

Radiative generation of neutrino mixing: degenerate masses and threshold corrections

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The neutrino mass spectrum

$$m_1 = m_0, \quad m_2 = \sqrt{m_0^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_0^2 + \Delta m_{31}^2}$$

- Oscillations:

[ν fit: www.nu-fit.org]

- $\Delta m_{21}^2 = 7.50_{-0.17}^{+0.19} \times 10^{-5} \text{ eV}^2$
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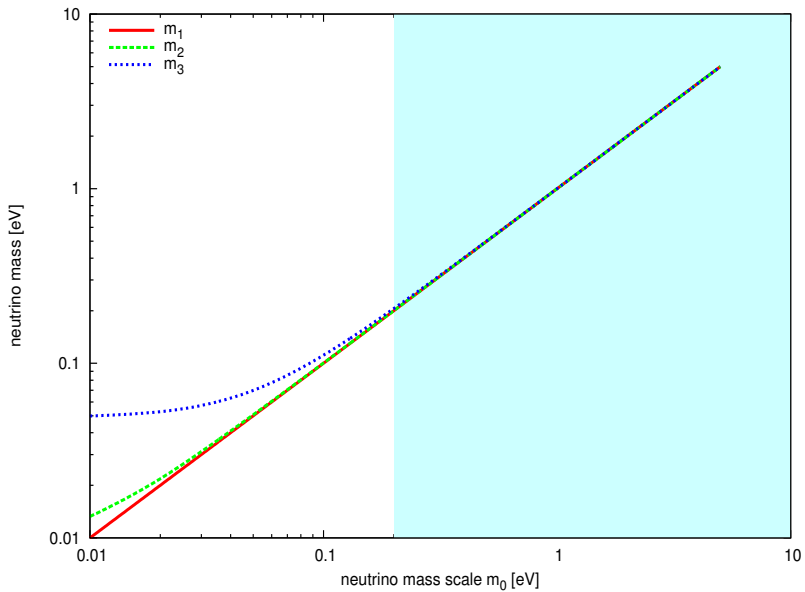
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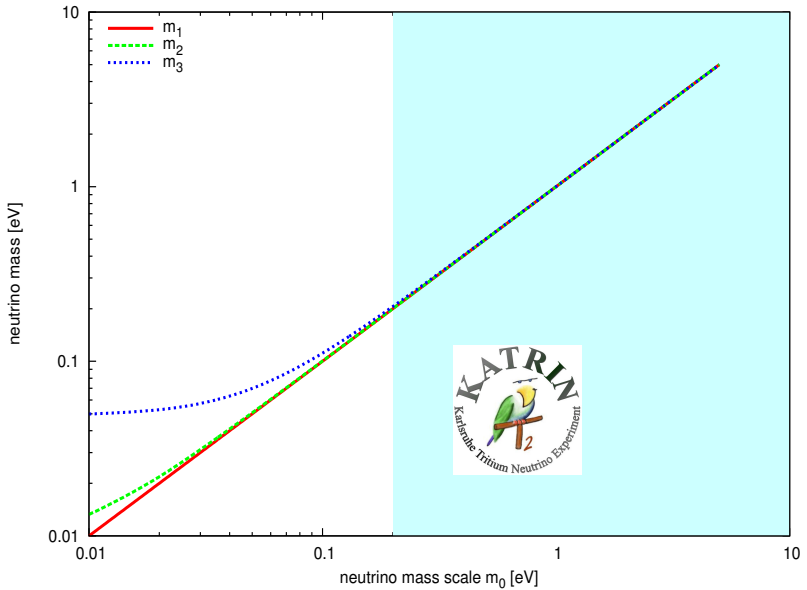


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Quasi-Degeneration



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Generic treatment

$$m_{AB}^\nu = m_{AB}^{(0)} + m_{AC}^{(0)} I_{CB} + I_{AC} m_{CB}^{(0)}$$

I : threshold correction $I \sim \frac{y^2}{16\pi^2} f(\ln(M^2/Q^2))$ (in the SM diagonal)

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Mass basis

$$m_{ab}^\nu = m_a^{(0)} \delta_{ab} + \left(m_a^{(0)} + m_b^{(0)} \right) I_{ab}$$

$$I_{ab} = \sum_{AB} I_{AB} U_{Aa}^{(0)} U_{Bb}^{(0)}$$

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- assumption: degenerate tree-level masses,
 $|m_1^{(0)}| = |m_2^{(0)}| = |m_3^{(0)}|$

Trivial mixing @ tree-level

$$\mathbf{m}^\nu = m_0 \mathbb{1} + m_0 \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{pmatrix}$$

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Generate e.g. tri-bimaximal mixing:

- requirements for I_{ij} :

- 1 $\theta_{23} \approx \pi/4$

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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- $I_{12} = I_{13} \Leftrightarrow \theta_{13} = 0$

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- get $m_{1,2}$ in terms of I_{ij} , $m_3 = m_0$

Deviations

- $\theta_{13} \neq 0 \quad \hookrightarrow I_{13} \neq I_{12}$

- $\theta_{23} \lesssim \frac{\pi}{2}$

$$I_{33} = I_{22} + \varepsilon$$

$$I_{13} = I_{12} + \delta$$

$$\begin{pmatrix} m_0 & & \\ & \sqrt{m_0^2 + \Delta m_{21}^2} & \\ & & \sqrt{m_0^2 + \Delta m_{31}^2} \end{pmatrix}$$

$$= m \mathbf{U}(\theta_{12}, \theta_{13}, \theta_{23})^T \begin{pmatrix} 1 + I_{11} & I_{12} & I_{12} + \delta \\ I_{12} & 1 + I_{22} & I_{22} \\ I_{12} + \delta & I_{22} & 1 + I_{22} + \varepsilon \end{pmatrix} \mathbf{U}(\theta_{12}, \theta_{13}, \theta_{23})$$

Inputs (central values),

$$m_0 = 0.35 \text{ eV} \quad \text{UNIVERSITÄT WÜRZBURG}$$

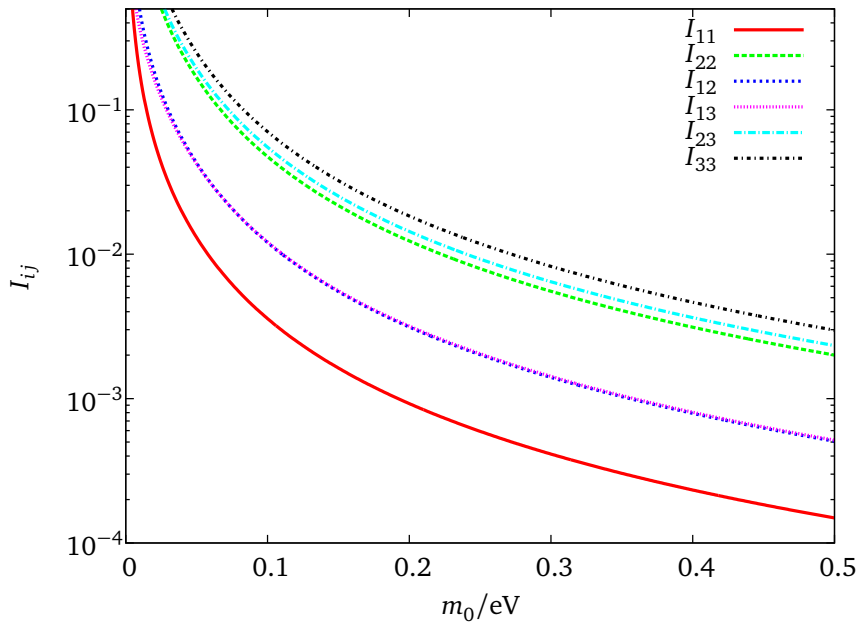
$$\theta_{12} \approx 31.8^\circ, \quad \theta_{13} \approx 8.5^\circ, \quad \theta_{23} \approx 39.2^\circ, \\ \Delta m_{21}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 \approx 2.458 \times 10^{-3} \text{ eV}^2$$

Outputs

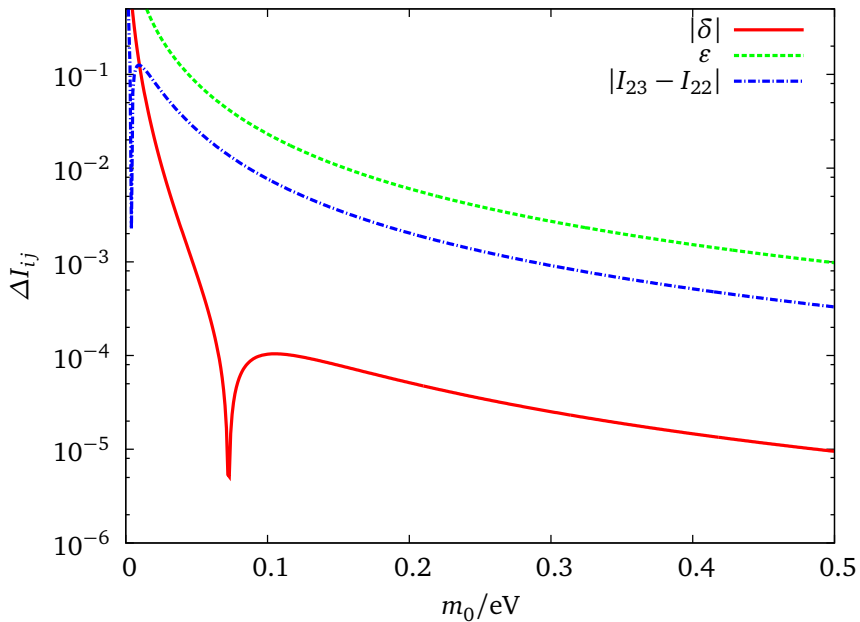
$$I_{11} \approx 3.00 \times 10^{-4}, \quad I_{22} \approx 4.01 \times 10^{-3}, \quad I_{12} \approx 1.02 \times 10^{-3}, \\ \delta \approx 1.56 \times 10^{-5}, \quad \varepsilon \approx 1.96 \times 10^{-2}$$

$$I = \begin{pmatrix} 0.30 & 1.02 & 1.03 \\ 1.02 & 4.01 & 4.67 \\ 1.03 & 4.67 & 5.97 \end{pmatrix} \times 10^{-3}$$

Dependency on neutrino mass



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MSSM with righthanded neutrinos

$$\mathcal{W} \supset \mu H_1 \cdot H_2 + Y_{ij}^\nu H_2 \cdot L_{L,i} N_{R,j} - Y_{ij}^\ell H_1 \cdot L_{L,i} E_{R,j} + \frac{1}{2} M_{ij}^R N_{R,i} N_{R,j}$$

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New soft SUSY breaking terms

$$V_{\text{soft}}^{\tilde{\nu}} = \left(m_{\tilde{L}}^2 \right)_{ij} \tilde{\nu}_{L,i}^* \tilde{\nu}_{L,j} + \left(m_{\tilde{R}}^2 \right)_{ij} \tilde{\nu}_{R,i} \tilde{\nu}_{R,j}^* \\ + \left(A_{ij}^\nu h_2^0 \tilde{\nu}_{L,i} \tilde{\nu}_{R,j}^* + (B^2)_{ij} \tilde{\nu}_{R,i}^* \tilde{\nu}_{R,j}^* + \text{h.c.} \right)$$

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- seesaw type I:

$$\mathbf{m}_\nu^{(0)} = -v_u^2 \mathbf{Y}_\nu^T \mathbf{M}_R^{-1} \mathbf{Y}_\nu + \mathcal{O}(v_u^4 / M_R^3)$$

- adding SUSY 1-loop

[Dedes, Haber, Rosiek 2007]

$$\left(\mathbf{m}_\nu^{1\text{-loop}} \right)_{ij} = (\mathbf{m}_\nu)_{ij} + \text{Re} \left[\Sigma_{ij}^{(\nu),S} + \frac{m_{\nu_i}}{2} \Sigma_{ij}^{(\nu),V} + \frac{m_{\nu_j}}{2} \Sigma_{ji}^{(\nu),V} \right]$$

Random scan

$$M_{\text{SUSY}} \in [500, 5000] \text{ GeV}$$

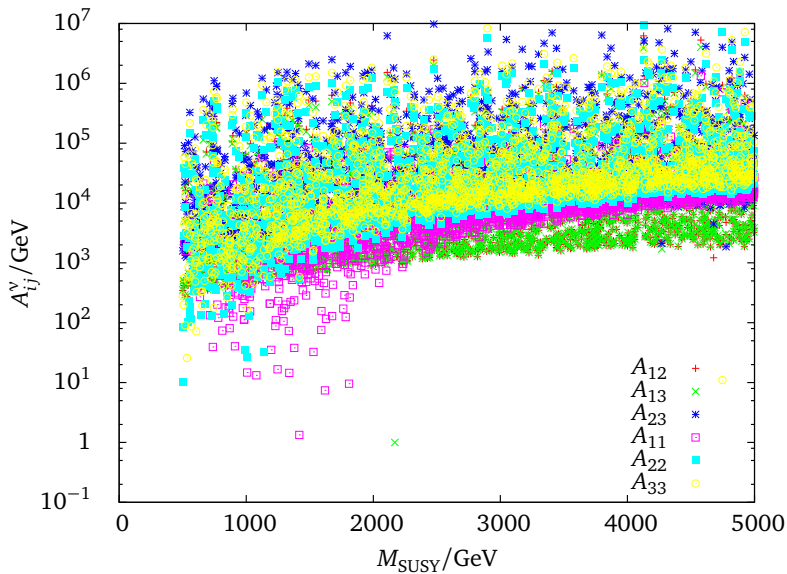
$$M_1 \in [0.3, 3] M_{\text{SUSY}}$$

$$M_2 \in [1, 5] M_{\text{SUSY}}$$

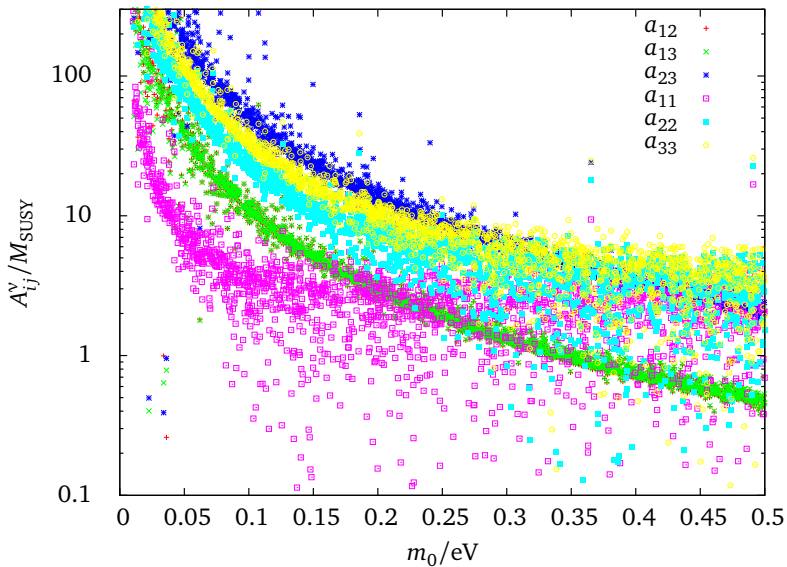
$$\mu \in [-15, 15] \text{ TeV}$$

$$\tan \beta \in [10, 60]$$

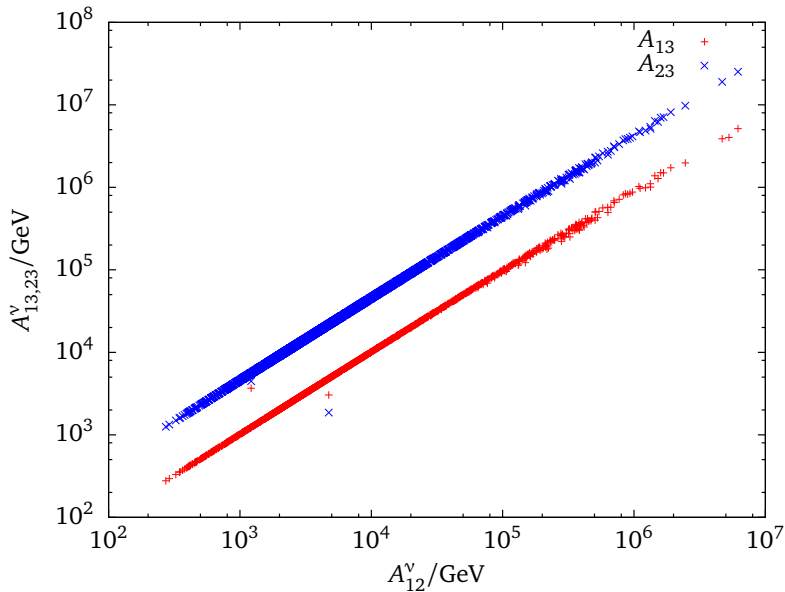
Constraints on A^ν



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- if tritium decay measures a neutrino mass: spectrum quasidegenerate
- exact degeneracy: $SO(3)$ or $SU(3)$ @ tree-level
- radiative breaking via threshold corrections
- threshold corrections have the power to completely determine the mixing and deviations from degenerate masses
- non-diagonal corrections needed: e.g. SUSY without MFV
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Backup

Slides

Renormalization Group Equation for ν masses and mixing

$$\frac{d}{dt} \mathbf{C} = -K\mathbf{C} - \kappa \left[\left(\mathbf{Y}_e^\dagger \mathbf{Y}_e \right)^T \mathbf{C} + \mathbf{C} \left(\mathbf{Y}_e^\dagger \mathbf{Y}_e \right) \right]$$

$$t = \frac{1}{16\pi^2} \ln \left(\frac{Q}{M_Z} \right)$$

SM: $\kappa = -\frac{3}{2}$ and $K = -3g_2^2 + 2 \text{Tr} \left(3\mathbf{Y}_u^\dagger \mathbf{Y}_u + 3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e \right) + 2\lambda$

MSSM: $\kappa = +1$ and $K = -6g_2^2 - 2g_Y^2 + 2 \text{Tr} \left(3\mathbf{Y}_u^\dagger \mathbf{Y}_u \right)$

Solving the RGE

$$\mathbf{C}(t) = I_K \mathcal{I} \mathbf{C}(0) \mathcal{I}, \quad \text{where } \mathcal{I} = \text{diag}(I_e, I_\mu, I_\tau) \text{ and}$$

$$I_K = \exp \left(- \int_0^t K(t') dt' \right), \quad I_{e_A} = \exp \left(-\kappa \int_0^t y_{e_A}^2(t') dt' \right).$$

- if $\mathbf{m}^{(0)} = m_0 \mathbb{1}$: $\mathbf{U}^{(0)T} \mathbf{m}^{(0)} \mathbf{U}^{(0)} = m_0 \mathbb{1}$ for any (real) $\mathbf{U}^{(0)}$
- if e.g. $\mathbf{m}^{(0)} = \text{diag}(1, -1, 1)$ this is not true
- in general: Majorana phases!
 - phase matrix $\mathbf{U}^{(0)} \rightarrow \mathbf{U}^{(0)} \mathbf{P}$ with $\mathbf{P} = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$
 - $\mathbf{m}^{(0)} \rightarrow \mathbf{P}^T \mathbf{U}^{(0)T} \mathbf{m}^{(0)} \mathbf{U}^{(0)} \mathbf{P} = m_0 \text{diag}(e^{2i\alpha_1} e^{2i\alpha_2}, 1)$
 - redefine masses
 - $m_1 = e^{2i\alpha_1} m_0$,
 - $m_2 = e^{2i\alpha_2} m_0$,
 - $m_3 = m_0$.
 - taking CP as good symmetry: $\alpha_{1,2} \in \{0, \pm \frac{\pi}{2}\}$
- choice: $m_1 = -m_2 = m_3$:

$$\mathbf{m}^\nu = m_0 \begin{pmatrix} 1 + 2U_{\alpha 1} U_{\beta 1} I_{\alpha \beta} & 0 & 2U_{\alpha 1} U_{\beta 3} I_{\alpha \beta} \\ 0 & -1 - 2U_{\alpha 2} U_{\beta 2} I_{\alpha \beta} & 0 \\ 2U_{\alpha 1} U_{\beta 3} I_{\alpha \beta} & 0 & 1 + 2U_{\alpha 3} U_{\beta 3} I_{\alpha \beta} \end{pmatrix}$$

update of [Chankowski, Pokorski 2002]

Brief review of [Chankowski, Pokorski 2002]

- degeneracy leaves freedom of rotation $U^{(0)} \rightarrow U^{(0)} R_{13}$

$$\sum_{\alpha\beta} U_{\alpha 1}^{(0)} U_{\beta 3}^{(0)} I_{\alpha\beta} = 0$$

- flavour diagonal corrections: $I_{\alpha\beta} = I_{\alpha} \delta_{\alpha\beta}$
- explain deviation from (tri-)bi-maximal mixing:

$$s_{13} = \sin \theta_{13} \approx 0$$

$$s_{13} = -\frac{s_{12}}{c_{12}} s_{23} c_{23} \frac{I_{\tau}}{I_e}, \quad \text{where } I_{\mu} = 0 \text{ and } I_e \gg I_{\tau}$$

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 $I_{\mu} \neq 0$

try to accommodate $s_{13} \approx 0.15$ and $\Delta m_{31}^2 / \Delta m_{21}^2 \approx 33$

$$s_{13} = c_{23} s_{23} \frac{s_{12}}{c_{12}} \frac{I_{\mu} - I_{\tau}}{I_e - s_{23}^2 I_{\mu} - c_{23}^2 I_{\tau}}$$

$$\Delta m_{ab}^2 = m^2 \left([1 + 2U_{\alpha a}^2 I_\alpha]^2 - [1 + 2U_{\alpha b}^2 I_\alpha]^2 \right)$$

- m^2 overall scale
- use relation for s_{13} to get correlation between I_e and I_μ, I_τ
- try to fit

$$\Delta m_{31}^2 / \Delta m_{21}^2 = \frac{([1 + 2U_{\alpha 3}^2 I_\alpha]^2 - [1 + 2U_{\alpha 1}^2 I_\alpha]^2)}{([1 + 2U_{\alpha 2}^2 I_\alpha]^2 - [1 + 2U_{\alpha 1}^2 I_\alpha]^2)}$$

the same follows from a special tree-level mass matrix

$$\begin{aligned}
 \mathbf{m}_{\text{tree}}^\nu &= x \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + z \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} x & y & y \\ y & x+z & z \\ y & z & x+z \end{pmatrix},
 \end{aligned}$$

which can be diagonalized by

$$\mathbf{U}_{\text{tree}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{with } s_{12} = \sin \theta_{12}, c_{12} = \cos \theta_{12}, \\
 \tan 2\theta_{12} = \sqrt{2} \frac{y}{z}$$

can be inverted

$$m_{\text{tree}}^\nu =$$

$$\begin{pmatrix} m_1 & \pm \frac{i}{\sqrt{2}} \sqrt{\frac{\Delta m_{31}^2}{m_1+m_3} \frac{\Delta m_{21}^2}{m_1+m_2}} & \pm \frac{i}{\sqrt{2}} \sqrt{\frac{\Delta m_{31}^2}{m_1+m_3} \frac{\Delta m_{21}^2}{m_1+m_2}} \\ \pm \frac{i}{\sqrt{2}} \sqrt{\frac{\Delta m_{31}^2}{m_1+m_3} \frac{\Delta m_{21}^2}{m_1+m_2}} & \frac{m_2+m_3}{2} & \frac{1}{2} (\sum_i m_i - 3m_1) \\ \pm \frac{i}{\sqrt{2}} \sqrt{\frac{\Delta m_{31}^2}{m_1+m_3} \frac{\Delta m_{21}^2}{m_1+m_2}} & \frac{1}{2} (\sum_i m_i - 3m_1) & \frac{m_2+m_3}{2} \end{pmatrix}$$

The philosophy behind threshold corrections

- exact degeneracy @ tree-level: trivial mass matrix
- $m^{(1)} = m^{(0)} + m^{(0)} I$, $I \sim \frac{1}{16\pi^2} \approx \frac{1}{100}$
- small perturbation