_3^{\text{tree}} &= \frac{g^2 + g'^2}{4}, \\
m_{22}^{2\,\text{tree}} &= |\mu|^2 + m_{H_u}^2, & \lambda_4^{\text{tree}} &= \frac{g^2}{2}, \\
m_{12}^{2\,\text{tree}} &= B_\mu, & \lambda_5^{\text{tree}} = \lambda_6^{\text{tree}} = \lambda_7^{\text{tree}} &= 0.
\end{aligned}
$$

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& + \frac{\lambda_1}{2}\left(H_d^\dagger H_d\right)^2 + \frac{\lambda_2}{2}\left(H_u^\dagger H_u\right)^2 \\
& + \lambda_3\left(H_u^\dagger H_u\right)\left(H_d^\dagger H_d\right) + \lambda_4\left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) + \{\lambda_5, \lambda_6, \lambda_7\}
\end{aligned}
$$

Unbounded from below requirements

$$
\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}
$$

and others. . . [Gunion, Haber 2003]

- always fulfilled in the MSSM @ tree

Extending the tree

- loop corrections? [Gorbahn, Jäger, Nierste, Trine 2011]

Effective 2HDM

[Gorbahn, Jäger, Nierste, Trine 2011]

- integrating out heavy SUSY particles
- requirement of large SUSY scale $M_{\mathrm{SUSY}} \gg M_A \sim v_{\mathrm{ew}}$
- effective theory: generic 2HDM, λ_i calculated from SUSY loops

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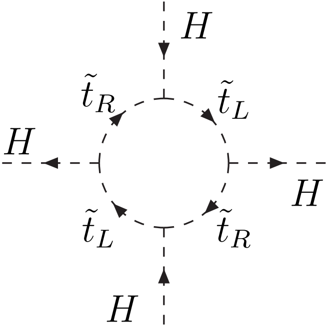
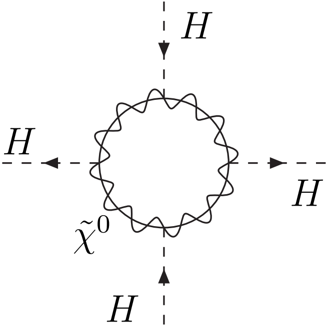




collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan\beta, \mu, M_1, M_2, \mathcal{M}^2_{\tilde{Q}}, \mathcal{M}^2_{\tilde{u}}, \mathcal{M}^2_{\tilde{d}}, \mathcal{M}^2_{\tilde{L}}, \mathcal{M}^2_{\tilde{e}}, A_u, A_d, A_e).$$

Triviality bounds on the effective 2HDM

simple check:

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1\lambda_2},$$

where now

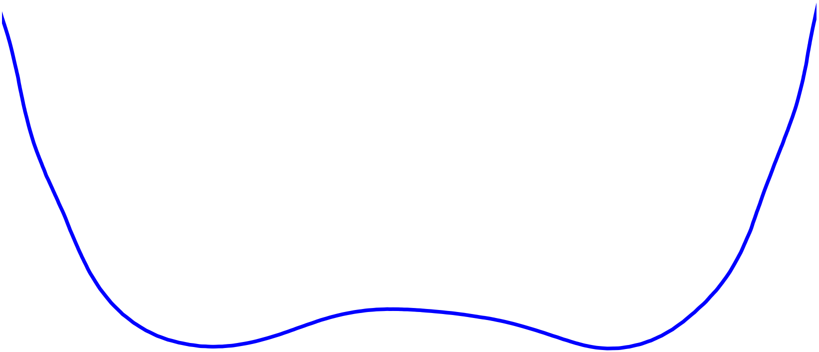
$$\lambda_i = \lambda_i^{\rm tree} + \frac{\lambda_i^{\rm ino} + \lambda_i^{\rm sferm}}{16\pi^2}\,.$$

Severe UFB limits

Bounds on $\lambda_{1,2,3}$ transfer into bounds on m_0, A_t, μ, ...

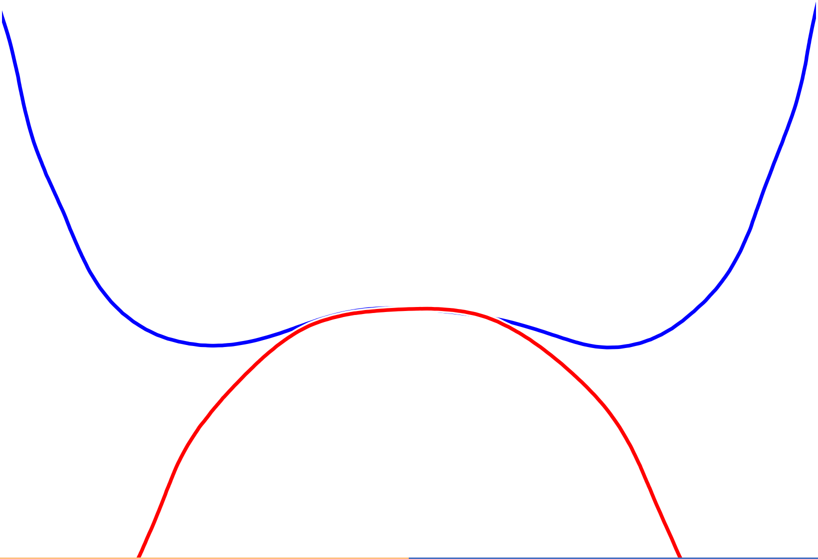
Recovery from unbounded from below???

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$



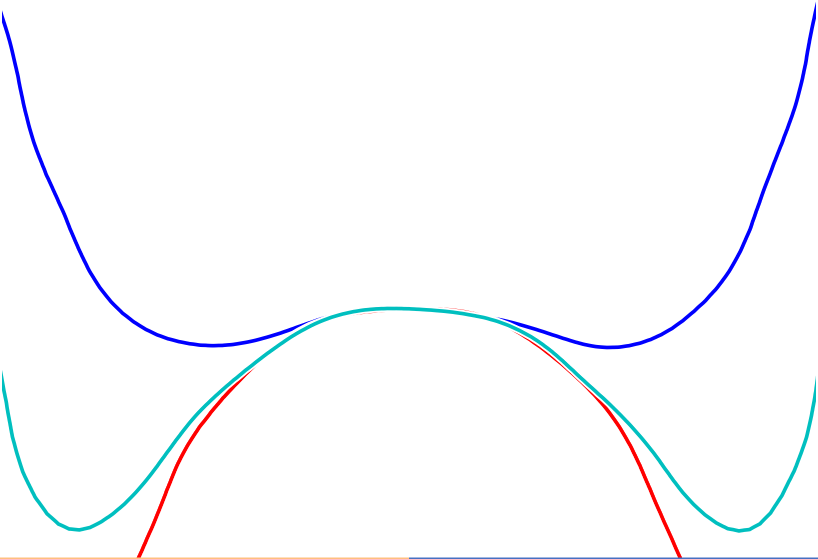
Recovery from unbounded from below???

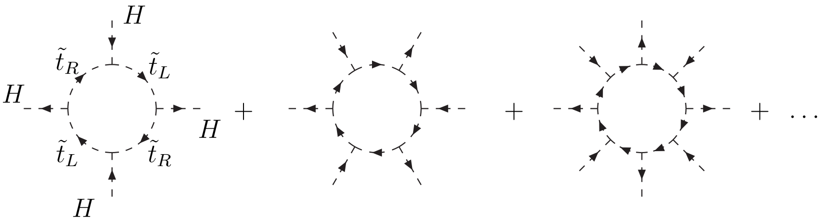
$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4$$



Recovery from unbounded from below???

$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4 + \lambda^{(6)}\phi^6$$

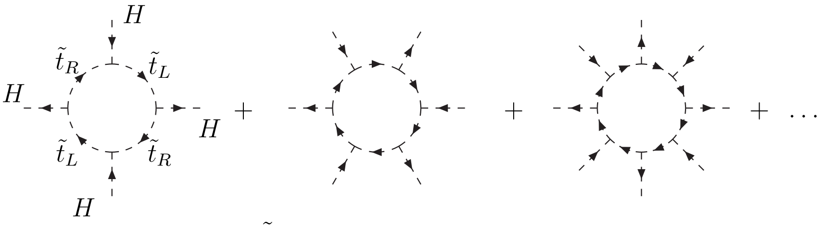


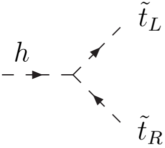


- most dominant contribution from top Yukawa y_t and A_t
- can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$
- 1-PI potential as generating function for 1-PI Green's functions

$$-V_{1-\mathrm{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\mathrm{ext}} = 0)\phi^n$$

- "classical" field value $\phi \to \langle 0|\phi|0\rangle$
- $\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$ determines ground state of the theory





$$h = h_d^{0\dagger} - \frac{A_t}{\mu^* Y_t} h_u^0$$

$$V_1 \sim \sum_n \frac{a_n}{n!^2} \left(h^\dagger h \right)^n , \qquad \frac{a_n}{n!^2} = \frac{1}{n(n-1)(n-2)}$$

$$V_1 = \frac{N_c M^4}{32\pi^2} \left[(1+x)^2 \log(1+x) + (1-x)^2 \log(1-x) - 3x^2 \right]$$

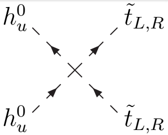
$Q^2 = M^2$

$x^2 = |\mu Y_t|^2 h^\dagger h / M^4$, $m^2_{\tilde{t}_L} = m^2_{\tilde{t}_R} = M^2$

Summing up (ctd.)

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$



- trilinear $\sim h(h_d^0, h_u^0)$, quadrilinear $\sim |h_u^0|^2$
- diagrams with mixed contributions

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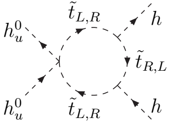

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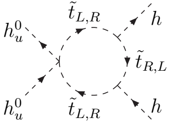

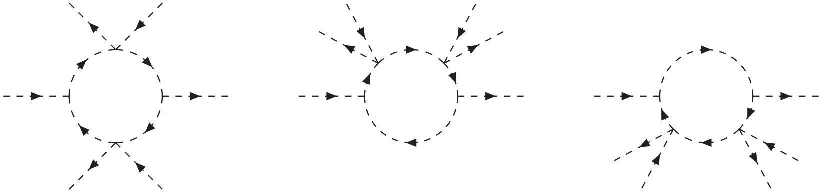

Field dependent stop mass

$$\mathcal{M}^2_{\tilde t}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde t_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde t_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

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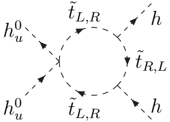

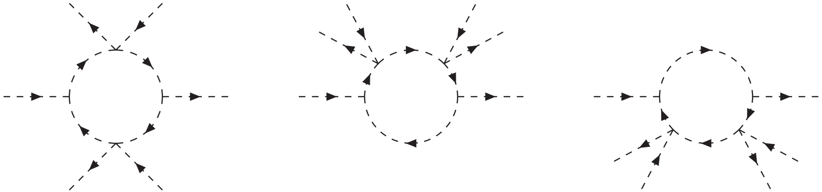

- gummi bear factor

$$\frac{(2n+k-1)!}{k!(2n-1)!}$$

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Field dependent stop mass

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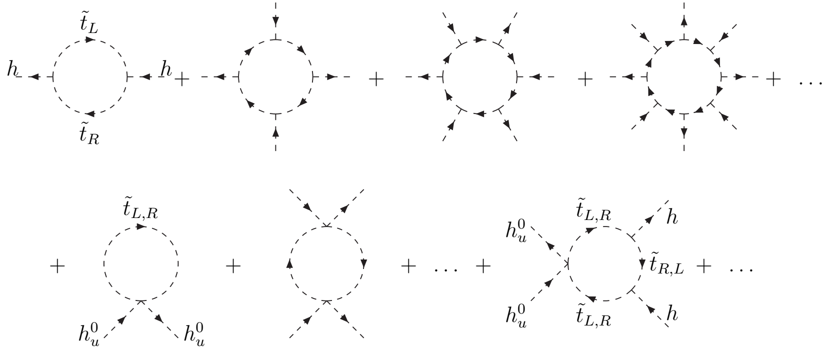

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$$V_1 \sim \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} a_{kn}\, x^{2n} y^k \,, \qquad x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, y = \frac{|Y_t h_u^0|^2}{M^2}$$

$$= \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{n(2n+k-1)(2n+k-2)} \frac{(2n+k-1)!}{k!(2n-1)!}\, x^{2n} y^k$$

$$= \Big[\, (1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y) \Big]$$

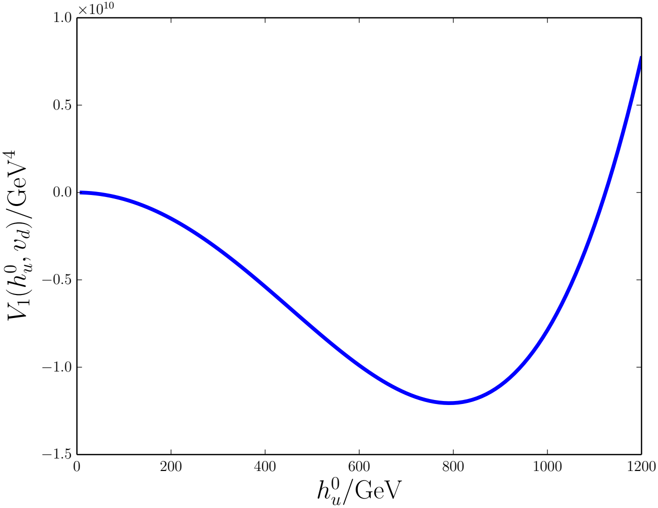
Features of the resummed series

$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2}\Big[(1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y)\Big]$$

$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4},\ h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters

$M_{\mathrm{SUSY}} = 1\,\mathrm{TeV},\ A_t = 1.5\,\mathrm{TeV},\ \mu = 3\,\mathrm{TeV},\ \tan\beta = 10.$



Radius of convergence, analytic continuation and imaginary part

- series of 1-PI Green's functions converges for $y, x < 1$
- imaginary part for $y - x > 1$ from logarithm
- take real part for stability discussion

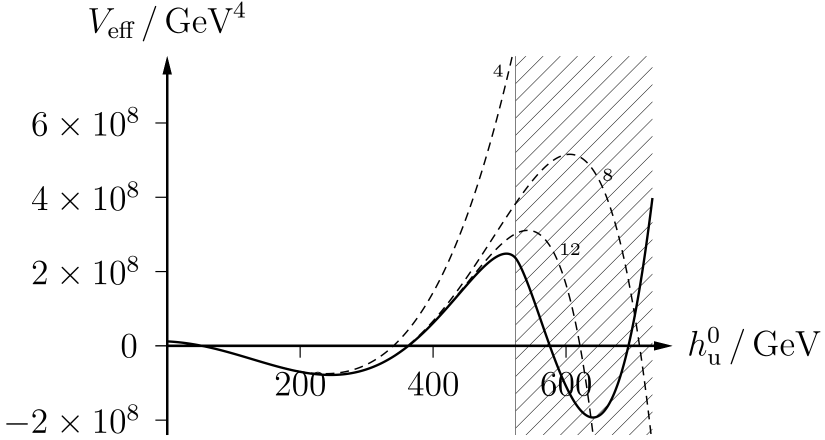
Constraint by radius of convergence

- stop discussion at large field values
- viability of the (effective) theory under consideration
- no physical interpretation possible

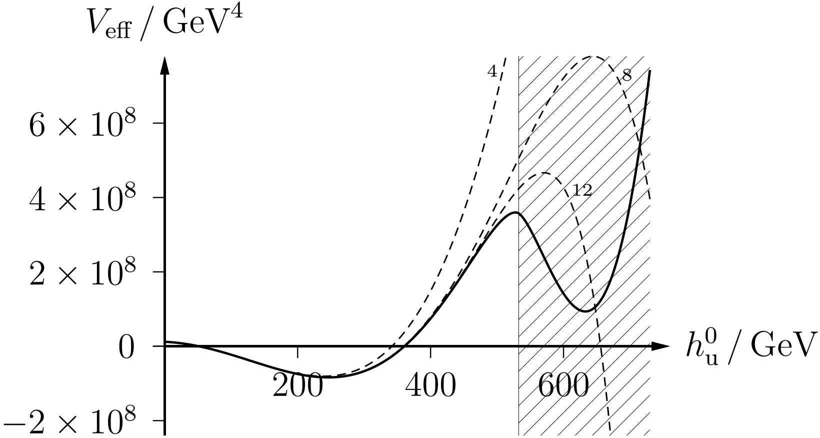
Analytic continuation: discussion of stability

- effective potential viable beyond "radius of convergence"
- 1-loop part drives deeper minimum
- interpretation via stability of the electroweak vacuum

- ignore imaginary part and interpretation [Weinberg, Wu 1987]



- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $A_t \simeq 1.5\,\text{TeV}$
- $\mu = 2.55\,\text{TeV}$



- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $A_t \simeq 1.5\,\text{TeV}$
- $\mu = 2.51\,\text{TeV}$

- UFB limits for effective 2HDM turn into bounds from the formation of a deeper minimum
- vacuum stability bounds on MSSM parameters
- constrained and simplified model discussed:
  - gluino and electroweak gauginos heavy
  - only third generation squarks light
  - A_t fixed by m_h, $A_b \equiv 0$ (for simplicity)
  - free parameters: $M_{\mathrm{SUSY}} = m_{\tilde{t}} = m_{\tilde{b}}$, $\tan\beta$, μ
  - no new insights if $m_{\tilde{t}_L,\tilde{b}_L} \neq m_{\tilde{t}_R,\tilde{b}_R}$
- instability of electroweak vacuum by deeper (global?) minimum in "standard model direction" $\sim v_u$

Backup

Slides

Minimum at the electroweak scale $v = 246\,\mathrm{GeV}$

$$m_{11}^{2\,\mathrm{tree}} = m_{12}^{2\,\mathrm{tree}} \tan\beta - \frac{v^2}{2}\cos(2\beta)\lambda_1^{\mathrm{tree}} - \frac{1}{v\cos\beta}\frac{\delta}{\delta\phi_d}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}},$$

$$m_{22}^{2\,\mathrm{tree}} = m_{12}^{2\,\mathrm{tree}} \cot\beta + \frac{v^2}{2}\cos(2\beta)\lambda_1^{\mathrm{tree}} - \frac{1}{v\sin\beta}\frac{\delta}{\delta\phi_u}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}}.$$

$m_h = 126\,\mathrm{GeV}$

- pseudoscalar mass m_A less dependent on higher loops
- using `FeynHiggs 2.10.0` to determine light Higgs mass by adjusting A_t
- connection to potential: m_A
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

- include sbottom loop as well, taking $A_b = 0$

- 1-loop effective potential [Coleman, Weinberg 1973]

$$V_1(h_u, h_d) = \frac{1}{64\pi^2} \, \mathrm{STr}\, \mathcal{M}^4(h_u, h_d) \left[\ln\left(\frac{\mathcal{M}^2(h_u, h_d)}{Q^2} \right) - \frac{3}{2} \right]$$

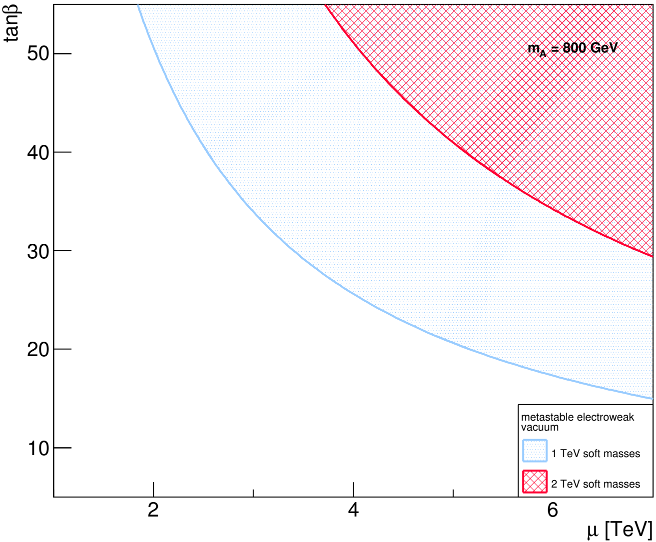
  - *field dependent mass* $\mathcal{M}(h_u, h_d)$
  - STr accounts for spin degrees of freedom

- same result can be obtained by the tadpole method

$$T \sim \frac{\partial}{\partial h} V_1(h) \quad \hookrightarrow \quad V_1(h) \sim \int \mathrm{d}h \, T(h)$$

[Lee, Sciaccaluga 1975]

- functional methods: effective potential for arbitrary number of scalars: $V_1(\phi_1, \phi_2, \ldots \phi_n)$ [Jackiw 1973]



m_A = 800 GeV
metastable electroweak vacuum
1 TeV soft masses
2 TeV soft masses
tanβ
50
40
30
20
10
2
4
6
μ [TeV]