About the stability of the vacuum in the MSSM

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Motivation and outline

- discovery of Higgs boson [ATLAS, CMS: 4th July 2012]
- $m_h = 126\,\mathrm{GeV}$: light SUSY (MSSM) Higgs

Stability of the electroweak vacuum

- ground state of the theory (global minimum)
- *unbounded from below* (UFB) limits
- further minima $\hookrightarrow$ vacuum decay or stable vacuum

This talk

- UFB directions and multiple minima
- access to charge and colour breaking minima
- new class of constraints from vacuum stability on SUSY (Higgs) parameters using 1-loop effective potential

Higgs potential of 2HDM type II

$$V = m_{11}^2\, H_d^\dagger H_d + m_{22}^2\, H_u^\dagger H_u + \left(m_{12}^2\, H_u \cdot H_d + \text{h.c.}\right)$$
$$+ \frac{\lambda_1}{2}\left(H_d^\dagger H_d\right)^2 + \frac{\lambda_2}{2}\left(H_u^\dagger H_u\right)^2$$
$$+ \lambda_3\left(H_u^\dagger H_u\right)\left(H_d^\dagger H_d\right) + \lambda_4\left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) + \{\lambda_5, \lambda_6, \lambda_7\}$$

In the MSSM: tree potential calculated from D-terms and $\mathcal{L}_{\text{soft}}$

$$m_{11}^{2\,\text{tree}} = |\mu|^2 + m_{H_d}^2, \qquad \lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = -\lambda_3^{\text{tree}} = \frac{g^2 + g'^2}{4},$$
$$m_{22}^{2\,\text{tree}} = |\mu|^2 + m_{H_u}^2, \qquad \lambda_4^{\text{tree}} = \frac{g^2}{2},$$
$$m_{12}^{2\,\text{tree}} = B_\mu, \qquad \lambda_5^{\text{tree}} = \lambda_6^{\text{tree}} = \lambda_7^{\text{tree}} = 0.$$

The Higgs sector of the MSSM and its stability

Higgs potential of 2HDM type II

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Unbounded from below requirements

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}$$

and others. . . [Gunion, Haber 2003]

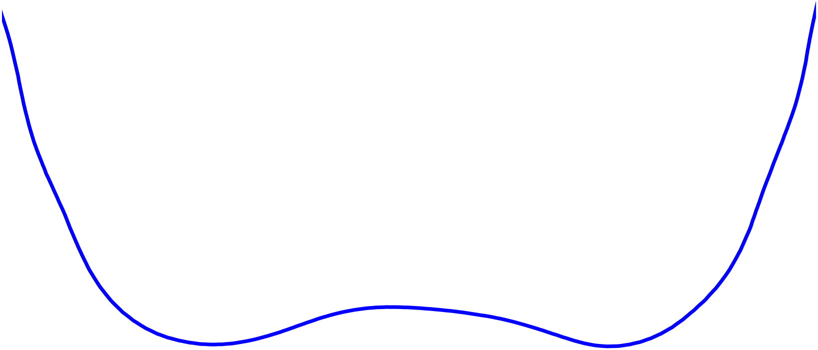
- always fulfilled in the MSSM @ tree

Extending the tree

- loop corrections? [Gorbahn, Jäger, Nierste, Trine 2011]

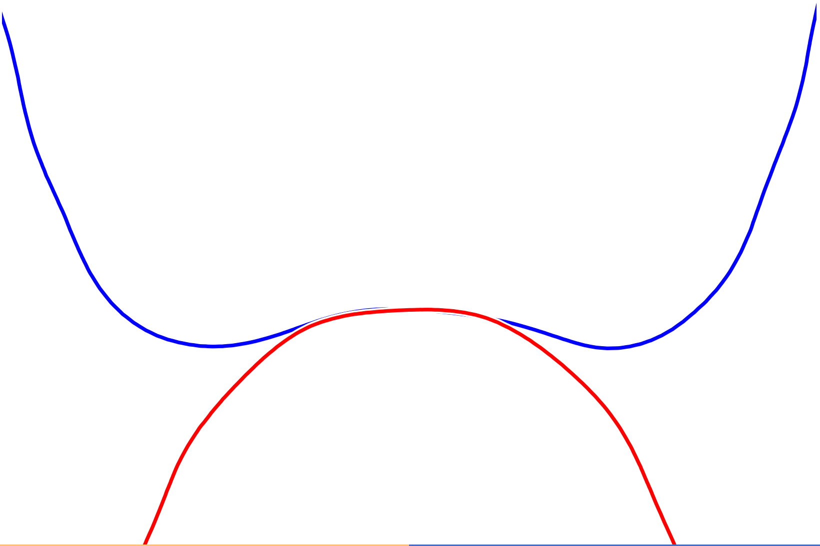
Recovery from unbounded from below???

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$



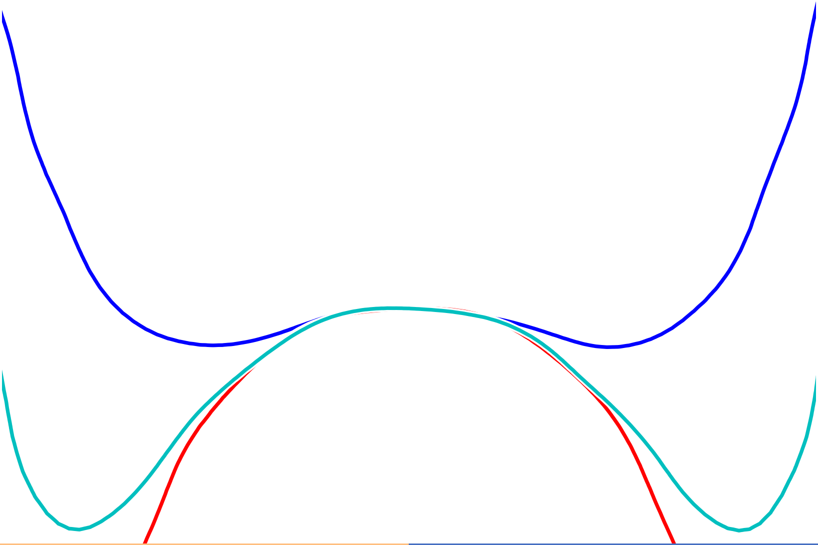
Recovery from unbounded from below???

$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4$$

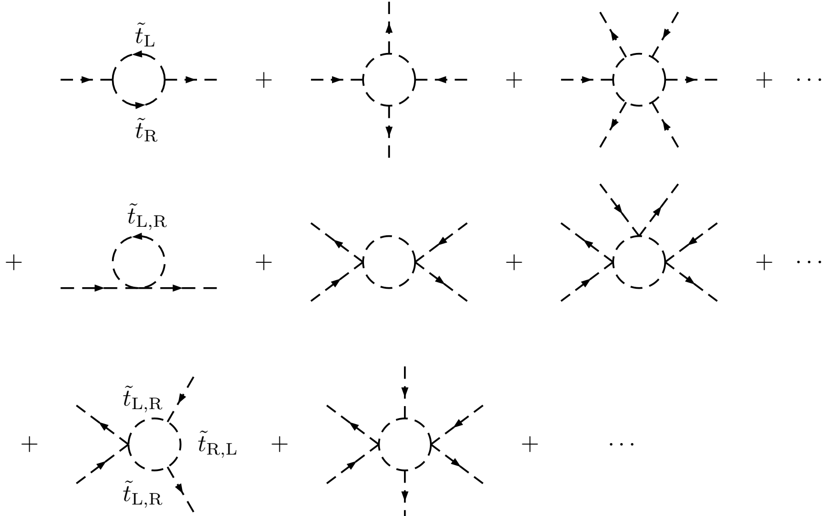


Recovery from unbounded from below???

$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4 + \lambda^{(6)}\phi^6$$



Calculating the 1-loop effeective potential



- quadrilinear couplings ($\sim |Y_{\mathsf{t}}|^2$)
- trilinear coupling to a linear combination ($\mu^* Y_{\mathsf{t}} h_{\mathsf{d}}^{\dagger} - A_{\mathsf{t}} h_{\mathsf{u}}^0$)

Check versus known results

- 1-loop effective potential [Coleman, Weinberg 1973]

$$V_1(h_u, h_d) = \frac{1}{64\pi^2}\,\mathrm{STr}\,\mathcal{M}^4(h_u, h_d)\left[\ln\left(\frac{\mathcal{M}^2(h_u, h_d)}{Q^2}\right) - \frac{3}{2}\right]$$

  - *field dependent mass* $\mathcal{M}(h_u, h_d)$
  - STr accounts for spin degrees of freedom

- same result can be obtained by the tadpole method

$$T \sim \frac{\partial}{\partial h} V_1(h) \quad \hookrightarrow \quad V_1(h) \sim \int \mathrm{d}h\, T(h)$$

[Lee, Sciaccaluga 1975]

- functional methods: effective potential for arbitrary number of scalars: $V_1(\phi_1, \phi_2, \ldots \phi_n)$ [Jackiw 1973]

Features of the resummed series

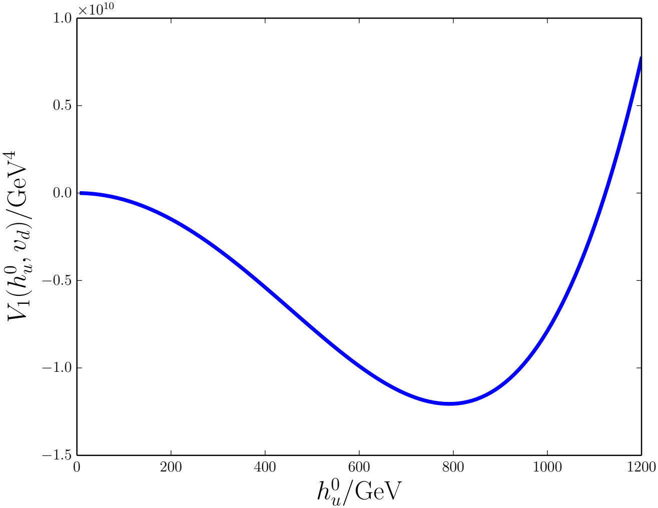
$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2}\Big[(1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y)\Big]$$

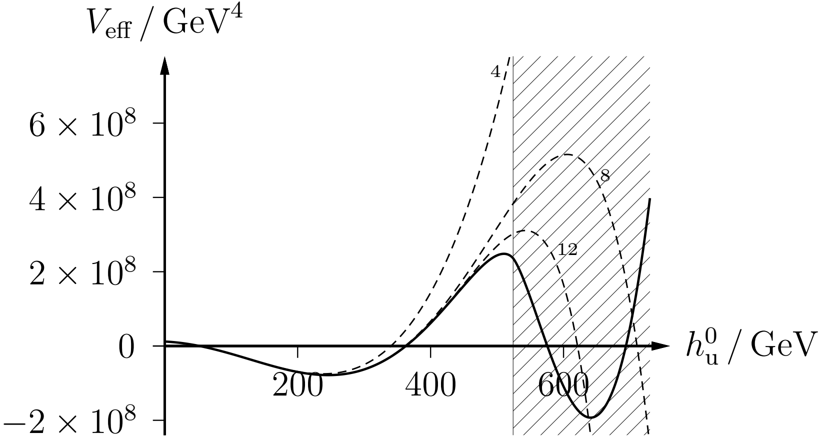
$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4},\ h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

- branch cut at $x - y = 1$: take real part (analytic continuation)
- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters

Features of the resummed series

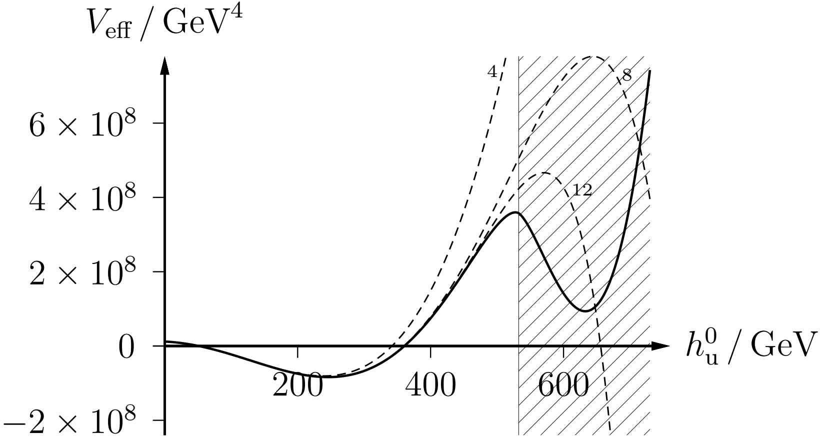
$$M_{\mathrm{SUSY}} = 1\,\mathrm{TeV},\ A_t = 1.5\,\mathrm{TeV},\ \mu = 3\,\mathrm{TeV},\ \tan\beta = 10.$$





- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $A_t \simeq 1.5\,\text{TeV}$
- $\mu = 2.55\,\text{TeV}$

Discussion of stability



- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $A_t \simeq 1.5\,\text{TeV}$
- $\mu = 2.51\,\text{TeV}$

Cooking up phenomenologically viable parameters

Minimum at the electroweak scale $v = 246\,\mathrm{GeV}$

$$m_{11}^{2\,\text{tree}} = m_{12}^{2\,\text{tree}} \tan\beta - \frac{v^2}{2}\cos(2\beta)\lambda_1^{\text{tree}} - \frac{1}{v\cos\beta}\;\frac{\delta}{\delta\phi_d}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}},$$

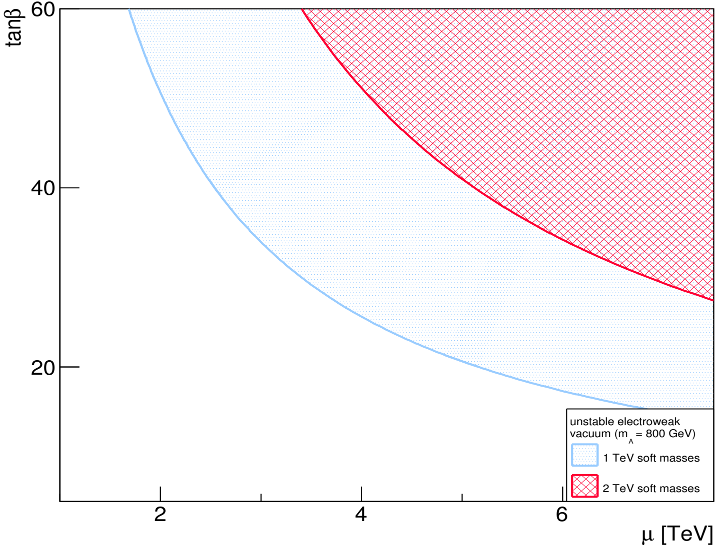
$$m_{22}^{2\,\text{tree}} = m_{12}^{2\,\text{tree}} \cot\beta + \frac{v^2}{2}\cos(2\beta)\lambda_1^{\text{tree}} - \frac{1}{v\sin\beta}\;\frac{\delta}{\delta\phi_u}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}}.$$

$m_h = 126\,\mathrm{GeV}$

- using `FeynHiggs 2.10.0` to determine light Higgs mass by adjusting A_t
- connection to potential: m_A
- pseudoscalar mass m_A less dependent on higher loops
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

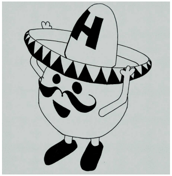
- include sbottom (drives minimum), take $A_b = 0$

Constraint in μ-$\tan\beta$



Conclusions

- formation of new minima at the 1-loop level
- stability of the electroweak vacuum: bounds on μ-$\tan\beta$
- instability of electroweak vacuum by second minimum in "standard model direction" $\sim v_u$: global CCB minimum
- squark contribution to the effective potential:
  - only third generation squarks light
  - A_t fixed by m_h, $A_b \equiv 0$ (for simplicity)
  - free parameters: $M_{\text{SUSY}} = m_{\tilde{t}} = m_{\tilde{b}}$, $\tan\beta$, μ
  - no new insights if $m_{\tilde{t}_L,\tilde{b}_L} \neq m_{\tilde{t}_R,\tilde{b}_R}$
  - gluino and electroweak gauginos heavy

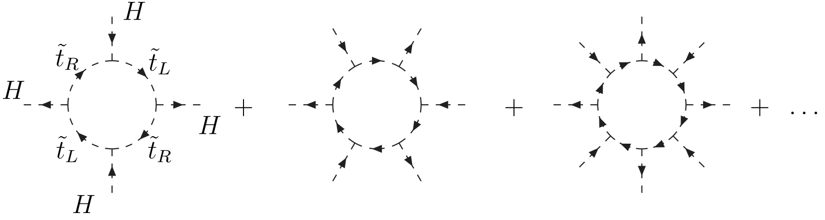


Greetings from Señor Higgs
(*courtesy of Jens Hoff*)

Backup

Slides

Summing up external legs

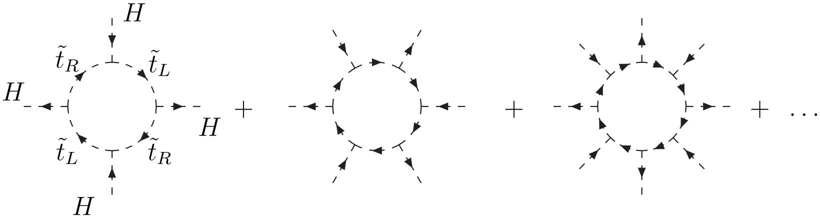


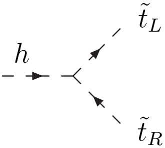
- most dominant contribution from top Yukawa y_t and A_t
- can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$
- 1-PI potential as generating function for 1-PI Green's functions

$$-V_{1-\mathrm{PI}}(\phi) = \Gamma_{1-\mathrm{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\mathrm{ext}} = 0)\phi^n$$

- "classical" field value $\phi \to \langle 0|\phi|0\rangle$
- $\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$ determines ground state of the theory

Summing up external legs





$$h = h_d^{0\dagger} - \frac{A_t}{\mu^* Y_t} h_u^0$$

$$V_1 \sim \sum_n \frac{a_n}{n!^2} \left(h^\dagger h\right)^n \,, \qquad \frac{a_n}{n!^2} = \frac{1}{n(n-1)(n-2)}$$

$$V_1 = \frac{N_c M^4}{32\pi^2} \left[(1+x)^2 \log(1+x) + (1-x)^2 \log(1-x) - 3x^2\right]$$

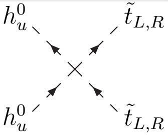
$Q^2 = M^2$ $\qquad$ $x^2 = |\mu Y_t|^2 h^\dagger h / M^4$, $m^2_{\tilde{t}_L} = m^2_{\tilde{t}_R} = M^2$

Summing up (ctd.)

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

- trilinear $\sim h(h_d^0, h_u^0)$, quadrilinear $\sim |h_u^0|^2$
- diagrams with mixed contributions



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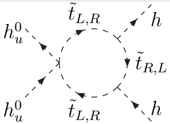

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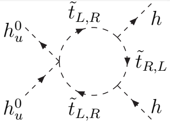

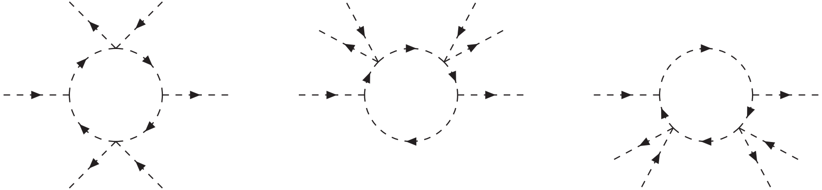

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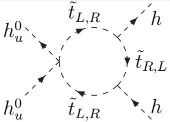

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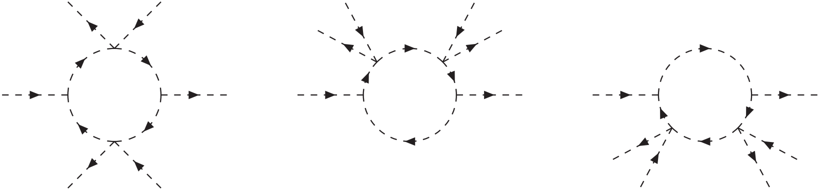
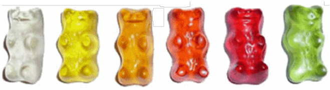


- gummi bear factor

$$\frac{(2n+k-1)!}{k!(2n-1)!}$$



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Summing up (ctd.)

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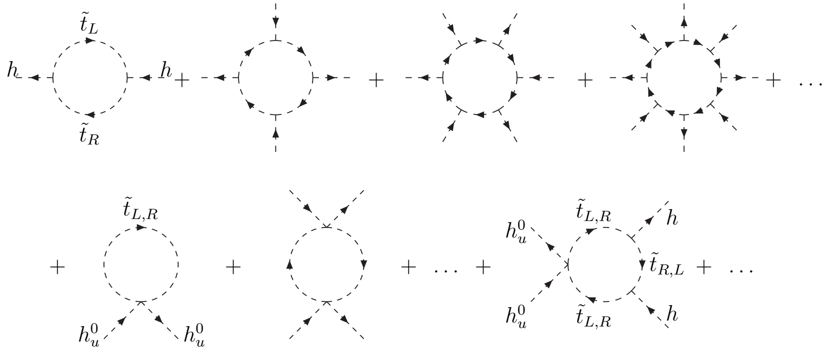

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$$\begin{aligned} V_1 &\sim \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{kn}\, x^{2n} y^k \,, \qquad x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, y = \frac{|Y_t h_u^0|^2}{M^2} \\ &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{1}{n(2n+k-1)(2n+k-2)} \frac{(2n+k-1)!}{k!(2n-1)!}\, x^{2n} y^k \\ &= \Big[\, (1+y+x)^2 \log(1+y+x) \\ &\qquad +(1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y)\Big] \end{aligned}$$

Effective 2HDM

[Gorbahn, Jäger, Nierste, Trine 2011]

- integrating out heavy SUSY particles
- requirement of large SUSY scale $M_{\mathrm{SUSY}} \gg M_A \sim v_{\mathrm{ew}}$
- effective theory: generic 2HDM, λ_i calculated from SUSY loops

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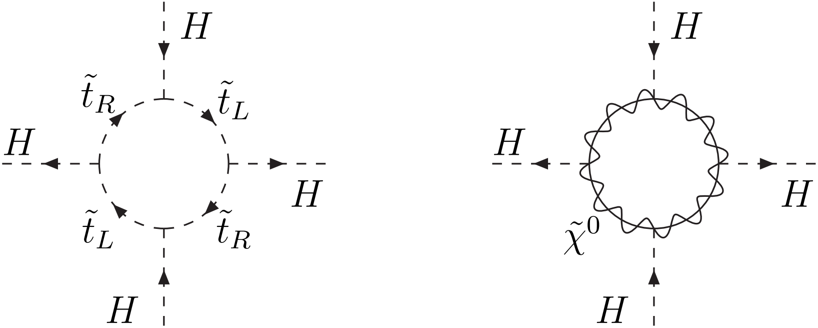

collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan\beta, \mu, M_1, M_2, \mathcal{M}^2_{\tilde{Q}}, \mathcal{M}^2_{\tilde{u}}, \mathcal{M}^2_{\tilde{d}}, \mathcal{M}^2_{\tilde{L}}, \mathcal{M}^2_{\tilde{e}}, A_u, A_d, A_e).$$

simple check:

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2},$$

where now

$$\lambda_i = \lambda_i^{\rm tree} + \frac{\lambda_i^{\rm ino} + \lambda_i^{\rm sferm}}{16\pi^2}.$$

Severe UFB limits

Bounds on $\lambda_{1,2,3}$ transfer into bounds on m_0, A_t, μ, ...